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Short Communication

A TECHNIQUE FOR INVESTIGATING NONLINEAR VIBRATIONS

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Abstract. In this short communication the main ideas of the method of equivalent linearization and dual conception are further extended to suggest a new technique for solving nonlinear differential equations. This technique allows improving the accuracy when the nonlinearity is strong and getting nonlinear features of responses. For illustration the Duffing oscillator is considered to demonstrate the effectiveness of the proposed technique.

Keywords: Equivalent linearzation method, dual conception, Duffing oscillator.

1. INTRODUCTION

Research on vibration phenomena in nonlinear systems has a long tradition. A great interest of researchers has been devoted to new methods for investigating nonlinear vibrations preferably applicable to wider classes of nonlinear systems including weak and strong nonlinearity, subject to deterministic and/or random excitations, see e.g. [1 - 5]. The method of equivalent linearization (MEL) is well known for analysis of nonlinear vibration phenomena and has been combined with the dual approach to give good approximate solutions for systems with larger nonlinearity [6, 7]. In this short communication the main ideas of MEL and the dual conception [6] are further extended to suggest a new technique for solving nonlinear differential equations. This technique allows improving the accuracy when the nonlinearity is strong and getting nonlinear features of responses. For illustration the Duffing oscillator is considered to demonstrate the effectiveness of the proposed technique.

2. COMBINATION OF MEL AND DUAL CONCEPTION

We consider the motion differential equation for a single degree of freedom (SDOF) system:

$$e(u) \equiv \ddot{u} + 2h\dot{u} + \omega_0^2 u + g(u, \dot{u}) - f(t) = 0 \tag{1}$$

where u is the displacement, $2h$ is damping coefficient, ω_o is natural frequency, $g(u, \dot{u})$ is a non-linear function, $f(t)$ is an excitation. According to the main idea of MEL for an approximate solution x we introduce linear terms in the left side of equation (1) as follows

$$e(x) \equiv \ddot{x} + 2h\dot{x} + \omega_0^2 x + b\dot{x} + \alpha x + \varepsilon g(x, \dot{x}) - b\dot{x} + \alpha x - f(t) \tag{2}$$

and let x be a solution of the linear equation

$$\ddot{x} + (2h + b)\dot{x} + (\omega_0^2 + \alpha)x - f(t) = 0 \tag{3}$$

Hence, we obtain the equation error

$$e(x) = g(x, \dot{x}) - b\dot{x} + \alpha x \tag{4}$$

The coefficients of linearization b, α are found by an optimal criterion. The most extensively used criterion is the mean - square error criterion which requires that the mean square of the error $e(x)$ be a minimum [1, 4],

$$< e^2(x) >=< (g(x, \dot{x}) - b\dot{x} - \alpha x)^2 > \to \min_{b,\alpha} \tag{5}$$

where $<\cdot>$ is the averaging operator in deterministic or random senses. Although in general the mean-square criterion (5) gives a good prediction, it was shown, however, by many authors, that in the case of major nonlinearity, the solution error may be unacceptable. In order to reduce the solution error we will use the dual conception [6] by representing the solution in the form

$$u = x + y \tag{6}$$

where x and y are so called basic and dual parts of solution, respectively. Substituting (6) into (1) and introducing new linear terms yield

$$\begin{aligned} e(x+y) &\equiv \ddot{x} + (2h + b + c)\dot{x} + (\omega_0^2 + \alpha + \beta)x - f(t) + [g(x, \dot{x}) - b\dot{x} - \alpha x] + \\ &+ \ddot{y} + (2h + p)\dot{y} + (\omega_0^2 + q)y + [g(x + y, \dot{x} + \dot{y}) - g(x, \dot{x}) - c\dot{x} - \beta x - p\dot{y} - qy] = 0 \end{aligned} \tag{7}$$

Let x and y be solutions of the following equations

$$\ddot{x} + (2h + b + c)\dot{x} + (\omega_0^2 + \alpha + \beta)x - f(t) = 0 \tag{8}$$

$$\ddot{y} + (2h + p)\dot{y} + (\omega_0^2 + q)y + [g(x, \dot{x}) - b\dot{x} - \alpha x] = 0 \tag{9}$$

we obtain then the equation error

$$e(x + y) = g(x + y, \dot{x} + \dot{y}) - g(x, \dot{x}) - c\dot{x} - \beta x - p\dot{y} - qy \tag{10}$$

According to the dual conception we require that the dual part is small is possible so one gets (5) and also the mean-square minimum of the equation error

$$< e^2(x + y) >=< (g(x + y, \dot{x} + \dot{y}) - g(x, \dot{x}) - c\dot{x} - \beta x - p\dot{y} - qy)^2 > \to \min_{c,\beta,p,q} \tag{11}$$

that gives

$$\begin{aligned} &\frac{\partial}{\partial c} < e^2(x + y) >= 0, \frac{\partial}{\partial \beta} < e^2(x + y) >= 0, \\ &\frac{\partial}{\partial p} < e^2(x + y) >= 0, \frac{\partial}{\partial q} < e^2(x + y) >= 0. \end{aligned} \tag{12}$$

Thus, we have 8 equations (5), (8), (9), (12) for 8 unknowns $x, y, b, c, \alpha, \beta, p$ and q. It is seen from (8), (9) that the basic part x is determined from the linear equation (8) while the dual part y from the nonlinear equation (9). This observation yields that the approximate solution (6) may keep nonlinear properties of the original nonlinear equation (1).

3. ILLUSTRATIVE EXAMPLE: DUFFING OSCILLATOR

For illustration of the effectiveness of the proposed technique we consider the Duffing oscillator

$$e(u) \equiv \ddot{u} + \omega_0^2 u + \gamma u^3 = 0 \tag{13}$$

Here we have $g(u, \dot{u}) = \gamma u^3$, $2h, b, c, p, f(t) = 0$ and the averaging operator $< \cdot >$ is taken as follows

$$< \cdot > = \frac{1}{2\pi} \int_0^{2\pi} (\cdot) d\varphi \tag{14}$$

From (8) one gets

$$x = a \cos \varphi, \quad \varphi = \Omega t + \theta \tag{15}$$

where

$$\Omega^2 = \omega_0^2 + \alpha + \beta \tag{16}$$

Eqs.(5) and (9) yield

$$\alpha = \frac{3}{4} \gamma a^2 \tag{17}$$

$$\ddot{y} + (\omega_0^2 + q) y + [\gamma a^3 \cos^3 \varphi - \frac{3}{4} \gamma a^3 \cos \varphi] = 0 \tag{18}$$

and then

$$y = ra \cos 3\varphi \tag{19}$$

where

$$r = \frac{\gamma a^2}{4(9\Omega^2 - \omega_0^2 - q)} \tag{20}$$

Substituting (15), (19) into (12) after some calculations one gets

$$\beta = \frac{3}{4} \gamma r a^4 + \frac{3}{2} \gamma r^2 a^6, \quad q = \frac{3}{2} \gamma a^2 + \frac{3}{4} \gamma r^2 a^6 \tag{21}$$

$$r = \frac{\gamma a^2}{32\omega_0^2 + 21\gamma a^2 + 27\gamma r a^2 + 51 \gamma r^2 a^2} \tag{22}$$

Thus, the approximate solution of Duffing oscillator (13) takes the form

$$u = a \cos \varphi + ra \cos 3\varphi, \quad \varphi = \Omega t + \theta \tag{23}$$

$$\Omega^2 = \omega_0^2 + \frac{3}{4} \gamma a^2 + \frac{3}{4} \gamma r a^2 + \frac{3}{2} \gamma r^2 a^2 \tag{24}$$

where r is determined from (22). In the case of small parameter $\gamma = \varepsilon$, one gets from (22)

$$r = \frac{\varepsilon a^2}{32\omega_0^2} + \varepsilon^2 ... \tag{25}$$

Substituting (24) into (23) yields

$$u = a \cos \varphi + \frac{\varepsilon}{32} a^3 \cos 3\varphi + \varepsilon^2 ..., \quad \varphi = \Omega t + \theta \tag{26}$$

$$\Omega^2 = \omega_0^2 + \frac{3}{4}\varepsilon a^2 + \frac{3}{128\omega_0^2}\varepsilon^2 a^4 + \varepsilon^3 ... \tag{27}$$

Comparison (26), (27) with the well - known solution [1, 2] yields that the proposed technique can give the exact expansion up to second order of small parameter.

4. CONCLUSION

In this short communication the main ideas of MEL and the dual conception are further extended to suggest a new technique for solving nonlinear differential equations. This technique allows improving the accuracy when the nonlinearity is strong and getting nonlinear features of responses. For illustration the Duffing oscillator is considered to demonstrate the effectiveness of the proposed technique.

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