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NONLINEAR ANALYSIS OF BUCKLING AND POSTBUCKLING FOR AXIALLY COMPRESSED FUNCTIONALLY GRADED CYLINDRICAL PANELS WITH THE POISSON'S RATIO VARYING SMOOTHLY ALONG THE THICKNESS

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Abstract. In this paper an approximate analytical solution to analyze the nonlinear buckling and postbuckling behavior of imperfect functionally graded panels with the Poisson's ratio also varying smoothly along the thickness is investigated. Based on the classical shell theory and von Karman's assumption of kinematic nonlinearity and applying Galerkin procedure, the equations for finding critical loads and load-deflection curves of cylindrical panel subjected to axial compressive load with two types boundary conditions, are given. Especially, the stiffness coefficients are analyzed in explicit form. Numerical results show various effects of the inhomogeneous parameter, dimensional parameter, boundary conditions on nonlinear stability of panel. An accuracy of present theoretical results is verified by the previous well-known results.

Keywords: Nonlinear, Postbuckling, Functionally graded materials, Cylindrical panel.

1. INTRODUCTION

Functionally graded structures such as cylindrical panels and cylindrical shells in recent years play the important part in the modern industries [1]. They are lightweight structures and are able to withstand high-temperature environments while maintaining their structural integrity. Therefore, researches on stability problems of functionally graded materials (FGMs) structures have received considerable attention. Some investigations on postbuckling of FGM cylindrical panels and cylindrical shells subjected to axial loading or pressure loading in thermal environments are presented by Shen and Noda [2, 3]. They employed singular perturbation techniques to determine the buckling loads and postbuckling equilibrium paths. Chang and Librescu [4] reported postbuckling of shear deformable flat and curved panels under combined loading conditions. The problem on structural stability of functionally graded panels subjected to aero-thermal loads is considered by Sohn and Kim [5]. The studies involving postbuckling of laminated cylindrical panels loaded by improved arc-length method can be found in the paper of Kweon and Hong [6]. Wilde et al. [7] presented investigation on critical state of an axially compressed cylindrical panel with

three edges simply supported and one edge free. Huang and Taucher [8] solved the problem on large deflection of laminated cylindrical and doubly-curved panels under thermal loading. By different methods, the authors Dennis et al. [9], Yamada and Croll [10], investigated instability, buckling behavior of pressure loaded cylindrical panels. Kabir and Chaudhur [11] presented a direct Fourier approach for the analysis of thin finite-dimensional cylindrical panels. Thermomechanical postbuckling of FGM cylindrical panels with temperature dependent properties is investigated by Yang et al. [12]. Geometrically nonlinear analysis of functionally graded shells are considered by Zhao and Liew [13]. Alijani and Aghdam [14], by applying the extended Kantorovich method, given a semi-analytical solution for stress analysis of moderately thick laminated cylindrical panels with various boundary conditions.

In the field of dynamic buckling of FGM structures, Sofiyev [15], Huang and Han [16], Ng et al. [17], Bich and Long [18], Dung and Nam [19] presented nonlinear dynamic buckling and postbuckling analysis of FGM shallow and cylindrical shells subjected to various loadings.

However, analytical investigations on nonlinear analysis of FGM cylindrical shells and panels under mechanical or thermal loading are limited in number, so it is necessary to be more accelerated in this area. Recently, the results on the nonlinear analysis of stability for functionally graded cylindrical panels under axial compression have been obtained by Duc and Tung [20]. They presented an analytical approach to obtain explicit expressions of buckling load and postbuckling load-deflection curves in the case Poisson's ratio ν being constant and boundary conditions being simply supported.

When Poisson's ratio ν depends on thickness z, there exists some investigations of Huang and Han [21, 22, 23]. These authors touched upon the problem on nonlinear buckling and postbuckling of imperfect functionally graded closed circular cylindrical shells subjected to different mechanical and thermal loadings with $\nu = \nu(z)$ in the power law of z, but the stiffness coefficients A_{ij} being still defined in the integrating form, not yet analyzed. Therefore, an aim of this present research is to extend the results of [20] considering Young's modulus $E = E(z)$ and Poisson's ratio $\nu = \nu(z)$, simultaneously for calculating and giving the stiffness coefficients A_{ij} of [21, 22, 23] in explicit form. Based on the classical shell theory and geometrical nonlinearity in von Karman sense, the approximate analytical solutions have been presented. The resulting equations are solved by Galerkin's procedure to obtain equations for finding critical loads and postbuckling load-deflection curves with two types of boundary conditions. In the case ν is a constant, the reached results return to ones of [20].

2. FUNCTIONALLY GRADED CYLINDRICAL PANELS AND FUNDAMENTAL RELATIONS

2.1. Functionally graded cylindrical panels

Let us consider a FGM cylindrical panel with uniform thickness h, mean radius R and length of straight edge a, of curved edge b. We choose a cylindrical coordinate $(x, y = R\theta, z)$ so that the axes x, y are in the longitudinal, circumferential directions

respectively and axe z is perpendicular to the middle surface and in the inward thickness direction ($-h/2 \leq z \leq h/2$) (see Fig. 1). The cylindrical panel is subjected to the uniform

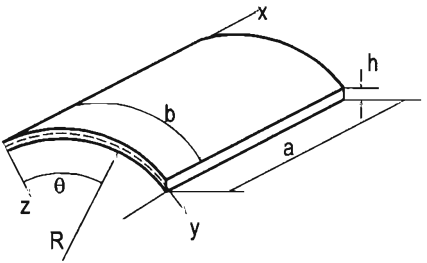


Fig. 1. FGM cylindrical panels

plane compressive loads of intensities r_0 on $x = 0, x = a$ and p_0 on $y = 0, y = b$ and an uniform radial load of intensity Q_0.

Assume that material properties vary through the thickness z with the power law as follows [21]

$$E = E(z) = E_m + (E_c - E_m)\left(\frac{2z+h}{2h}\right)^k \equiv E_m + E_{cm}r^k \tag{1}$$

$$\nu = \nu(z) = \nu_m + (\nu_c - \nu_m)\left(\frac{2z+h}{2h}\right)^{k_1} \equiv \nu_m + \nu_{cm}r^{k_1} \tag{2}$$

in which

$$E_{cm} = E_c - E_m, \quad r = \frac{2z+h}{2h}, \quad \nu_{cm} = \nu_c - \nu_m, \quad k \geq 0, k_1 \geq 0. \tag{3}$$

The quantities E_m, E_c and ν_m, ν_c are Young's moduli and Poisson's ratios of metal (m) and ceramic (c), respectively.

2.2. Governing relations and equations

The strain components on the middle surface of imperfect cylindrical panel based upon the von Karman's theory are [21]

$$\begin{aligned}
\varepsilon_x^0 &= u_{,x} + \frac{1}{2}w_{,x}^2 + w_{,x}w_{,x}^*, \\
\varepsilon_y^0 &= v_{,y} - \frac{w}{R} + \frac{1}{2}w_{,y}^2 + w_{,y}w_{,y}^*, \\
\gamma_{xy}^0 &= u_{,y} + v_{,x} + w_{,x}w_{,y} + w_{,x}w_{,y}^* + w_{,y}w_{,x}^*.
\end{aligned} \tag{4}$$

where $u = u(x,y), v = v(x,y), w = w(x,y)$ are the displacements along x, y and z axes respectively. The quantity $w^* = w^*(x,y)$ is an initial imperfection of panel and assumed smaller than thickness of panel.

The strain components across the panel thickness at a distance z from the mid-plane are in the form

$$\begin{aligned} &\varepsilon_x = \varepsilon_x^0 + zk_x, \quad \varepsilon_y = \varepsilon_y^0 + zk_y, \quad \gamma_{xy} = \gamma_{xy}^0 + 2zk_{xy}, \\ &k_x = -w_{,xx}, \quad k_y = -w_{,yy}, \quad k_{xy} = -w_{,xy}. \end{aligned} \tag{5}$$

Note that the subscript $A_{,i}$ in this paper indicates the partial derivative of A for i.

The stress-strain relationships of cylindrical panel are defined by Hookian law, as

$$(\sigma_x, \sigma_y) = \frac{E}{1-\nu^2}\left[(\varepsilon_x, \varepsilon_y) + \nu(\varepsilon_y, \varepsilon_x)\right], \quad \sigma_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}. \tag{6}$$

The force and moment resultants are expressed by

$$\{(N_x, N_y, N_{xy}), (M_x, M_y, M_{xy})\} = \int_{-h/2}^{h/2} \{\sigma_x, \sigma_y, \sigma_{xy}\}(1, z)\, dz. \tag{7}$$

Substituting Eqs (4)÷(6) into (7), we get

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{10} & A_{20} & 0 & A_{11} & A_{21} & 0 \\ A_{20} & A_{10} & 0 & A_{21} & A_{11} & 0 \\ 0 & 0 & A_{30} & 0 & 0 & A_{31} \\ A_{11} & A_{21} & 0 & A_{12} & A_{22} & 0 \\ A_{21} & A_{11} & 0 & A_{22} & A_{12} & 0 \\ 0 & 0 & A_{31} & 0 & 0 & A_{32} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x \\ k_y \\ 2k_{xy} \end{bmatrix} \tag{8}$$

where the stiffness coefficients A_{ij} $(i = 1,2,3; j = 0,1,2)$ are defined by the formulae

$$\begin{aligned} A_{1j} &= \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu^2(z)} z^j dz, \quad A_{2j} = \int_{-h/2}^{h/2} \frac{E(z)\nu(z)}{1-\nu^2(z)} z^j dz, \\ A_{3j} &= \int_{-h/2}^{h/2} \frac{E(z)}{2|1+\nu(z)|} z^j dz = \frac{1}{2}(A_{1j} - A_{2j}). \end{aligned} \tag{9}$$

The explicit analytical expressions of A_{ij} are calculated and given in the Appendix. The equilibrium equations of imperfect cylindrical panel are derived from [21]

$$N_{x,x} + N_{xy,y} = 0, \tag{10}$$

$$N_{xy,x} + N_{y,y} = 0, \tag{11}$$

$$\begin{aligned} M_{x,xx} + 2M_{xy,xy} + M_{y,yy} + \frac{N_y}{R} + N_x\left(w_{,xx} + w_{,xx}^*\right) + \\ + 2N_{xy}\left(w_{,xy} + w_{,xy}^*\right) + N_y\left(w_{,yy} + w_{,yy}^*\right) + Q_0 = 0. \end{aligned} \tag{12}$$

The geometrical compatibility equation deduced from (4) and (5), assuming $w^*_{,ij}$ small so the quadratic terms of $w^*_{,ij}$ may be omitted, becomes

$$\varepsilon^0_{x,yy} + \varepsilon^0_{y,xx} - \gamma^0_{xy,xy} = -\frac{1}{R}w_{,xx} + w^2_{,xy} - w_{,xx}.w_{,yy} + 2w_{,xy}w^*_{,xy} - \\ - w_{,xx}.w^*_{,yy} - w_{,yy}w^*_{,xx} \tag{13}$$

Introducing Airy's stress function $f = f(x, y)$ so that

$$N_x = f_{,yy}, \quad N_y = f_{,xx}, \quad N_{xy} = -f_{,xy}. \tag{14}$$

It is easy seen that two equations (10), (11) are automatically satisfied.

Substituting these functions (14) into N_{ij} of relations (8) and solving conversely, we obtain

$$\begin{aligned} \varepsilon^0_x &= J_0\left(A_{10}f_{,yy} - A_{20}f_{,xx} + J_1 w_{,xx} + J_2 w_{,yy}\right) \\ \varepsilon^0_y &= J_0\left(A_{10}f_{,xx} - A_{20}f_{,yy} + J_1 w_{,yy} + J_2 w_{,xx}\right) \\ \gamma^0_{xy} &= \frac{1}{A_{30}}\left(2A_{31}w_{,xy} - f_{,xy}\right), \end{aligned} \tag{15}$$

where

$$J_0 = \frac{1}{A^2_{10} - A^2_{20}}, \quad J_1 = A_{10}A_{11} - A_{20}A_{21}, \quad J_2 = A_{10}A_{21} - A_{20}A_{11}. \tag{16}$$

Substituting once again Eq. (15) into the expressions of M_{ij} in (8), then M_{ij} into the equation (12) and taking into account (14), leads to

$$C_3\nabla^4 f + \frac{1}{R}f_{,xx} + C_4\nabla^4 w + f_{,yy}\left(w_{,xx} + w^*_{,xx}\right) - 2f_{,xy}\left(w_{,xy} + w^*_{,xy}\right) \\ + f_{,xx}\left(w_{,yy} + w^*_{,yy}\right) + Q_0 = 0 \tag{17}$$

where

$$C_3 = J_0 J_2, \quad C_4 = J_0\left(A_{11}J_1 + A_{21}J_2\right) - A_{12}, \\ \nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}.$$

The equation (17) includes two unknowns functions w and f, so it is necessary to find a second equation relating to these two functions. For this aim, substituting expression (15) into the compatible equation (13), after some calculations, we get

$$\nabla^4 f + C_1\nabla^4 w - C_2\left(w^2_{,xy} - w_{,xx}w_{,yy} - \frac{w_{,xx}}{R} + 2w_{,xy}w^*_{,xy} - w_{,xx}w^*_{,yy} - w_{,yy}w^*_{,xx}\right) = 0 \tag{18}$$

where $C_1 = \dfrac{J_2}{A_{10}}, C_2 = \dfrac{1}{J_0 A_{10}}$.

Two equations (17) and (18) are the governing equations used to investigate the nonlinear stability of imperfect FGM cylindrical panels.

Remarks

i) If $R \to \infty$, the equations (17) and (18) return to the basic stability equations for imperfect FGM plates.

ii) In the case $w^* = 0$, from (17) and (18) we obtain the governing equations for perfect cylindrical panels.

iii) Eqs. (15), (17) and (18) are similar to ones of [21], but the stiffness coefficients A_{ij}, in this paper, are analyzed in the explicit form.

iv) If $\nu = \text{const}$, Eqs.(17) and (18) coincide with ones of [20].

3. BOUNDARY CONDITIONS AND SOLUTION OF THE PROBLEM

3.1. Boundary conditions

Suppose that two cases boundary conditions will be considered below

Case (1). Four edges of cylindrical panel are simply supported

$$\begin{aligned} &w = M_x = N_{xy} = 0, \quad N_x = -r_0 h \quad at \quad x = 0, \quad x = a \\ &w = M_y = N_{xy} = 0, \quad N_y = -p_0 h \quad at \quad y = 0, \quad y = b \end{aligned} \tag{19}$$

Case (2). Two edges loaded $x = 0$ and $x = a$ are simply supported, the remaining two edges $y = 0, y = b$ being unloaded and clamped. So we have

$$\begin{aligned} &w = M_x = N_{xy} = 0, \quad N_x = -r_0 h \quad at \quad x = 0, \quad x = a \\ &w = \frac{\partial w}{\partial y} = N_y = N_{xy} = 0 \quad at \quad y = 0, \quad y = b. \end{aligned} \tag{20}$$

3.2. Solving for FGM cylindrical panel with simply supported four edges

Based on mentioned boundary conditions (19), the deflection w and function f are chosen in the form [20]

$$w = W \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}, \quad f = F\left[\sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} - \theta(x) - \lambda(y)\right], \tag{21}$$

in which $F\dfrac{d^2\theta(x)}{dx^2} = p_0 h, \quad F\dfrac{d^2\lambda(y)}{dy^2} = r_0 h, \quad m, n = 1, 2, 3, ...$

For the initial imperfection $w^* = w^*(x, y)$, we assume it has the form like the deflection w, i.e.

$$w^* = \xi h \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}, \quad m, n = 1, 2, 3, ... \tag{22}$$

where the coefficient $\xi \in [0, 1]$ expresses an imperfection size of panel.

Now substituting Eqs. (21) and (22) into Eqs. (17) and (18) and applying Galerkin's procedure, leads to nonlinear algebraic two equations for F and W as

$$F = -C_1 W + \frac{C_2 W}{R\left[\left(\dfrac{m\pi}{a}\right)^2 + \left(\dfrac{n\pi}{b}\right)^2\right]^2}\left[\left(\frac{m\pi}{a}\right)^2 - \frac{16Rmn\pi^2}{3a^2b^2}(W + 2\xi h)\delta_1\delta_2\right], \tag{23}$$

$$\begin{aligned} &\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^2 (C_3 F + C_4 W) + F\left[\frac{32}{3}(W + \xi h)\frac{mn\pi^2}{a^2b^2}\delta_1\delta_2 - \frac{1}{R}\left(\frac{m\pi}{a}\right)^2\right] + \\ &+ (W + \xi h)\left[r_0 h\left(\frac{m\pi}{a}\right)^2 + p_0 h\left(\frac{n\pi}{b}\right)^2\right] + \frac{16}{mn\pi^2}\delta_1\delta_2\left[Q_0 - \frac{p_0 h}{R}\right] = 0. \end{aligned} \tag{24}$$

Herein, note that $\delta_1 = \dfrac{1}{2}[1 - (-1)^m]$, $\delta_2 = \dfrac{1}{2}[1 - (-1)^n]$, so $\delta_1\delta_2 = 1$ if m and n being odd numbers while $\delta_1\delta_2 = 0$ if m or n even.

By eliminating F in Eqs. (23), (24), after some calculations we obtain

$$\left[(C_4 - C_3C_1)\,\pi^4(m^2B_a^2+n^2)^4 + \frac{(C_1+C_2C_3)}{R}\,(m\pi bB_a)^2\,(m^2B_a^2+n^2)^2 - \right.$$
$$\left. - \frac{C_2\,(mbB_a)^4}{R^2}\right] W - \frac{16mn\pi^2B_a^2}{3}(m^2B_a^2+n^2)^2\left[(C_2C_3+2C_1)\,W + 2\,(C_2C_3+\right.$$
$$\left.+C_1)\times\xi h\right] W\delta_1\delta_2 + \frac{16m^3n b^2B_a^4}{3R}C_2W(3W+4\xi h)\delta_1\delta_2 - \frac{512m^2n^2B_a^4}{9}C_2W\times \tag{25}$$
$$\times\,(W+\xi h)\,(W+2\xi h)\delta_1^2\delta_2^2 + \pi^2b^2r_0h(m^2B_a^2+n^2)^2\,(m^2B_a^2+\beta n^2)\,(W+\xi h) +$$
$$+\frac{16b^4}{mn\pi^2}(m^2B_a^2+n^2)^2\left[Q_0 - \frac{\beta r_0h}{R}\right]\delta_1\delta_2 = 0$$

where $B_a = b/a, \quad \beta = p_0/r_0$.

The equation (25) establishes the relation of load-deflection, so it is used to analyze buckling and postbuckling behavior of imperfect FGM cylindrical panels subjected to loads r_0, p_0 and Q_0.

Since the aim of present study is only to consider cylindrical panel subjected to axial compressive load, thus taking $N_{x0} = -r_0h, N_{y0} = -p_0h = 0, Q_0 = 0$, Eq. (25) yields

$$r_0 = \frac{\overline{W}}{(\overline{W}+\xi)}\left[\frac{\pi^2(m^2B_a^2+n^2)^2}{B_h^2m^2B_a^2}\left(\overline{D}+\overline{C}_3\overline{C}_1\right) - \frac{\left(\overline{C_1}+\overline{E_1}h^2\overline{C_3}\right)}{R} + \frac{\overline{E_1}R_a^2m^2B_a^4}{\pi^2(m^2B_a^2+n^2)^2}\right]$$
$$+\frac{16nh}{3b^2m}\frac{\overline{W}}{(\overline{W}+\xi)}\left[\left(\overline{E_1}h^2\overline{C_3}+2\overline{C}_1\right)\overline{W} + 2\left(\overline{E_1}h^2\overline{C_3}+\overline{C}_1\right)\xi\right] * \delta_1\delta_2 - \tag{26}$$
$$-\frac{16mnR_aB_a^3\overline{E_1}}{3\pi^2B_h(m^2B_a^2+n^2)^2}\frac{\overline{W}(3\overline{W}+4\xi)}{(\overline{W}+\xi)}\delta_1\delta_2 + \frac{512n^2B_a^2\overline{E_1}}{9\pi^2B_h^2(m^2B_a^2+n^2)^2}\overline{W}(\overline{W}+2\xi)\delta_1^2\delta_2^2,$$

where

$$\overline{D} = -\frac{C_4}{h^3}, \quad \overline{E_1} = \frac{C_2}{h}, \quad B_h = \frac{b}{h},$$
$$R_a = \frac{a}{R}, \quad \overline{W} = \frac{W}{h}, \quad \overline{C_1} = \frac{C_1}{h}, \quad \overline{C_3} = \frac{C_3}{h^2}. \tag{27}$$

For a perfect cylindrical panel ($\xi = 0$) subjected to only axial compressive load r_0, Eq. (24) leads to

$$r_0 = \frac{\pi^2(m^2B_a^2+n^2)^2}{B_h^2m^2B_a^2}\left(\overline{D}+\overline{C}_3\overline{C}_1\right) - \frac{\left(\overline{C_1}+\overline{E_1}h^2\overline{C_3}\right)}{R} + \frac{\overline{E_1}R_a^2m^2B_a^4}{\pi^2(m^2B_a^2+n^2)^2} + \frac{16nh}{3b^2m}\times \tag{28}$$
$$\times\left(\overline{E_1}h^2\overline{C_3}+2\overline{C}_1\right)\overline{W}\delta_1\delta_2 - \frac{16mnR_aB_a^3\overline{E_1}}{\pi^2B_h(m^2B_a^2+n^2)^2}\overline{W}\delta_1\delta_2 + \frac{512n^2B_a^2\overline{E_1}}{9\pi^2B_h^2(m^2B_a^2+n^2)^2}\overline{W^2}\delta_1^2\delta_2^2$$

from which upper buckling compressive load may be obtained with $W \to 0$ as

$$r_{0\,\text{upper}} = \frac{\pi^2(m^2B_a^2+n^2)^2}{B_h^2m^2B_a^2}\left(\overline{D}+\overline{C}_3\overline{C}_1\right) - \frac{\left(\overline{C_1}+\overline{E_1}h^2\overline{C_3}\right)}{R} + \frac{\overline{E_1}R_a^2m^2B_a^4}{\pi^2(m^2B_a^2+n^2)^2}. \quad (29)$$

Note that the relation (29) may also be deduced from Eq. (28) with m or n being even numbers.

Now we are interested in finding a lower buckling load. For that, consider $r_0 = r_0(\overline{W})$ and m, n odd numbers and calculating $\frac{dr_0}{d\overline{W}} = 0$ we get

$$\overline{W}_{th} = \frac{9mR_aB_hB_a}{64n} - \frac{3\pi^2B_h(m^2B_a^2+n^2)^2}{64bmnB_a^2\overline{E_1}}\left(\overline{E_1}h^2\overline{C_3}+2\overline{C_1}\right).$$

In addition $\left.\frac{d^2r_0}{d\overline{W}^2}\right|_{\overline{W}=\overline{W}_{th}} > 0$, the value of lower buckling load is

$$\begin{aligned} r_{0\text{lower}} &= \frac{\pi^2(m^2B_a^2+n^2)^2}{B_h^2m^2B_a^2}\left(\overline{D}+\overline{C}_3\overline{C}_1\right) - \frac{\left(\overline{C_1}+\overline{E_1}h^2\overline{C_3}\right)}{R} + \frac{\overline{E_1}R_a^2m^2B_a^4}{\pi^2(m^2B_a^2+n^2)^2} + \\ &+ \frac{3R_aB_hB_ah}{4b^2}\times\left(\overline{E_1}h^2\overline{C_3}+2\overline{C_1}\right) - \frac{\pi^2hB_h(m^2B_a^2+n^2)^2}{4b^3m^2B_a^2\overline{E_1}}\left(\overline{E_1}h^2\overline{C_3}+2\overline{C_1}\right)^2 - \\ &- \frac{9m^2R_a^2B_a^4\overline{E_1}}{4\pi^2(m^2B_a^2+n^2)^2} + \frac{3R_aB_a}{4b}\left(\overline{E_1}h^2\overline{C_3}+2\overline{C_1}\right) + \\ &+ \frac{B_a^2\overline{E_1}}{8\pi^2(m^2B_a^2+n^2)^2}\left[3mR_aB_a - \frac{\pi^2(m^2B_a^2+n^2)^2}{bmB_a^2\overline{E_1}}\left(\overline{E_1}h^2\overline{C_3}+2\overline{C_1}\right)\right]^2 \end{aligned} \quad (30)$$

Quantities $r_{0\text{upper}}$ and $r_{0\text{lower}}$ given by (29) and (30) still depend on values of m and n, therefore one must minimize these expressions with respect to m and n to obtain critical values of axial compressive load.

Remarks

i) If Poisson's ratio $\nu = \text{const}$, i.e. $k_1 = 0$, m, n are odd numbers and $N_x = -r_0h$, $Q_0 = 0$, $N_y = -p_0h = 0$ then $\overline{C}_1 = \overline{C}_3 = 0$.

Eq. (24) becomes

$$\begin{aligned} r_0 = &\frac{\overline{W}}{(\overline{W}+\xi)}\left[\frac{\overline{D}\pi^2(m^2B_a^2+n^2)^2}{B_h^2m^2B_a^2} + \frac{\overline{E_1}R_a^2m^2B_a^4}{\pi^2(m^2B_a^2+n^2)^2}\right] - \\ &- \frac{16mnR_aB_a^3\overline{E_1}}{3\pi^2B_h(m^2B_a^2+n^2)^2}\frac{\overline{W}(3\overline{W}+4\xi)}{(\overline{W}+\xi)} + \frac{512n^2B_a^2\overline{E_1}}{9\pi^2B_h^2(m^2B_a^2+n^2)^2}\overline{W}(\overline{W}+2\xi) \end{aligned} \quad (31)$$

ii) If $\nu = \text{const}$, $\xi = 0$ and m, n odd numbers then Eq. (28) gives us

$$\begin{aligned} r_0 = &\frac{\overline{D}\pi^2(m^2B_a^2+n^2)^2}{B_h^2m^2B_a^2} + \frac{\overline{E_1}R_a^2m^2B_a^4}{\pi^2(m^2B_a^2+n^2)^2} \\ &- \frac{16mnR_aB_a^3\overline{E_1}}{\pi^2B_h(m^2B_a^2+n^2)^2}\overline{W} + \frac{512n^2B_a^2\overline{E_1}}{9\pi^2B_h^2(m^2B_a^2+n^2)^2}\overline{W}^2 \end{aligned} \quad (32)$$

Taking $\overline{W} \to 0$ then Eq. (32) gives the upper buckling load for a perfect panel

$$r_0 = \frac{\overline{D}\pi^2(m^2B_a^2 + n^2)^2}{B_h^2 m^2 B_a^2} + \frac{\overline{E_1}R_a^2 m^2 B_a^4}{\pi^2(m^2B_a^2 + n^2)^2}. \tag{33}$$

Eqs. (31), (32) and (33) coincide with ones given in the paper [20].

iii) If cylindrical panel is perfect and isotropic and $\overline{W} \to 0$ then

$$E_1 = Eh, \quad E_2 = 0, \quad E_3 = \frac{Eh^3}{12}, \quad \overline{E_1} = \frac{E_1}{h} = E,$$

$$\overline{D} = \frac{E_1 E_3 - E_2^2}{E_1(1-\nu^2)h^3} = \frac{E}{12(1-\nu^2)}.$$

Noting $B_a = \dfrac{b}{a}$, $B_h = \dfrac{b}{h}$, $R_a = \dfrac{a}{R}$, Eq. (33) leads to

$$r_0 = \frac{E\pi^2(m^2\dfrac{b^2}{a^2} + n^2)^2 a^2 h^2}{12(1-\nu^2)b^4 m^2} + \frac{Eb^4 m^2}{\pi^2 R^2 a^2 (m^2\dfrac{b^2}{a^2} + n^2)^2}. \tag{34}$$

The minimum value of r_0, in this case, is

$$r_{0cr} = \frac{Eh}{R\sqrt{3(1-\nu^2)}}. \tag{35}$$

This is result can be found in [24].

3.3. Solving for FGM cylindrical panel with simply supported loaded two edges and clamped unloaded two edges

Suppose a FGM cylindrical panel is only subjected to uniform axial compressive load of intensity r_0 on $x = 0$, $x = a$ and uniform compressive radial load of intensity Q_0. Two edges $y = 0$, $y = b$ are clamped and unloaded, while the remaining two edges are simply supported.

In this case, the approximate solutions satisfying boundary conditions (20) are follow as

$$\begin{aligned} w &= W \sin\frac{m\pi x}{a}\left(1 - \cos\frac{2n\pi y}{b}\right) \\ f &= F\left[\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} - \lambda(y)\right], \quad F\lambda''(y) = r_0 h \\ w^* &= \xi h \sin\frac{m\pi x}{a}\left(1 - \cos\frac{2n\pi y}{b}\right), \quad m, n = 1, 2, 3, \ldots \end{aligned} \tag{36}$$

By the same method in the part 3.2, we substitute Eq. (36) into left side of Eqs. (17) and (18), and then apply Galerkin's procedure, we get

$$F = \frac{1}{A}\left\{-W\frac{4C_1}{3n\pi}\delta_2\left[3\left(\frac{m\pi}{a}\right)^4 + B\right] + C_2\left[\frac{W}{R}\frac{16m^2\pi}{3na^2}\delta_2 - W(W + 2\xi h)\frac{1024mn\pi^2}{45a^2b^2}\delta_1\delta_2\right]\right\}, \tag{37}$$

$$\left[C_3A-\frac{1}{R}\left(\frac{m\pi}{a}\right)^2\right]\frac{4ab}{3n\pi}\delta_2F+C_4W\frac{ab}{4}\left[2\left(\frac{m\pi}{a}\right)^4+B\right]+F(W+\xi h)\left(\frac{mn\pi^2}{ab}\right)^2\times$$
$$\times\frac{512ab}{45mn\pi^2}\delta_1\delta_2+r_0h(W+\xi h)\left(\frac{m\pi}{a}\right)^2\frac{3ab}{4}+Q_0\frac{2ab}{m\pi}\delta_1=0, \tag{38}$$

where $A=\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]^2$, $B=\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{2n\pi}{b}\right)^2\right]^2$

Eliminating F in Eqs. (37) and (38), leads to

$$\alpha_1W+\alpha_2W(W+2\xi h)+\alpha_3W(W+\xi h)+\alpha_4W(W+\xi h)(W+2\xi h)+$$
$$+r_0h(W+\xi h)\frac{3bm^2\pi^2}{4a}+\frac{2ab}{m\pi}\delta_1Q_0=0 \tag{39}$$

in which

$$\alpha_1=\left[C_3-\frac{1}{RA}\left(\frac{m\pi}{a}\right)^2\right]\left\{-\frac{16abC_1}{9n^2\pi^2}\delta_2^2\left[3\left(\frac{m\pi}{a}\right)^4+B\right]+\frac{64m^2bC_2}{9n^2aR}\delta_2^2\right\}+$$
$$+\frac{abC_4}{4}\left[2\left(\frac{m\pi}{a}\right)^4+B\right],$$
$$\alpha_2=-\left[C_3-\frac{1}{RA}\left(\frac{m\pi}{a}\right)^2\right]\frac{4096m\pi C_2}{135ab}\delta_1\delta_2^2, \tag{40}$$
$$\alpha_3=\left\{-\frac{2048m\pi C_1}{135abA}\left[3\left(\frac{m\pi}{a}\right)^4+B\right]+\frac{8192m^3\pi^3C_2}{135a^3bRA}\right\}\delta_1\delta_2^2,$$
$$\alpha_4=-\frac{524288m^2n^2\pi^4C_2}{2025a^3b^3A}(\delta_1\delta_2)^2.$$

Now, consider the case $Q_0=0$, i.e. the panel only is subjected to axial compressive load, from Eq. (39), deduces

$$r_0=\frac{-4a}{3bm^2\pi^2}\left[\alpha_1\frac{\overline{W}}{h(\overline{W}+\xi)}+\alpha_2\frac{\overline{W}(\overline{W}+2\xi)}{(\overline{W}+\xi)}+\alpha_3\overline{W}+\alpha_4h\overline{W}(\overline{W}+2\xi)\right]. \tag{41}$$

Herein denote $\overline{W}=W/h$.

If a cylindrical panel is perfect ($\xi=0$), Eq. (41) becomes

$$r_0=\frac{-4a}{3bm^2\pi^2}\left[\frac{\alpha_1}{h}+(\alpha_2+\alpha_3)\overline{W}+\alpha_4h\overline{W}^2\right]. \tag{42}$$

From this relation, let $\overline{W}\to 0$, we obtain the expression of upper buckling compressive load as

$$r_{0\text{upper}}=\frac{-4a}{3bm^2\pi^2h}\alpha_1=\left[C_3-\frac{1}{RA}\left(\frac{m\pi}{a}\right)^2\right]\times$$
$$\times\left\{\frac{64a^2C_1}{27m^2n^2\pi^4h}\delta_2^2\left[3\left(\frac{m\pi}{a}\right)^4+B\right]-\frac{256C_2}{27n^2\pi^2Rh}\delta_2^2\right\}-\frac{a^2C_4}{3m^2\pi^2h}\left[2\left(\frac{m\pi}{a}\right)^4+B\right] \tag{43}$$

In order to find a lower buckling compressive load, from (42) calculate $\frac{dr_0}{d\overline{W}} = 0$, it is easy to receive

$$\overline{W}_{\text{th}} = \frac{-(\alpha_2 + \alpha_3)}{2\alpha_4 h}. \tag{44}$$

In addition $\left.\frac{d^2 r_0}{d\overline{W}^2}\right|_{\overline{W}=\overline{W}_{\text{th}}} > 0$, so yields to

$$r_{0\text{lower}} = \frac{-4a}{3bm^2\pi^2}\left[\frac{\alpha_1}{h} - \frac{(\alpha_2 + \alpha_3)^2}{4\alpha_4 h}\right]. \tag{45}$$

4. NUMERICAL CALCULATIONS AND DISCUSSIONS

The FGMs considered, in this section, are made from two constituent materials: Stainless steel (SUS304) and Silicon nitride (Si_3N_4) with the properties given by Shariyat [25] $E_c = 322.2$ (GPa), $E_m = 207.7$ (GPa), $\nu_c = 0.24$, $\nu_m = 0.3177$.

The accuracy of proposed approach and effects of dimensional parameters, of power law indexes k and k_1, of imperfection and boundary conditions on buckling and postbuckling behavior are presented by numerical results.

For the sake of simplifying calculations A_{ij}, the sum in Appendix taken values from 0 to 5.

As a first example, the buckling load $N_x/(ER)$, $N_x = r_0 h$, r_0 given by Eq. (34) for isotropic cylindrical panels under axial compression (with $\nu = 0.3$) studied by Turvey (1977) and Shen (2002) in [2] are reexamined and are compared in Table 1.

Table 1. Comparisons of buckling load $N_x/(ER)$ for isotropic cylindrical panels under axial compression

	Calculated by			
Geometrical parameters	Turvey	Shen	Present	Percent (%)
a/b=0.4, a/R=1.0, b/h=25	0.73675e-4	0.71410e-4 (m,n)=(3,1)	0.77355e-4 (m,n)=(3,1)	4.99
a/b=1.333, a/R=1.0, b/h=75	0.60523e-4	0.58737e-4 (m,n)=(2,2)	0.60756e-4 (m,n)=(2,2)	0.38

The axial buckling loads $P_{cr} = r_{0\text{upper}} bh$, $r_{0\text{upper}}$ in Eq. (29), for perfect Si_3N_4 - SUS304 cylindrical panels with different values of volume fraction indexes k and k_1 are given and compared with results of Shen [2] in Table 2.

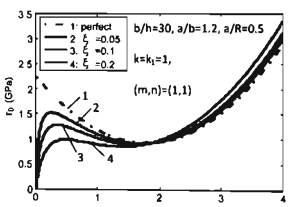
Two above comparisions show that the results from the present proposed method agree well with the comparator solutions.

For further examples see below, the graphs from Fig. 2 to Fig. 7 are traced according to Eq.(26) for simply supported panels and Eq.(41) for clamped panels respectively.

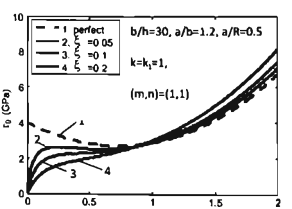
As part of the effects of imperfections, the postbuckling load-deflection curves for FGM cylindrical panels shown in Fig. 2a and Fig. 2b. As can be observed, when the deflection exceeds a specific value, the curves become higher when ξ is increased.

Table 2. Comparisons of buckling loads P_{cr} of perfect FGM simply supported cylindrical panels

	$k_1 = k$	P_{cr} (MN)		Percent (%)
		Shen (2002)	Present	
	∞	4.9565	5.2719	5.98
	5.0	5.4489	5.9632	8.62
(m, n) = (1, 1)	2.0	5.8836	6.3736	7.69
b = 0.3 m, a/b = 1.2	1.0	6.2758	6.7490	7.01
T_0 = 300 K	0.5	6.6488	7.1451	6.95
n/R = 0.5, b/h = 30	1/3	6.8431	7.3522	6.92
	0.2	7.0594	7.5636	6.67
	1/8	7.2280	7.7057	6.20
	0.1	7.2968	7.7575	5.94
	∞	10.290	10 544	2.41
	5.0	11.314	11 930	5.17
(m,n)=(3,1)	2.0	12.215	12 747	4.18
b=0.3m; a/b=1 2	1 0	13.026	13.494	3.47
T_0 = 300K	0.5	13.797	14.284	3.41
a/R=1.0; b/h=30	1/3	14.199	14 698	3.40
	0.2	14.648	15.121	3.13
	1/8	14.998	15.405	2.64
	0.1	15.142	15 509	2.37
	∞	1.2968 (m, n) = (1, 3)	1.3180 (m, n) = (3, 1)	1.61
	5.0	1.4261	1.4913	4.37
b=0.3m; a/b=1.2	2.0	1.5396	1.5934	3.38
T_0 = 300K	1.0	1.6413	1.6867	2.69
a/R = 0.5, b/h = 60	0.5	1.7377	1.7855	2.68
	1/3	1.7881	1.8373	2.68
	0.2	1.8445	1.8901	2.41
	1/8	1.8889	1.9256	1.91
	0.1	1.9071	1.9386	1.62



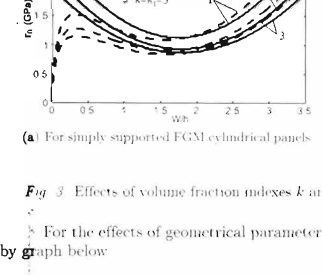
(a) For simply supported FGM cylindrical panels



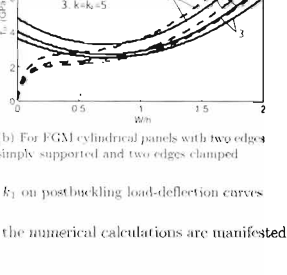
(b) For FGM cylindrical panels with two edges simply supported and two edges clamped

Fig. 2. Effects of imperfection ξ on postbuckling load-deflection curves

Fig. 3a and Fig. 3b illustrate the effects of volume fraction indexes k and k_1. Three values of $k = k_1 = 0, 1, 5$ are used. As shown, the postbuckling load-deflection curves are gradually lower when the values of $k = k_1$ increase, i.e. the load carrying capacity of structure decreases with the greater percentage of metal. The prime reason for the fall of the critical loads is that the higher value of k corresponds to a metal-richer cylindrical panel which usually has less stiffness than a ceramic-richer one.



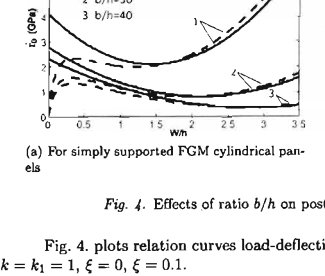
(a) For simply supported FGM cylindrical panels



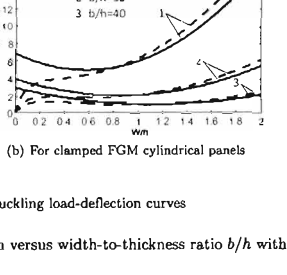
(b) For FGM cylindrical panels with two edges simply supported and two edges clamped

Fig. 3 Effects of volume fraction indexes k and k_1 on postbuckling load-deflection curves

For the effects of geometrical parameters, the numerical calculations are manifested by graph below



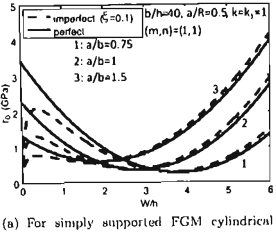
(a) For simply supported FGM cylindrical panels



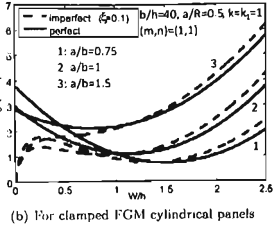
(b) For clamped FGM cylindrical panels

Fig. 4. Effects of ratio b/h on postbuckling load-deflection curves

Fig. 4. plots relation curves load-deflection versus width-to-thickness ratio b/h with $k = k_1 = 1$, $\xi = 0$, $\xi = 0.1$.



(a) For simply supported FGM cylindrical panels

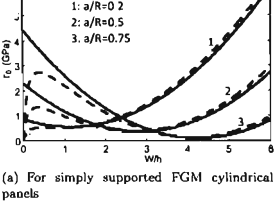


(b) For clamped FGM cylindrical panels

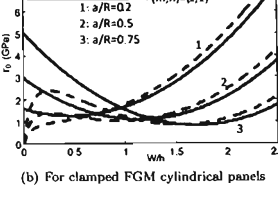
Fig. 5. Effects of ratio a/b on postbuckling load-deflection curves

Fig. 5. plots these relation curves versus length-to-width ratio a/b with $k = k_1 = 1$, $\xi = 0$, $\xi = 0.1$.

Fig. 6. shows the effect of length-to-radius ratio a/R on $(W/h, r_0)$ relation curves. It is obvious, from these figures, that the buckling loads and postbuckling load bearing



(a) For simply supported FGM cylindrical panels

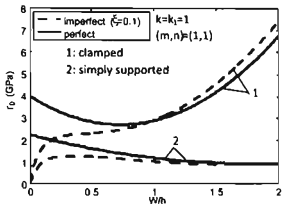


(b) For clamped FGM cylindrical panels

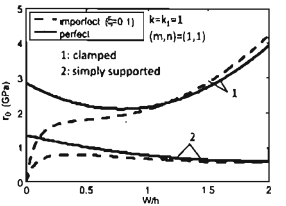
Fig. 6. Effects of ratio a/R on postbuckling load-deflection curves

capacity of imperfect FGM cylindrical panels are considerably reduced when b/h ratios increase (Fig. 4). The values of r_0 when the deflection is still small, are decrease when a/b increase, and they only increase together a/b when W/h ratio exceeds special value (Fig. 5). In Fig. 6, can be seen that the $(W/h, r_0)$ relation curves graduate higher according to a/R with W/h being still small and they become gradually lower when the W/h ratios are greater than a any special value.

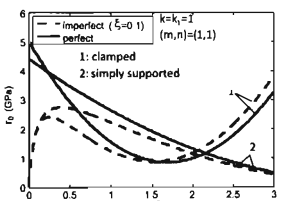
The influence of two types boundary conditions on stability behavior has been also carried out. The numerical results are represented by graph in Fig. 7 (a, b, c). It can be seen that the critical buckling loads when panels are simply supported, are smaller than ones when those structures are clamped. These results correspond to the facts.



(a) For FGM cylindrical panels with $a/b = 1.2$, $b/h = 30$, $a/R = 0.5$



(b) For FGM cylindrical panels with $a/b = 1.5$, $b/h = 40$, $a/R = 0.5$



(c) For FGM cylindrical panels with $a/b = 1$, $b/h = 40$, $a/R = 0.75$

Fig. 7. Effects of boundary conditions on postbuckling load-deflection curves

The influence of boundary conditions, indexes k and k_1, buckling mode (m, n) on critical loads $r_{0\text{upper}}$ and $r_{0\text{lower}}$ for imperfect FGM cylindrical panel with $b/h = 30$, $a/b = 1.2$, $a/R = 0.5$, again are confirmed by numerical calculation in Table 3.

For this example, it can be seen that the critical buckling loads of simply supported FGM cylindrical panels correspond to the buckling mode (m,n)=(1,1), while the critical buckling loads of clamped FGM panels reached with (m,n)=(3,1).

Table 3. The influence of boundary conditions, of indexes k and k_1, of buckling mode (m,n) on critical loads r_{0cr}

Boundary conditions	r_{0cr} (Gpa)	(m,n)=(1,1)	(m,n)=(1,3)	(m,n)=(3,1)	(m,n)=(3,3)
Simply supported	$r_{0upper}(k_1 = k = 0)$	**2.6632**	42.335	3.3023	11 781
	$r_{0upper}(k_1 = k = 0.5)$	**2.3817**	37.073	2.9357	10.348
	$r_{0upper}(k_1 = k = 10)$	**1.9046**	31.241	2.4179	8.7018
	$r_{0upper}(k_1 = k = \infty)$	**1.7573**	28.611	2.2109	7.9575
	$r_{0lower}(k_1 = k = 0)$	**1.1207**	42.288	2.5439	11.609
	$r_{0lower}(k_1 = k = 0.5)$	**0.9733**	36.970	2.2184	10.146
	$r_{0lower}(k_1 = k = 10)$	**0.8337**	31.178	1.8779	8.5583
	$r_{0lower}(k_1 = k = \infty)$	**0.7627**	28.581	1.7220	7.8470
Simply supported $x = 0, x = a$ Clamped $y = 0, y = b$	$r_{0upper}(k_1 = k = 0)$	4.7677	202.13	**3.7008**	31.070
	$r_{0upper}(k_1 = k = 0.5)$	4.2414	176.76	**3.2834**	27.176
	$r_{0upper}(k_1 = k = 10)$	3.4704	149.05	**2.7130**	22.913
	$r_{0upper}(k_1 = k = \infty)$	3.1813	136.61	**2.4811**	20.999
	$r_{0lower}(k_1 = k = 0)$	3.2858	20.212	**2.9723**	31.052
	$r_{0lower}(k_1 = k = 0.5)$	2.8558	176.73	**2.5926**	27.150
	$r_{0lower}(k_1 = k = 10)$	2.4239	149.03	**2.1933**	22.895
	$r_{0lower}(k_1 = k = \infty)$	2.2259	136.61	**2.0114**	20.987

5. CONCLUSIONS

This paper deals with the nonlinear buckling and postbuckling problem of axially compressed imperfect FGM cylindrical panels by using the nonlinear deflection shell theory taking into account the Poisson's ratio $\nu = \nu(z)$. The stiffness coefficients A_{ij} defined in the integrating form in [21, 22, 23] are analyzed in explicit form.

Approximate analytical solutions for two types boundary conditions are given and applying Galerkin's procedure are obtained the explicit relations finding critical buckling loads and postbuckling load-deflection curves.

The nonlinear stability problem of simply supported-clamped FGM cylindrical panels which has not been considered in [20] is solved too here.

The effects of inhomogeneous parameter, dimensional parameter, boundary conditions, initial imperfections, buckling mode on nonlinear stability behavior of FGMs cylindrical panels are investigated.

The comparisions of results of present paper with the ones other authors [2, 20, 24] have affirmed the reliability and accuracy of the proposed approach.

In the case $\nu = \text{const}$, the present results return to ones of [20].

In this paper, we does not compare results with these ones of [21, 22, 23] because those articles were only considered for the FGM closed circular cylindrical shell while our results are obtained for FGM cylindrical panels.

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APPENDIX

$$A_{10} = \frac{h}{2}\sum_{n\geq 0}\left(\frac{E_m}{nk_1+1} + \frac{E_{cm}}{k+nk_1+1}\right)c_n$$

$$A_{11} = \frac{h^2}{2}\sum_{n\geq 0}\left[E_m\left(\frac{1}{nk_1+2} - \frac{1}{2(nk_1+1)}\right) + E_{cm}\left(\frac{1}{k+nk_1+2} - \frac{1}{2(k+nk_1+1)}\right)\right]c_n$$

$$A_{12} = \frac{h^3}{2}\sum_{n\geq 0}\left[E_m\left(\frac{1}{nk_1+3} - \frac{1}{nk_1+2} + \frac{1}{4(nk_1+1)}\right)\right.$$

$$\left.+E_{cm}\left(\frac{1}{k+nk_1+3} - \frac{1}{k+nk_1+2} + \frac{1}{4(k+nk_1+1)}\right)\right]c_n$$

$$A_{20} = \frac{h}{2}\sum_{n\geq 0}\left[\frac{E_m\nu_m}{nk_1+1} + \frac{E_m\nu_{cm}}{(n+1)k_1+1} + \frac{E_{cm}\nu_m}{k+nk_1+1} + \frac{E_{cm}\nu_{cm}}{k+(n+1)k_1+1}\right]c_n$$

$$A_{21} = \frac{h^2}{2}\sum_{n\geq 0}\left[E_m\nu_m\left(\frac{1}{nk_1+2} - \frac{1}{2(nk_1+1)}\right) + E_m\nu_{cm}\left(\frac{1}{(n+1)k_1+2} - \frac{1}{2|(n+1)k_1+1|}\right)\right.$$

$$\left.+E_{cm}\nu_m\left(\frac{1}{k+nk_1+2} - \frac{1}{2(k+nk_1+1)}\right) + E_{cm}\nu_{cm}\left(\frac{1}{k+(n+1)k_1+2} - \frac{1}{2[k+(n+1)k_1+1]}\right)\right]c_n$$

$$A_{22} = \frac{h^3}{2}\sum_{n\geq 0}\left[E_m\nu_m\left(\frac{1}{nk_1+3} - \frac{1}{nk_1+2} + \frac{1}{4(nk_1+1)}\right) + E_m\nu_{cm}\left(\frac{1}{(n+1)k_1+3} - \frac{1}{(n+1)k_1+2}\right.\right.$$

$$\left.+\frac{1}{4|(n+1)k_1+1|}\right) + E_{cm}\nu_m\left(\frac{1}{k+nk_1+3} - \frac{1}{k+nk_1+2} + \frac{1}{4(k+nk_1+1)}\right) +$$

$$\left.+E_{cm}\nu_{cm}\left(\frac{1}{k+(n+1)k_1+3} - \frac{1}{k+(n+1)k_1+2} + \frac{1}{4|k+(n+1)k_1+1|}\right)\right]c_n$$

$$A_{30} = \frac{1}{2}(A_{10} - A_{20}),\ A_{31} = \frac{1}{2}(A_{11} - A_{21}),\ A_{32} = \frac{1}{2}(A_{12} - A_{22})$$

$$a_n = \frac{\nu_{cm}^n}{(1-\nu_m)^{n+1}}, c_n = \left[\frac{1}{(1-\nu_m)^{n+1}} + \frac{(-1)^n}{(1+\nu_m)^{n+1}}\right]\nu_{cm}^n$$

$$E_1 = E_m h + \frac{E_{cm}h}{(k+1)}, E_2 = E_{cm}h^2\left[\frac{1}{k+2} - \frac{1}{2k+2}\right]$$

$$E_3 = \frac{E_m h^3}{12} + E_{cm}h^3\left[\frac{1}{k+3} - \frac{1}{k+2} + \frac{1}{4k+4}\right], D = \frac{E_1E_3 - E_2^2}{E_1(1-\nu^2)}.$$