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ON THE INVERSE KINEMATICS OF AN UNDERWATER VEHICLE - MANIPULATOR SYSTEM

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Abstract. The inverse kinematics plays an important role in the trajectory planning and the control of underwater vehicle-manipulator system. The solutions of this problem have an important influence on the motion quality of end-effectors. This paper presents an improved method based on the jacobian matrix and the error feedback. By using this method, the accuracy of the solution of inverse kinematics for the vehicle-manipulator system is improved. In addition, one of the advantages of a redundant system is exploited to avoid impact on joint limitations. Numerical simulations in software Matlab are carried out to verify the efficiencies of the proposed method.

Key words: Vehicle-manipulator system, inverse kinematics, numerical simulation.

1. INTRODUCTION

Nowadays, underwater remotely operated vehicles (ROV) equipped with manipulators have been applied in many areas, such as ocean research and monitoring, checking and maintaining underwater structure in offshore industries [15]. The usefullnes of an on - board manipulalaror in applications of underwater vehicle has made the vehicle-manipulator mechanism very popular in recent years and attracted several researchers [2, 12]. Normally, the vehicle - manipulator system is redundant, because it has a number degree of freedom (DOF) being relatively larger than the DOF of the end - effector. The number DOFs are the same as those of the manipulator. If the motion of the vehicle and the manipulator are controlled independently, so the advantage of the redundancy of the system is not be exploited.

The motion of the end - effector depends on those of the vehicle and the manipulator, or system configurations changed to time. In order to keep the end-effector along a desired trajectory, the problem of kinematics is required to solve with as high accuracy as possible. The forward kinematics has been solved effectively by several methods such as Denavit - Hartenberg parameters and homogeneous transformation matrix [1, 3, 8, 14]. And the results are analytical formulae that describes the relationship of position and orientation of the end - effector in depending on vehicle position and joint coordinates of manipulator.

On the contrary, there are not available general methods for solving the inverse kinematics of vehicle - manipulator. The solution of inverse kinematics in closed form can be obtained in some special cases. In other cases, the numerical methods are a useful tool.

Normally, this kind of method based on the jacobian matrix that gives the linear relationship between the velocities of the end - effector and the derivatives of joint coordinates respect to time [8, 9, 13]. The joint coordinates can be obtained by integrating its derivatives, which are the solution of linear equations. Simplicity is one of the advantages of this method. However, errors may appear during the integration process due to rounding and integral method. Such errors are accumulated and therefore the end - effector is not able to track the desired trajectory with a high accuracy.

This paper presents an improved method based on jacobian matrix and kinematic error feedback. By using this method, the accuracy of the solution of inverse kinematics for the vehicle - manipulator system is improved. Besides, one of the advantages of redundant system is exploited to avoid impact on joint limitations by using the nullspace technique. This paper is organized as follows: Section 2 presents a forward kinematics and method for inverse kinematics of a vehicle - manipulator system. Some numerical simulations are shown in Section 3. Finally, the conclusion is given in Section 4.

2. KINEMATICS OF THE VEHICLE - MANIPULATOR SYSTEM

2.1. Forward kinematics

Let's consider a vehicle with n_v degree of freedom (DOF) ($n_v \leq 6$) and a manipulator with n_m DOF mounted on the vehicle. The number DOF of the total system is $n = n_v + n_m$. Let's introduce two coordinates systems: a earth - fixed $(Oxyz)_0$ and a vehicle-fixed ones $(Oxyz)_R$ (Fig. 1). The position and orientation of the vehicle are given by the vector

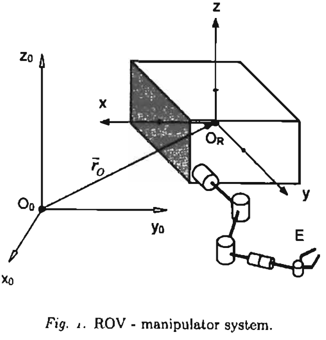


Fig. 1. ROV - manipulator system.

$\vec{r}_O = \overrightarrow{O_oO_R}$ and Euler angles or Roll-Pitch-Yaw angles. Let $\eta \in \mathrm{R}^{n_v}$ be the generalized coordinate vector of the vehicle. In case of spatial motion $n_v = 6$, we have

$$\eta = \left[\eta_1^T, \eta_2^T\right]^T = [x, y, z, \phi, \theta, \psi]^T,$$

with $\eta_1 = \mathbf{r}_O = [x, y, z]^T$ and the rotation matrix depending on the Roll-Pitch-Yaw ($\eta_2 = [\phi, \theta, \psi]^T$) is given as, [4]

$$\mathbf{A} = \begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\varphi + c\psi s\theta c\phi \\ c\theta s\psi & c\psi c\phi + s\psi s\theta s\phi & -c\psi s\varphi + s\psi s\theta c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} =: \mathbf{A}(\eta_2) \tag{1}$$

If the vehicle moves in the horizontal plane ($n_v = 3$), so $\eta = \left[\eta_1^T, \eta_2^T\right]^T = [x, y, \psi]^T$ and the rotation matrix becomes

$$\mathbf{A} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} =: \mathbf{A}(\eta_2) \tag{2}$$

Let $\mathbf{q}_m \in \mathrm{R}^{n_m}$ be the vector containing of joint variables of the vehicle. The position and posture of the end-effector respect to the vehicle can be determined by methods like ones of Denavit-Hartenberg, Denavit - Hartenberg - Craig. The results of the forward kinematics of the manipulator respect to the vehicle are given as follows

$$\mathbf{r}_E^{(V)} = \mathbf{r}_E^{(V)}(\mathbf{q}_m), \quad \mathbf{A}_E^{(V)} = \mathbf{A}_E^{(V)}(\mathbf{q}_m) \tag{3}$$

So the position and orientation of the end-effector respect to the fixed frame are given as

$$\mathbf{r}_E^{(0)}(\eta, \mathbf{q}_m) = \eta_1 + \mathbf{A}(\eta_2)\mathbf{r}_E^{(V)}(\mathbf{q}_m), \quad \mathbf{A}_E^{(0)}(\eta_2, \mathbf{q}_m) = \mathbf{A}(\eta_2)\mathbf{A}_E^{(V)}(\mathbf{q}_m). \tag{4}$$

Here $\mathbf{r}_E^{(V)}$ and $\mathbf{A}_E^{(V)}$ denote the position and rotation matrix of the end - effector relative to the vehicle, respectively. The vector $\mathbf{r}_E^{(0)}$ and matrix $\mathbf{A}_E^{(0)}$ denote the position and rotation matrix of the end-effector respect to the fixed coordinate systems.

Putting $\mathbf{q} = \left[\eta^T, \mathbf{q}_m^T\right]^T$ be a vector of generalized coordinates of the system including vehicle and manipulator, the formulae (4) can be rewritten as

$$\mathbf{r}_E^{(0)} = \mathbf{r}_E^{(0)}(\mathbf{q}) = \eta_1 + \mathbf{A}(\eta_2)\mathbf{r}_E^{(V)}(\mathbf{q}_m), \quad \mathbf{A}_E^{(0)} = \mathbf{A}_E^{(0)}(\mathbf{q}) = \mathbf{A}(\eta_2)\mathbf{A}_E^{(V)}(\mathbf{q}_m) \tag{5}$$

or in the compact form

$$\mathbf{x} = \mathbf{f}(\mathbf{q}), \quad \mathbf{x}, \mathbf{f} \in \mathrm{R}^m, \mathbf{q} \in \mathrm{R}^n \tag{6}$$

where $\mathbf{x} = [x_E, y_E, z_E, \varphi_E, \theta_E, \phi_E]^T$ is a vector containing position and orientation of the end - effector in the fixed frame and $m \leq 6$ is the number DOF of the end - effector. The equation (6) is nonlinear algebraic equations with the number of unknowns larger than the number of equations, $m > n$.

2.2. Inverse kinematics

In the problem of inverse kinematics, the vector of generalized coordinates $\mathbf{q} \in R^n$ need to be determined so that the end-effector tracks a desired trajectory given by $\mathbf{x}_d = \mathbf{x}_d(t)$. It is very difficult to get an analytical solution of the equation (6) even in the case of planar motion of the system. In the following, the method based on jacobian matrix is presented to solve this problem. Differentiating (6) with respect to time, one obtains

$$\dot{\mathbf{x}} = \frac{\partial \mathbf{f}}{\partial \mathbf{q}}\dot{\mathbf{q}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}, \tag{7}$$

where

$$\mathbf{J}(\mathbf{q}) = \frac{\partial \mathbf{f}}{\partial \mathbf{q}} = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \dots & \frac{\partial f_1}{\partial q_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial f_m}{\partial q_1} & \frac{\partial f_m}{\partial q_2} & \dots & \frac{\partial f_m}{\partial q_n} \end{bmatrix}. \tag{8}$$

Assuming that the jacobian $m \times n$ - matrix $\mathbf{J}(\mathbf{q})$ has a rank of m. If $\dot{\mathbf{x}}$ and $\mathbf{q}$ are known, equation (7) is a set of m linear algebraic equations with n unknown, which is a vector of generalized velocity $\dot{\mathbf{q}}$. In the infinite set of solutions satisfying (7), we could find a solution with a minimal magnitude. In order to find this solution, a quadratic cost functional of joint velocities is introduced

$$C = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{W}\dot{\mathbf{q}}, \tag{9}$$

where $\mathbf{W}$ is a symmetric positive definite weighting matrix with suitable size, $n \times n$. This problem can be solved with the method of Lagrangian multipliers. Considering the modified cost functional

$$L(\dot{\mathbf{q}}, \lambda) = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{W}\dot{\mathbf{q}} + \lambda^T[\dot{\mathbf{x}} - \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}], \tag{10}$$

differentiating $\mathcal{L}$ with respect to $\dot{\mathbf{q}}$ one yields

$$\frac{\partial}{\partial \dot{\mathbf{q}}}\mathcal{L}(\dot{\mathbf{q}}, \lambda) = \mathbf{W}\dot{\mathbf{q}} - \mathbf{J}^T(\mathbf{q})\lambda = 0$$

Solving for $\dot{\mathbf{q}}$ one yields

$$\dot{\mathbf{q}} = \mathbf{W}^{-1}\mathbf{J}^T(\mathbf{q})\lambda \tag{11}$$

which, substituted into (7), gives the sought optimal solution

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{J}(\mathbf{q})\mathbf{W}^{-1}\mathbf{J}(\mathbf{q})^T\lambda. \tag{12}$$

With the assumption that $\mathbf{J}(\mathbf{q})\mathbf{W}^{-1}\mathbf{J}(\mathbf{q})^T$ is non - singular, solving for λ yields

$$\lambda = [\mathbf{J}(\mathbf{q})\mathbf{W}^{-1}\mathbf{J}(\mathbf{q})^T]^{-1}\dot{\mathbf{x}}. \tag{13}$$

So we get an optimal solution for $\dot{\mathbf{q}}$ as

$$\begin{aligned} \dot{\mathbf{q}} &= \mathbf{W}^{-1}\mathbf{J}^T(\mathbf{q})[\mathbf{J}(\mathbf{q})\mathbf{W}^{-1}\mathbf{J}^T(\mathbf{q})]^{-1}\dot{\mathbf{x}} \\ &\doteq \mathbf{J}(\mathbf{q})^+_W\dot{\mathbf{x}}. \end{aligned} \tag{14}$$

The matrix

$$\mathbf{J}(\mathbf{q})_W^+ = \mathbf{W}^{-1}\mathbf{J}^T(\mathbf{q})[\mathbf{J}(\mathbf{q})\mathbf{W}^{-1}\mathbf{J}^T(\mathbf{q})]^{-1} \tag{15}$$

is called as the weighting pseudo - inverse of $\mathbf{J}(\mathbf{q})$ [6, 11]. In particular cases, if the weighting matrix $\mathbf{W}$ is chosen to be the unit matrix, one obtains

$$\mathbf{J}(\mathbf{q})^+ = \mathbf{J}^T(\mathbf{q})[\mathbf{J}(\mathbf{q})\mathbf{J}^T(\mathbf{q})]^{-1} \tag{16}$$

Matrix $\mathbf{J}^+(\mathbf{q})$ is pseudo - inverse of $\mathbf{J}(\mathbf{q})$ and

$$\dot{\mathbf{q}} = \mathbf{J}^+(\mathbf{q})\dot{\mathbf{x}}. \tag{17}$$

Integrating $\dot{\mathbf{q}}$ one gets

$$\mathbf{q}(t) = \mathbf{q}(0) + \int_0^t \dot{\mathbf{q}}(\tau)d\tau \tag{18}$$

Considering the nullspace of the jacobian matrix $\mathbf{J}(\mathbf{q})$, the solution of linear equation (7) can be written in the following form:

$$\dot{\mathbf{q}} = \mathbf{J}_W^+(\mathbf{q})\dot{\mathbf{x}} + [\mathbf{E} - \mathbf{J}_W^+(\mathbf{q})\mathbf{J}(\mathbf{q})]\mathbf{z}_0 \tag{19}$$

where $\mathbf{z}_0 \in \mathrm{R}^n$ is an arbitrary vector and $\mathbf{E}$ is a unit matrix with size of $n \times n$.

The advantages of the redundancy such as avoiding obtacles, singularities in configuration, impact with joint limitation are exploited by choosing vector $\mathbf{z}_0$. In this paper, the vector $\mathbf{z}_0$ is chosen as

$$\mathbf{z}_0 = -\alpha\frac{\partial S(\mathbf{q})}{\partial \mathbf{q}} \tag{20}$$

with constant α and

$$S(\mathbf{q}) = \frac{1}{2}\sum_{i=1}^{n} c_i\left(\frac{q_i - \bar{q}_i}{q_{iM} - q_{im}}\right)^2 \tag{21}$$

where q_{iM}, q_{im} and $\bar{q}_i$ are maximal, minimal and average values of joint variables respectively; $c_i \geq 0$ are weighting coefficients.

The values of $\dot{\mathbf{q}}$ getting from (14) satisfy only equation (7), but the values $\mathbf{q}(t)$ getting from (18) after integrating may not satistfy (6) due to accumulated error. In order to overcome this problem, let's introduce a dynamic equation of error

$$\dot{\mathbf{e}} = -\mathbf{K}\mathbf{e}, \tag{22}$$

with

$$\mathbf{e} = \mathbf{x} - \mathbf{f}(\mathbf{q}), \tag{23}$$

and $\mathbf{K}$ is a positive definite matrix with the size of $m \times m$. In case choosing $\mathbf{K}$ be a diagonal matrix, $k_{ii} = \lambda_i > 0$, so the solution of equation (22) has a form as following

$$e_i(t) = e_i(0)e^{-\lambda_i t}, i = 1, 2, .., m. \tag{24}$$

Formula (24) shows that $e_i(0) = 0 \Rightarrow e_i(t) = 0$, otherwhile $e_i(0) \neq 0 \Rightarrow e_i(t) \to 0$ when t is large enough.

Differetiating (23) respect to time one gets

$$\dot{\mathbf{e}} = \dot{\mathbf{x}} - \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} \tag{25}$$

Combining (25) and (22) we have

$$\dot{\mathbf{e}} = \dot{\mathbf{x}} - \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = -\mathbf{K}\mathbf{e}$$

so

$$\dot{\mathbf{q}} = \mathbf{J}^+_W(\mathbf{q})\,[\dot{\mathbf{x}} + \mathbf{K}\mathbf{e}] = \mathbf{J}^+_W(\mathbf{q})\,[\dot{\mathbf{x}} + \mathbf{K}(\mathbf{x} - \mathbf{f}(\mathbf{q}))]\,. \tag{26}$$

If the nullspace of jacobian matrix $\mathbf{J}(\mathbf{q})$ is considered, equation (26) can be rewritten as

$$\dot{\mathbf{q}} = \mathbf{J}^+_W(\mathbf{q})\,[\dot{\mathbf{x}} + \mathbf{K}(\mathbf{x} - \mathbf{f}(\mathbf{q}))] + [\mathbf{E} - \mathbf{J}^+_W(\mathbf{q})\mathbf{J}(\mathbf{q})]\mathbf{z}_0 \tag{27}$$

Based on equations (27) and (18), the inverse kinematics of the vehicle - manipulator system can be presented as a block diagram shown in Fig. 2.

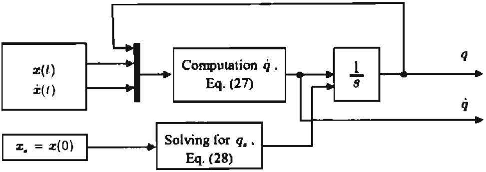


Fig. 2. Block diagram of the algorithm.

2.3. Determination of q(0) from the starting point x(0)

The initial configuration $\mathbf{q}_o$ coresponding to the depature position of the end - effector at the beginning time $t_o = 0$, $\mathbf{x}_o = \mathbf{x}(t_o)$, is one of solutions of the nonlinear algebraic equation

$$\mathbf{x}_o - \mathbf{f}(\mathbf{q}_o) = 0 \tag{28}$$

Because (28) has a number of equations that is smaller than the number of unknowns, so it has several solutions. So, to get the best solution some additional conditions need to be considered:

- Keeping the joint variables of the manipulator in their limitations

$$q_m^{(i)} \le q^{(i)} \le q_M^{(i)},\ i = 1, 2, \ldots \tag{29}$$

- Minimization of the square sum of distances to the middle position of joint variables

$$S(\mathbf{q}_o) = \frac{1}{2}\sum_{i=1}^{3} c_i \left(\frac{q_{oi} - \bar{q}_i}{q_{iM} - q_{im}}\right)^2 \to \min \tag{30}$$

where q_{iM}, q_{im} and $\bar{q}_i$ are maximal, minimal and average values of joint variables respectively; $c_i > 0$ are weighting coefficients.

The optimal problem (30) with constraints (28) and (29) can be solved by several methods presented in [5, 10].

3. NUMERICAL SIMULATION

In this section, some simulations in universal software Matlab is implemented to illustrate the presented algorithm. The manipulator with 3 DOF is mounted on the vehicle moving in a horizontal plane (Fig. 3). The number degree of freedom of all system is 6 ($n = 6$) while the number DOF of the end-effector is $m = 3$. The vehicle has a length of $2b$ and a width of $2a$, the position of joint A on the vehicle frame is $\mathbf{r}_A^{(V)} = [a, -b, 0]^T$. The length of three links of the manipulator are l_1, l_2, l_3; respectively. Some parameters of the system are given in Table 1.

Table 1. Parameters of the ROV-manipulator system

ROV		
width	a[m]	0.4
length	b[m]	0.6
Manipulator		
Link i [m]	1	0.4
	2	0.4
	3	0.3

The joint varibales of the manipulator have the limitations with:

+ Joint 1: $q_M = \frac{3}{2}\pi, q_m = 0, \bar{q} = \frac{3}{4}\pi$.

+ Joint 2 and 3: $q_M = \frac{3}{2}\pi, q_m = -\frac{3}{2}\pi, \bar{q} = 0$.

From the forward kinematics we get the position and orientation of the end - effector respect to the fixed frame Oxy as follows

$$x_E = x_O + a\cos\psi + b\sin\psi + l_1\sin(\psi + q_1) + l_2\sin(\psi + q_1 + q_2) + l_3\sin(\psi + q_1 + q_2 + q_3)$$
$$y_E = y_O + a\sin\psi - b\cos\psi - l_1\cos(\psi + q_1) - l_2\cos(\psi + q_1 + q_2) - l_3\cos(\psi + q_1 + q_2 + q_3)$$
$$\phi_E = (\psi + q_1 + q_2 + q_3) - \frac{1}{2}\pi$$

or in the compact form as

$$\mathbf{x} = \mathbf{f}(\mathbf{q}), \mathbf{x} = [x_E, y_E, \phi_E]^T, \mathbf{q} = [x_O, y_O, \psi, q_1, q_2, q_3]^T$$

where $\mathbf{x} = [x_E, y_E, \phi_E]^T$ is a vector containing position (x_E, y_E) and orientation ϕ_E of the end-effector; $\mathbf{q} = [x_O, y_O, \psi, q_1, q_2, q_3]^T$ is a vector containing position of the vehicle and joint angles of the manipulator.

In this simulation, the end - effector will be forced to move at the velocity of 0.5 m/s along a circular trajectory with a radius of 4 m, center at (0, 2), departure position at (0, -2), while its orientation is constant, $\phi = 1.0$ rad. Total time of the motion is 190 seconds, acceleration and deacceleration time is 5 seconds. The velocity profile respect to time and the desired motion of the end-effector $\mathbf{x}_d = [x_d(t), y_d(t), \phi_d(t)]^T$ are shown in Figs. 4a. and 4b.

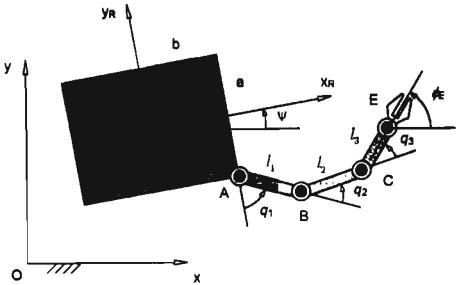
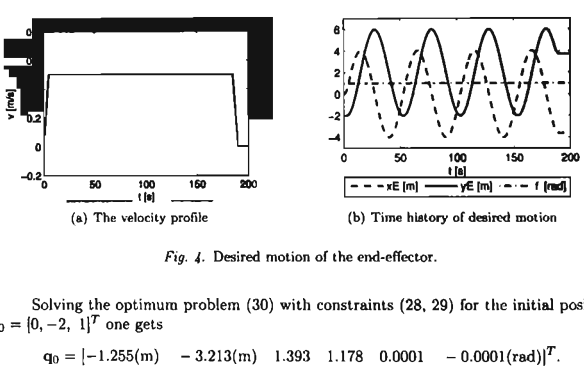


Fig. 3. Planar ROV-manipulator system.



(a) The velocity profile

(b) Time history of desired motion

Fig. 4. Desired motion of the end-effector.

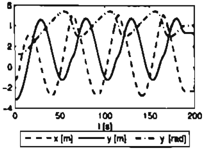
Solving the optimum problem (30) with constraints (28, 29) for the initial position $\mathrm{x}_0 = [0, -2, \ 1]^T$ one gets

$$\mathrm{q}_0 = [-1.255(\mathrm{m}) \quad -3.213(\mathrm{m}) \quad 1.393 \quad 1.178 \quad 0.0001 \quad -0.0001(\mathrm{rad})]^T.$$

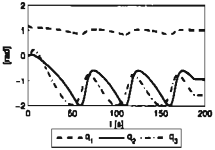
In the simulation some parameters are choosen as

$$\mathbf{K} = \mathrm{diag}([1 \ 1 \ 1]); \ c_1 = c_2 = c_3 \ = \ 1; \ \alpha = 10.$$

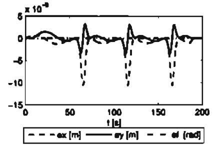
In order to show the role of the weighting matrix $\boldsymbol{W}$, simulations are implemeted with different values of $\boldsymbol{W}$. The simulation results are given in the form of time history of generalized coordinates as presented in Figs. 5, 6, 7, and 8.



(a) Position, Orientation of the Vehicle

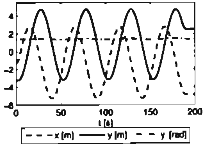


(b) Joint vairiable of the manipulator

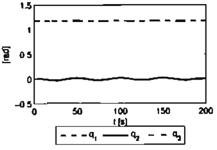


(c) Error $\mathbf{e} = \mathbf{x}(t) - \mathbf{x}_d(t)$

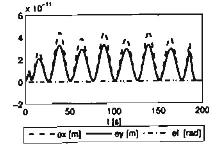
Fig. 5. Simulation results with W = diag([1, 1, 1, 1, 1, 1]).



(a) Position, Orientation of the Vehicle

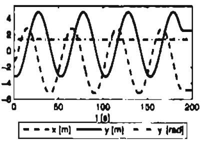


(b) Joint vairiable of the manipulator

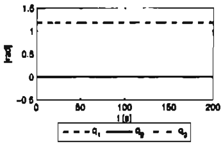


(c) Error $\mathbf{e} = \mathbf{x}(t) - \mathbf{x}_d(t)$

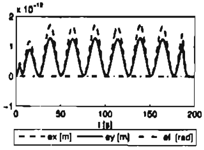
Fig. 6. Simulation results with W = diag([1, 1, 1, 100 ,100, 100]).



(a) Position, Orientation of the Vehicle

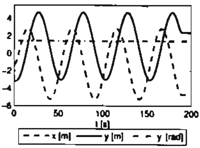


(b) Joint vairiable of the manipulator

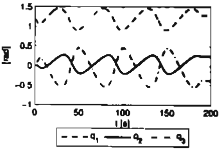


(c) Error $e = x(t) - x_d(t)$

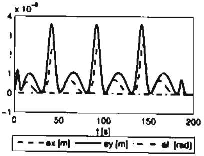
Fig. 7. Simulation results with W = diag([1, 1, 1, 500, 500, 500]).



(a) Position, Orientation of the Vehicle



(b) Joint vairiable of the manipulator



(c) Error $e = x(t) - x_d(t)$

Fig. 8. Simulation results with W = diag([1, 1, 500, 1, 1, 1]).

The simulation results show that the joint variables of the manipulator stay within their limitations and they change periodically corresponding to the motion of the end - effector. The position errors are relative small, about 10^{-8}. The weighting matrix $\mathbf{W}$ affects on the motion of the vehicle and manipulator. The motion of manipulator is small if their weigting values are large. Fig. 7 shows that manipulator stays almost at rest relative to the vehicle. The orientation of the vehicle is nearly constant, (Fig. 8). So the motion of the vehicle and the manipulator can be coordinated by changing the elements of the weighting matrix.

4. CONSLUSION

This paper presents an algorithm based on jacobian matrix to solve the inverse kinematics of a system of vehicle and manipulators. The position error has been used as a feedback signal in order to guarantee the convergence of the solution. In addition, the nullspace of the jacobian matrix is also exploited to avoid the impact with the limitations of joint variable of the manipulator. The effectiveness of the proposed method is demonstrated by means of numerical experiments with the vehicle - manipulator system moving in the horizontal plane.

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