TEACHING OF MATHEMATICAL MODELING AT HIGH SCHOOL IN LAO PEOPLE'S DEMOCRATIC REPUBLIC

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ARTICLE INFO	ABSTRACT
	This paper presents an empirical research about implementing mathematical modelling in some high schools in Lao People's Democratic Republic (PDR) in the context of reforming the general education program. The paper uses survey methods by student questionnaires and in-depth interviews with some experienced math teachers. The data from practical survey have shown that there were some cognitive barriers in applying modelling to the classroom and designing real world models for teaching mathematics. The paper also proposes a model of modelling process in teaching mathematics. Moreover, we designed mathematical modelling activities to support the students better understanding about the application of school mathematics in real life and make a contribution to develop their problem-solving skills. The research results have shown that modelling teaching approach meets the requirements of renovating methods of teaching and learning mathematics in Lao PDR.

TỔ CHỨC DẠY HỌC MÔ HÌNH HÓA Ở TRƯỜNG TRUNG HỌC PHỔ THÔNG NƯỚC CỘNG HÒA DÂN CHỦ NHÂN DÂN LÀO

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THÔNG TIN BÀI BÁO	TÓM TẮT
	Bài viết trình bày nghiên cứu về tổ chức dạy học mô hình hóa ở trường trung học phổ thông nước Cộng hòa Dân chủ Nhân dân Lào trong bối cảnh đổi mới chương trình giáo dục phổ thông. Bài viết sử dụng phương pháp điều tra, khảo sát bằng bảng hỏi học sinh và phỏng vấn sâu một số giáo viên toán có kinh nghiệm giảng dạy. Số liệu nghiên cứu từ khảo sát thực tiễn cho thấy những khó khăn trong việc vận dụng phương pháp mô hình hóa và thiết kế các mô hình thực tiễn trong dạy học môn Toán. Bài viết cũng đề xuất quy trình dạy học mô hình hóa toán học. Từ đó, chúng tôi thiết kế một số hoạt động mô hình hóa để hỗ trợ học sinh hiểu sâu hơn về ứng dụng của toán học trong thực tiễn và góp phần phát triển kỹ năng giải quyết vấn đề cho các em. Kết quả nghiên cứu cho thấy dạy học mô hình hóa toán học đáp ứng được yêu cầu đổi mới phương pháp dạy học môn Toán ở trường phổ thông.

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1. Introduction

One of the central themes of mathematics education over the past three decades has been mathematical modelling and its application in real life. More generally, it is the relationship between mathematics and reality (the world outside of mathematics). Modelling in formal mathematics education first appeared at Freudenthal's Conference in 1968 [1], [2]. At the conference, mathematics educators raised many problems related to modelling. Teaching mathematics needs to help students be able to apply mathematics to simple situations in life. The connection between mathematics and modelling continued to be addressed at conferences of German-speaking countries that included discussions on aspects of applied mathematics in education [3]-[6]. Modelling in teaching mathematics was introduced into schools after Pollak's research in 1979. According to Pollak, mathematics education must teach students how to use mathematical knowledge in daily life. Since then, modelling teaching and learning in schools has become a prominent topic on a global scope [7], [8]. For example, research by the program for international student assessment (PISA) emphasizes that the purpose of mathematics education is to develop students' ability to use mathematics in life [9], [10].

In teaching mathematics in high schools, the model used can be drawings, tables, functions, graphs, equations, diagrams, charts, symbols or virtual models on electronic computers. Modelling in teaching mathematics is a method to help students learn and explore situations arising from reality using mathematical tools and language with the support of teaching software. Using this method in teaching will help teachers promote students' active learning, help students answer the question "What is the application of mathematics in practice and what role does it play in the classroom to interpret real phenomena?". This has great significance in motivating students to learn from the beginning stage [11]-[13]. The process of modelling real-life situations shows the relationship between practice and textbook problems from a mathematical perspective. Therefore, it requires students to master mathematical thinking operations such as analysis, synthesis, comparison, generalization, and abstraction. In high school, this approach makes mathematics learning more practical and meaningful for students, creating motivation and passion for learning mathematics [10], [14]. In the Lao PDR, the practical applications of mathematics in curricula and textbooks, as well as in the practice of teaching mathematics, have not been given adequate and regular attention.

Some problems need to be solved such as epistemology and the relationship between mathematics and the world; the meaning of the mathematical model and its components; the difference between pure mathematics and applied mathematics; modelling and application in teaching mathematics; compatibility between modelling operations and other mathematical operations; describe students' modelling competence; identify the most important mathematics competencies students need, and how modelling and application activities can contribute to building these competencies; appropriate pedagogical principles and strategies to develop modelling competence; the role of technology in teaching modelling and applying mathematics; the role of modelling and application in daily math teaching; promote the use of model examples in everyday classrooms; component assessment of modelling competence; appropriate strategies for implementing the assessment methods in practice [15]-[17]. Modelling and application in educational mathematics will be of interest to mathematics educators, educators, educational administrators, teachers and students.

This study focuses on analyzing the mathematics textbook program of the Lao PDR, assessing students' mathematical modeling competence, difficulties and challenges in applying mathematical modeling in teaching high school mathematics. As a result, the study has proposed a modeling process in teaching mathematics and illustrated with some appropriate real world situations.

2. Research methods

In order to investigate the real situation on modelling teaching in Lao PDR, we conducted a survey in seven high schools during December 2020 to September 2021. A questionnaire was

designed to assess mathematical modelling competence of high school students. Participants of the survey were 200 high school students of 10th grade. Moreover, the content of the survey was also examined the current situation of mathematical modelling competence and the development of mathematical modelling competence of high school students who participated in the survey. In-depth interviews with 12 mathematics teachers were also recorded and analysed to understand students' difficulties during mathematical modelling process. As a result, some recommendations in this study are based on these teachers' and educational experts' points of view.

3. Results and dicussion

3.1. Analysing Mathematics Textbooks in Lao PDR

According to the content of Algebra mentioned in the textbook of the educational program of Lao PDR, currently, problems and exercises have very few practical problems. The exercises and examples in the high school mathematics textbooks are mainly divided into two categories: "pure mathematics" problems and problems with practical situations in which the problem has a realistic situation (but most of them are problems with hypothetical situations). There are very few realistic problems for students to apply mathematical knowledge, but we find there are many lessons learned in the Algebra section that we can ask questions or apply to problem solving in real situations such as calculations in sets, constant and first-order functions, quadratic equations, quadratic inequalities, counting rules,... We also found that only about 5% of problems have practical content, of which there are 8 practical examples and students can describe these problems. In the questions and exercises in the mathematics textbook, there are only 10 practical exercises for students to build the mathematical models. Moreover, most teachers use the system of examples and exercises in textbooks without focusing on realistic situations during the process of teaching mathematics.

In Table 1, we see that the contents of the 10th grade Algebra section have practical problems and can develop modelling competence such as judgment function common guess, existential judgment and inference (three exercises with practical situations); calculations in sets (two examples and four practical exercises); set of numbers (only one example); constant and first-order functions (two examples and two practical exercises); quadratic function (one example and two practical exercises); quadratic equation (two examples); cubic function (two exercises). Thus, in the 10th grade textbook program, there are more content on Algebra than in 11th grade, so students following the 10th grade program will have the opportunity to be exposed to different types of problems.

Table 1. *The content of Algebra with realistic problems in 10th and 11th grade mathematics textbooks*

Math textbooks	The content of Algebra	Number of realistic situations	
		Example	Exercise
Grade 10	Judgment function, common judgment, existence judgment, and inference	0	3
	Calculations in the set	2	4
	Set of numbers	1	0
	Constant function, first-order function, quadratic functions, cubic function	5	6
Grade 11	How to apply the exponential function	2	3
	Counting rules	6	14
	Total	**8**	**17**

Teaching mathematics in high school is aimed at helping students develop an understanding of basic mathematical skills and apply mathematical knowledge into practical life as well as in other subjects. Students could use mathematical modelling in learning the following topics in Algebra Grade 10: equations and systems of equations; inequalities and systems of inequalities; trigonometric equations and inequalities; graphs of quadratic, cubic, rational, logarithmic, exponential, and parabolic functions; area of the graph, the volume of the graph rotation through the coordinate axis; plane geometry and spatial geometry; statistical problems.

3.2. Mathematical Modelling Competence of High School Students

In this research, we have conducted a survey with 200 students in high schools. The purpose of the survey is to assess students' mathematical modelling competence as well as their difficulties in solving modelling problems from students' viewpoints. We used a questionnaire for students to self-assess on the components of mathematical modelling competence at four levels from 1 (low level) to 4 (high level). The mean score is calculated to determine the student's level of achievement for each component of this competence.

Table 2. *Students' self-assessment of mathematical modelling competence*

Content of the survey	Competence levels				Mean score
	1	**2**	**3**	**4**	
Understanding the application of mathematics in real life	37%	19%	33%	11%	2.18
Experience in solving practical problems	28%	44%	15%	13%	2.13
Excited to learn new knowledge through mathematical modelling activities	23%	28%	39%	10%	2.36
Difficulties encountered in the process of mathematical modelling from practical problems	16%	49%	27%	8%	2.27
Competence in understanding problems in a practical context	30%	41%	20%	4%	1.93
Degree of natural language understanding in real world problems	30%	31%	30%	9%	2.18
The ability to construct a mathematical model from a real model or from relevant contexts	30%	40%	19%	6%	1.91
Problem solving ability	32%	48%	27%	3%	1.91
Competence to solve problems in mathematical modelling	33%	35%	27%	5%	2.04
The ability to interpret mathematical results in real-life situations	26%	59%	17%	8%	2.07

The survey results shown in Table 2 showed that 74 students (37%) knew about real-world problems through the teacher's introduction, 38 students (19%) knew the real-world problem through the teacher's introduction and read reference books. For experience in solving practical problems, we found that 56 students (28%) could not solve the problems from the real context. However, there are also only 30 students (15%) who can learn the relationship between the hypotheses and set the variables of the realistic problem. Learn the relationship between assumptions and set variables. Apply mathematical knowledge to solve problems

For the excitement of learning new knowledge through mathematical modelling activities, there are 78 students (39%) who are passionate and curious to discover the relationship between mathematics and problems in daily life and 20 students (10%) said that they think and find ways to apply mathematical knowledge to solve rea-life problems. With the difficulty encountered in the process of mathematical modelling from practical problems, there are 98 students (49%) said that they sometimes model mathematics from real life problems, but often do not finish because they do not know how to connect mathematical knowledge with real problems. With 54 students (27%) saying that they often mathematically model life's problems, they already know the assumptions from real-life problems, set variables, establish mathematical relationships between variables, but sometimes make mathematical mistakes. Only 16 students (8%) regularly perform mathematical modelling for real-life problems and solve problems in many ways, but sometimes they lack ability to evaluate the solution in real context. Especially, there are only 8 students (4%) said that they had knowledge from experience, understanding connecting mathematical knowledge to problem solving from real-world contexts. Regarding the ability to construct a mathematical model from the real model or from relevant contexts, there are 60 students (30%) said that they are not able to build mathematical models from real models or related contexts and only 12 students (6%) said that they could establish the correspondence of objects from the real model to mathematical model. The survey results also showed that 34 students (27%) who proactively detecting problems, predicting conditions for problems arising and commenting on how to approach and solve problems. Only 6 students (3%) know how to mobilize their own knowledge and experiential skills to solve problems.

Through in-depth interviews with 12 mathematics teachers, we have realized some students' difficulties during mathematical modelling process. Students do not realize all the important information of the situation needed to convert into mathematical language, often misrepresent the relationships, and misunderstood or unclear requirements of the situation. Moreover, students have difficulty in simplifying the problem, dealing with the conditions of the problem, establishing the mathematical problem from the real world situation, clarifying the problem's objective. In other words, the students have difficulty in identifying appropriate variables, parameters, relevant constants, finding relationships between variables, collecting real data to provide more information about the situation, eliminating non-mathematical factors and convert real world problems into mathematical language. Students often lack practical knowledge related to the situation because they are less likely to participate in practical activities, the ability to relate interdisciplinary knowledge in the problem-solving process is weak, as well as lack of experiences to create and select mathematical models.

3.3. Modelling Process in Teaching Mathematics

According to Edwards and Hamson (2001) [18], mathematical modelling is the process of transforming a real-world problem into a mathematical problem by establishing and solving mathematical models, expressing and evaluating solution in a real-world context, improve the model if the solution is unacceptable. To be more specific, mathematical modelling is the entire process of converting a real problem to a mathematics problem and vice versa, with everything involved in that process, from reconstructing the real situation to reality, deciding on an appropriate mathematical model, working in a mathematical environment, interpreting the results in relation to a real-world situation and sometimes needing to adjust the models, repeating the process many times until when a reasonable result is obtained. Thus, mathematical modelling is about describing real-world phenomena, answering questions about the world around them, explaining real-world phenomena, testing ideas, and making predictions about the world around them. The surrounding world is mentioned in relation to engineering, physics, biology, ecology, chemistry, economics, sports, etc. However, in short, mathematical modelling is the process of solving real-world problems using mathematical tools and languages.

The transformation step from the actual model to the mathematical model in the modelling process is called is mathematization [1], [2], [19]. When students enter the process of mathematization, the real situation has been specialised, idealized, at this time students need to convert non-mathematical objects and relations into mathematical objects and relations, convert the question posed in the real situation to a mathematical question, the goal is to represent the actual model in the language of mathematics. In other words, mathematization from this point of view is an activity or process associated with the modelling process in order to represent or explain the actual model by mathematical means [20]-[22]. Thus, the concept of mathematization presented in the PISA study is essentially the entire modelling process. In this paper, we are interested in the concept of mathematization from this point of view of PISA. In the modelling process, reality and mathematics are viewed as two separate worlds, and modelling will involve some transformation between the two environments as well as within each environment to solve the given situation.

The process of mathematical modelling is the process of applying mathematical knowledge to the study of real-life problems, first of all converting the problem to be studied into a mathematical problem, then using mathematical tools and methods to solve real-world problems initially to get results. In other words, it is the process of establishing a mathematical model for the problem to be studied, solving the problem in that model, then expressing and evaluating the solution, and improving the model if the solution is unacceptable [12]. Researchers often use different diagrams, depending on the approach, the complexity of the real situation under consideration, or the purpose of the research to show the nature of the modelling process.

However, all diagrams are intended to illustrate the main steps in an iterative process, starting with a real situation and ending with a solution or repeating the process to achieve better results. In order to flexibly apply the above modelling process in the process of teaching mathematics, teachers need to help students understand the specific requirements of each of the following steps in the process:

Step 1 (*mathematization*): Understand the real problem, build hypotheses to simplify the problem, describe and express the problem using mathematical tools and language.

Step 2 (*solving problem*): Use appropriate mathematical tools and methods to solve mathematized problems or problems.

Step 3 (*understanding*): Understand the meaning of the solution of the problem for the realistic situation (the original problem), in which it is necessary to recognize the limitations and possible difficulties when applying the results into a realistic situation.

Step 4 (*reflection*): Review the hypotheses, learn the limitations of the mathematical model as well as the solution of the problem, review the used mathematical tools and methods, compare the reality practice to improve the built model.

Example 1. The teacher showed an image of an overpass at the Vientiane-Vengung highway with a parabolic shape of 40 *m* in length and 12 *m* in height from the bridge deck to the highest peak. Draw a graph and determine the highest point of that overpass (see Figure 1).

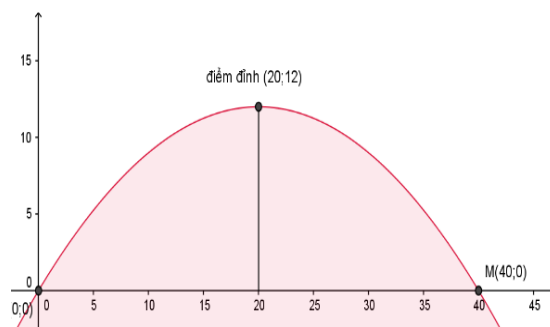**Figure 1.** *Overpass in Vientiane-Vengung highway*

Figure 2. *Graph of the function of the overpass at the expressway*

Solution. Select the origin to coincide with the beginning of one side of the bridge and the other end at point $M(40; 0)$. Draw a graph of the bridge (see Figure 2). Given the required function of the form $y = ax^2 + bx + c$. We have:

$$f(0) = a.0^2 + b.0 + c = 0 \Leftrightarrow c = 0 \quad f(20) = 400a + 20b = 12; f(40) = 1600a + 40b = 0$$

Then we have a system of equations: $\begin{cases} 100a + 5b = 3 \\ 40a + b = 0 \end{cases} \Leftrightarrow \begin{cases} a = -\dfrac{3}{100} \\ b = \dfrac{6}{5} \end{cases}$

So, the required function is an equation of the form $y = -0.03x^2 + 1.2x$.

This example helps high school students develop modeling competence through identifying problems in practice, the ability to set real models to mathematical models, the ability to represent mathematical models in the form of quadratic functions and apply mathematical knowledge to solve practical problems.

Example 2. The grid is shaped like a parabola, the lowest grid is hung on a 30.25 *m* high power pole, we know the two poles are 150 *m* apart. Suppose we set up an *Oxy* coordinate such that one pole is located directly *Oy* (*x* and *y* in meters), the second pole is at position (150; 0). Know a point *M* on a wire with coordinates (10; 27.45). Find the function whose graph shows the

shape of the wire mesh, and find the height from the ground to the lowest point of the wire mesh, rounding the result to the units) (see Figure 3).

Figure 3. *Power grid at Namnguam hydroelectric power station*

Solution.

Step 1 *(mathematization):* The teacher asked the students to observe the wire mesh. Students discuss and make predictions that the grid shape is like a parabola. Then the teacher asked the students to find the representation of that parabola. Students discuss and come up with a way to determine the representation equation.

Step *2 (solving problem):* Students based on observations and given data to find the representation of the parabola as a quadratic function. Students discuss and give the required function of the form $y = ax^2 + bx + c; (a \neq 0)$ satisfying the following conditions: $f(0) = 30,25 \Rightarrow c = 30,25$;

$f(10) = 100a + 10b + 30,25 = 27,45; f(150) = (150)^2 a + 150b + 30,25 = 30,25$ or they obtained the equation $150a + b = 0$

Solve the system of equations:

$$\begin{cases} 100a + 10b = -2,8 \\ 150a + b = 0 \end{cases} \Leftrightarrow \begin{cases} 100a + 10(-150a) = -2,8 \\ b = -150a \end{cases} \Leftrightarrow \begin{cases} -1400a = -2,8 \\ b = -150a \end{cases} \Leftrightarrow \begin{cases} a = 0,002 \\ b = -0,3 \end{cases}$$

Therefore, the equation of the parabola is $y = 0,002x^2 - 0,3x + 30,25$. Then, the group of students plotted the function they just found and the lowest point of the wire mesh (see Figure 4).
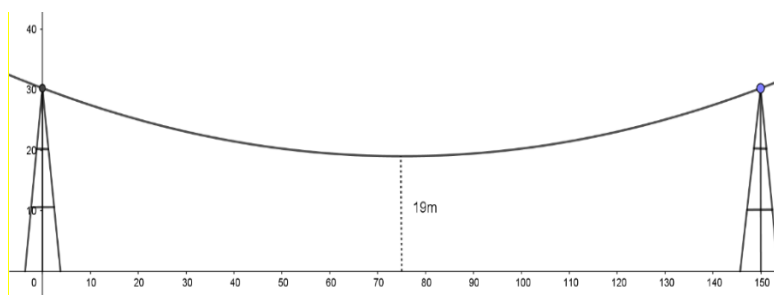

Figure 4. *The parabola represents the shape of the wire mesh*

Finally, students observe the graph just drawn and draw the conclusion that the lowest point of the wire mesh is:

$$h = f(150/2) = f(75) = 0,002(75)^2 - 0,3(75) + 30,25 = 11,25 - 22,5 + 30,25 = 19$$

So the lowest point of the grid is the point $(75;19)$.

Step 3 *(understanding):* The height from the ground to the lowest top of the wire mesh is 19 *m*.

Step 4 *(reflection):* In fact, there are many structures designed with the same shape as the wire mesh. The results found are satisfactory and consistent with practice.

Hence, teaching mathematical modelling in the classroom would help students develop the ability to apply mathematics to real-life problems. The students have ability to take mathematics out of the classroom by using real-world context as a key component of the modeling process. In other words, the students can make the transition from the real environment to the math environment and vice versa. Consequently, it can be said that mathematical modeling is an approach to help the teachers create learning motivation, enhance interdisciplinary and applicability of mathematics in learning and teaching high school mathematics.

4. Conclusions

The modelling teaching is still quite new for teachers when teaching mathematics in high schools in Lao PDR and there have not been many studies on the application of this approach in teaching and learning mathematics at high schools. A number of recent studies in some countries have shown the role of modelling in teaching mathematics in helping students become familiar with the use of different types of data representation, solve realistic problems by selecting and using appropriate mathematical tools and methods. This research also shows that this method helps students' mathematics learning become more meaningful through teaching activities that clarify mathematical elements in real life. In particular, the modelling method helps to improve the spirit of cooperation in learning, enhances the independence and confidence of students through group exchanges, and uses teaching mathematical software to support the problem-solving process, modelling and improving the realistic matching model. These results will be the basis for further studies on the possibility of using modelling method in teaching mathematics in high schools, especially the teaching approach to bring practical problems into educational curriculum and mathematics textbooks.

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