OPTIMAL CONTROLLER DESIGN FOR ACTIVE SUSPENSION SYSTEM ON CARS

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ABSTRACT

Suspension system is one of the most important parts when designing a car, playing a key element in comfort of drivers and passengers (comfort criteria) and keep the contact of the tyres with road surface (road holding criteria). This paper presents a two-degree-of-freedom quarter car model using active suspension system with two optimal controllers: Linear Quadratic Regulator and Linear Quadratic Gaussian. By using the Kalman-Bucy observer, the number of sensors used to measure the input signals of the linear quadratic regulator controller has been minimized to only conventional sensors such as the sprung mass acceleration. In order to evaluate the effectiveness, the comfort and road holding criteria when using those controllers are compared to the case of the passive suspension system through the sprung mass displacement and its acceleration. The simulation results clearly show that the root mean square value of the sprung mass acceleration with the linear quadratic regulator, linear quadratic gaussian controllers has been reduced by about 20% when compared to a car using a passive suspension system.

Keywords: *Vehicle dynamics; suspension system; active suspension system; optimal control; Kalman observer design*

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THIẾT KẾ BỘ ĐIỀU KHIỂN TỐI ƯU CHO HỆ THỐNG TREO TÍCH CỰC TRÊN Ô TÔ

Vũ Văn Tấn
Trường Đại học Giao thông Vận tải

TÓM TẮT

Hệ thống treo là một trong những bộ phận quan trọng nhất trong thiết kế ô tô và là yếu tố quyết định đến sự thoải mái của lái xe, hành khách (độ êm dịu) và giữ được bám giữa lốp và mặt đường (độ an toàn). Bài báo này giới thiệu một mô hình ¼ ô tô có 2 bậc tự do sử dụng hệ thống treo chủ động với hai bộ điều khiển tối ưu: linear quadratic regulator và linear quadratic gaussian (linear quadratic regulator kết hợp với bộ quan sát Kalman-Bucy). Bằng cách sử dụng bộ quan sát Kalman-Bucy, số lượng cảm biến dùng để đo đạc các tín hiệu đầu vào của bộ điều khiển linear quadratic regulator đã được giảm thiểu tối đa chỉ còn các cảm biến thông thường như gia tốc của khối lượng được treo. Độ êm dịu và an toàn chuyển động khi ô tô sử dụng hệ thống treo chủ động được so sánh với ô tô sử dụng hệ thống treo bị động thông thường thông qua dịch chuyển của khối lượng được treo và gia tốc của nó. Kết quả mô phỏng đã thể hiện rõ giá trị sai lệch bình phương trung bình của gia tốc dịch chuyển thân xe với hệ thống treo tích cực điều khiển tối ưu linear quadratic regulator, linear quadratic gaussian đã giảm khoảng 20% so với ô tô sử dụng hệ thống treo bị động.

Từ khóa: *Động lực học ô tô; hệ thống treo; hệ thống treo chủ động; điều khiển tối ưu; thiết kế bộ quan sát Kalman*

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1. Introduction

In 1770, the first car running by steam engine was introduced to public by Nicolas Joseph Cugnot. As the world is developing rapidly over the last centuries, the automobiles have become more helpful and necessary for transportation and trading, contributing to the increase of competitiveness in car trade market. Therefore, car producers around the world need to improve the performance of their products in order to occupy more market share. Apart from increasing the power of the car, refinements of comfort and safety of the vehicles are also taken into consideration. An efficient suspension system plays a vital role in passenger comfort and road holding. Almost of modern cars manufactured recently are equipped with a passive suspension system. However, the coefficients of spring stiffness and damping of this type are designed to be nearly constant or negligibly change within a narrow range, meaning that the passive suspension system tends to be inadequate and not adaptable to some infrequent situations of traveling on the roads. As a result, an active suspension system with optimal controller is becoming popular to apply on luxurious vehicles to minimize possibilities of collision between the tyres and the vehicle frame and extinguishing quickly the fluctuation of the vehicles when moving, leading to enhancements of ride comfort and road holding criteria.

In fact, there are a large number of works discussing thoroughly about the active suspension system. Due to the high energy consumption characteristics, most active suspension systems are used in cars. Therefore, most of the types of car models have been used in the studies of this system such as quarter, pitch, roll and full car models [1]. In addition, the actuator's properties were also taken into consideration in building the integrated model [2]. There are many control methods that have been applied to the active suspension system and are summarized in some relevant studies as follows: Yoshimura et al. presented the construction of an active suspension control of a one-wheel car model using fuzzy reasoning and a disturbance observer [3]. Mouleeswaran et al. presented work aiming at developing an active suspension system for the quarter car model of a passenger car to improve its performance by using a proportional integral derivative (PID) controller [4]. Advanced control methods have also been applied to the active suspension system such as robust control [5], sliding mode control [6], linear parameter varying control [7], and even nonlinear control methods [8].

The Linear Quadratic Regulator (LQR) control method is also used for the active suspension system [9]-[12]. In all the studies mentioned above, the authors focus mostly on improving the ride comfort and a few priorities on road safety. However, a major difficulty in applying this control method is that the number of sensors is often very large because it covers all the state variables in the state vector of the system, thus leading to the cost of the system often increases a lot. One of the most solution to overcome the drawback of the LQR control method is that it can combine with one observer in order to receive the equivalent signals for the control inputs. An Linear Quadratic Gaussian (LQG) is established by combining an LQR controller with a Kalman observer. Research article of Hui Pang and his colleagues investigated LQG control design for active suspension without considering road input signals [13].

On the basis of studies that have been done for the active suspension system, in this study the author performs the following main tasks: 1) The two optimal controllers LQR and LQG are synthesized to improve the quality of car vibrations through the ride comfort (sprung mass displacement and its acceleration), the road safety (unsprung displacement) and the suspension travel. By considering the above criteria at the same time, the controllers have met the control design goal. 2) The LQG controller is designed on the basis of the designed LQR controller and a Kalman-Bucy observer. Instead of using four sensor signals that are difficult to do in real cars, two sensors are used here for the acceleration of sprung and unsprung masses. Using the Kalman-Bucy observer in this manner allows it to be easily applied to more complex models and to actual cars while ensuring the control quality. Therefore, one of the outstanding advantages of this paper is to present an interesting and practical idea to be able to perform the optimal control method for the active suspension with the full car model and actual cars.

2. Vehicle modelling

In the areas of designing and researching to refine the performance of automobiles, there are three types of vehicle model that are regularly used: full model, half model and quarter model. Even though a full vehicle model captures almost essential dynamic features of an automobile, this article will only consider the quarter model (Figure 1) in order to be simplified. Specifically, with a model of ¼ vehicle, it is assumed that the effects of movements of passengers and vibration of engine are ignored, thereby, the unique disturbance is the road roughness. Additionally, this model has two degrees of freedom: (1) vertical motion of sprung mass stood by Z_S and (2) vertical motion of the unsprung mass expressed by Z_u. Based on the model, the position, velocity and acceleration of sprung mass, tyre and suspension space can be identified. However, as to achieve the simplicity of calculation and analysis, the ¼ vehicle model takes into account of the vertical dynamics with the road profile as the only source disturbance. Table 1 given below shows the symbols of parameters associated to the considered model.

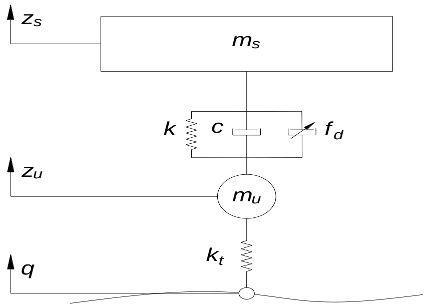

Figure 1. *Quarter vehicle model*

Based on Newton's Second Law, the motion differential equations of the quarter vehicle model are formalized as follow:

$$\begin{cases} m_s\ddot{z}_s = -c\left(\dot{z}_s - \dot{z}_u\right) - k\left(z_s - z_u\right) - f_d \\ m_u\ddot{z}_u = c\left(\dot{z}_s - \dot{z}_u\right) + k\left(z_s - z_u\right) - k_t\left(z_u - q\right) + f_d \end{cases} \quad (1)$$

Equations (1) can be transformed to state-space representation:

$$\begin{cases} \dot{X} = A.X + B_1.W + B_2.U \\ Y = C.X + D_1.W + D_2.U \end{cases} \quad (2)$$

With the state vector $X = \left[Z_s \quad Z_u \quad \dot{Z}_s \quad \dot{Z}_u\right]^T$, the exogenous disturbance $W = \left[q\right]$, the control input $U = \left[f_d\right]$ and the output vector $Y = \left[\ddot{Z}_s \quad Z_s \quad Z_u \quad Z_s - Z_u\right]$. It should be emphasized here that the choice of output signals like this allows considering simultaneously the most important criteria in the study of car oscillation. For other purposes, it is entirely possible to choose other output signals, depending on the designer.

These matrices below are results of the combination of equations (1) and (2):

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{-k}{m_s} & \dfrac{k}{m_s} & \dfrac{-c}{m_s} & \dfrac{c}{m_s} \\ \dfrac{k}{m_u} & \dfrac{-\left(k+k_t\right)}{m_u} & \dfrac{c}{m_u} & \dfrac{-c}{m_u} \end{bmatrix};\ B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{k_t}{m_u} \end{bmatrix};$$

$$B_2 = \begin{bmatrix} 0 \\ 0 \\ \dfrac{-1}{m_s} \\ \dfrac{1}{m_u} \end{bmatrix};\ C = \begin{bmatrix} \dfrac{-k}{m_s} & \dfrac{k}{m_s} & \dfrac{-c}{m_s} & \dfrac{c}{m_s} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix};$$

$$D_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \ D_2 = \begin{bmatrix} \frac{-1}{m_s} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Table 1. *Symbols and parameters of quarter vehicle model* [9]

Description	Symbols	Value	Unit
Sprung mass	ms	500	kg
Unsprung mass	mu	50	kg
Stiffness coefficient	k	20000	N/m
Stiffness of tyre	kt	120000	N/m
Damping coefficient	c	1000	Ns/m

3. Observer design

In this section, a Kalman observer is developed. The input *W* (the sensor signals) is considered. The main purpose of Kalman observer in this article is to estimate unknown signals. The Kalman filter (KF) is a discrete filter over time. In fact, many cases require estimating state parameters that are not able to design a continuous filter over time to change the KF filter to continuously calculate the system's state parameters, Kalman-Bucy Filter (KBF) is the continuous filter over time of KF filter.

Figure 2 depicts a linear system that changes continuously over time with the process noise vector *w(t)* and the measurement noise *v(t)* (assuming the normal Gaussian distribution rules with zero and ghosts). The covariance matches are Q and R, input vector *u(t)*, state vector *x(t)* (observable but not measurable; actual output vector of the *y(t)* and the measured output vector $\tilde{y}(t)$.

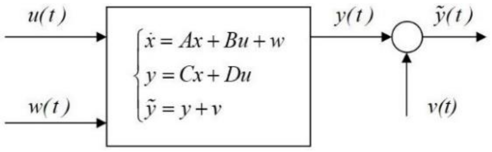

Figure 2. *Linear system changes continuously over time with noise and measurement noise*

Given the input parameters, measurable output and process noise assumptions, the purpose of the KBF filter is to identify non-zero state parameters (assuming they are observable) and actual output vector. Estimates of the state and output vectors of the KBF filter are depicted in Figure 3.

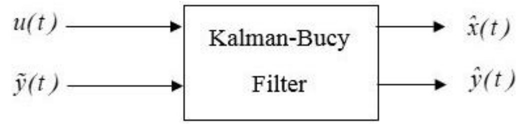

Figure 3. *Input and output signals of Kalman-Bucy Filter (KBF)*

Different from the KF filter that uses "Prediction" and "Correction" algorithms, the KBF filter requires Riccati differential equations to be continuously integrated over time. Mathematically, the filter update equations are represented as follows:

$$\begin{cases} K = PC^T R^{-1} \\ \dot{\hat{x}} = A\hat{x} + Bu + K[\tilde{y} - (C\hat{x} + Du)] \\ \hat{y} = C\hat{x} + Du \\ \dot{P} = AP + PA^T - PC^T R^{-1} CP + Q \end{cases} \tag{3}$$

where P: an estimate of covariance of the measurement error; K: Kalman - Bucy gain; R: weight matrix (covariance matrix) of measurement noise; Q: weight matrix (covariance matrix) of process noise (state).

After several tests based on the simulation model presented above, the values of the matrices Q, R for Kalman-Bucy filter estimates the inertial parameters of the car body selected as follows:

$$Q_n = diag\left(\begin{bmatrix} 10^{-1} & 10^{-5} & 10^{-4} & 10^{-1} \end{bmatrix}\right)$$

$$R_n = diag\left(\begin{bmatrix} 10^{-2} & 10^{-3} \end{bmatrix}\right)$$

4. Optimal controller design

The objectives of optimal control systems are to improve comfort and road holding performance of the vehicles. This section will present two types of optimal controller: Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG).

4.1. LQR controller design

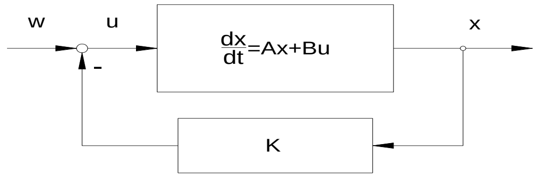

Figure 4. *Feedback controller design diagram*

The LTI model is described by equation:

$$\begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = Cx + D_1 w + D_2 u \end{cases} \quad (4)$$

For the controller design, it is supposed that all the states are available from measurements or can be estimated. Then, consider the state feedback control law [14]:

$$u = -Kx \quad (5)$$

where K is the state feedback gain matrix. The optimization procedure consists of determining the control input u which minimizes some performance index J. This index includes the performance characteristic requirement as well as the controller input limitations, usually expressed by:

$$J = \int_0^\infty (x^T Qx + u^T Ru + 2x^T Nu)dt \quad (6)$$

where Q, R, and N are positive definite weighting matrices. To achieve a solution for the optimal controller (5), the LTI system must be stablisable, which is true for the system (4). The gain K minimizing (6) has the following form:

$$K = R^{-1} B^T P \quad (7)$$

where the matrix P is the solution of the algebraic Riccati equation:

$$AP + A^T P - PBR^{-1} B^T P + Q = 0 \quad (8)$$

The optimal closed-loop system is obtained from Equations (4), (5) and (7) as follows:

$$\dot{x} = (A - B_2 K)x + B_1 \mathrm{w} \quad (9)$$

***Remark 01**:* The choice of the state vector x and the control input u will greatly affect the finding of matrices Q, R, N.

Since the purposes of the optimal controllers are to enhance comfort and road holding of the vehicles, which are mentioned above, the index J is chosen as below:

$$J = \int_0^\infty \left(\rho_1 \ddot{Z}_s^2 + \rho_2 Z_s^2 + \rho_3 Z_u^2 + \rho_4 f_d^2 \right) dt \quad (10)$$

Where $\ddot{Z}_s$, Z_s are criteria to evaluate comfort, and Z_u is representative for road holding criteria.

Remark 02: $\rho_1, \rho_2, \rho_3, \rho_4 \geq 0$ are the weighting parameters, impacting considerably the value of the index J. The values of weighting parameters show the preference to particular criteria. Specifically, if the comfort is preferred, ρ_1 and ρ_2 need increasing. Meanwhile, in case road holding is preference, the value of ρ_3 is necessary to be raised. The author would like to emphasize that with the LQR controller, the control input needs 4 variables of the state vector such as the sprung, unsprung masses displacement and their derivation.

4.2. LQG control design

However, such states of the system as displacements and velocities of the sprung mass and unsprung mass tend to be difficult to measure. Therefore, using Kalman filter to estimate the signals and combining them with LQR would form another type of optimal control design called Linear Quadratic Gaussian (LQG) as shown in Figure 5 [15], [16].

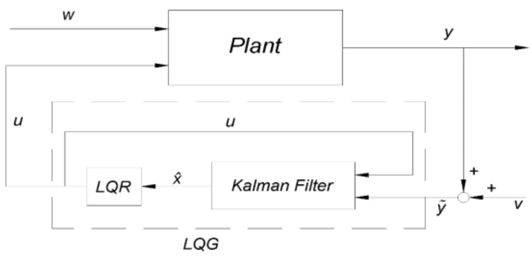

Figure 5. *LQG controller diagram*

This control method has the following state models:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Gw(t) \\ y(t) = Cx(t) + Du(t) + Hw(t) + v(t) \end{cases} \quad (11)$$

with $x(t)$ is state vector, $y(t)$ is output vector, $u(t)$ is input-manipulating vector, A and B are state matrices, C and D are output matrices, G and H are noise matrices, $w(t)$ is input noise vector, $v(t)$ is output noise vector.

Root of (11) is:

$$K(s) = \frac{U(s)}{Y(s)} = -K_c \left(sI - A + BK_c - K_f C - K_f DK_c \right)^{-1} K_f \quad (12)$$

Remark 03: The author would like to emphasize that with the LQG controller, the control input needs only 2 signals such as the sprung, unsprung masses acceleration, which are easy to measure by the normal sensors. This approach is very practical for the active suspension system on real cars and there is a big difference compared to the studies mentioned above.

5. Simulation results analysis

In this paper, the system is simulated by road profile of sine wave and step, and there are comparisons among the simulation results in the three cases: LQR, LQG and Passive systems. Those results are displayed in the time domain in the figures below. Looking at the Figure (6), it is can be seen that the shapes of the LQG controller in both cases are really close to the case of LQR controller. Moreover, Table (2) shows that the values of Root-Mean-Square of the two control designs are also not much of difference. Both two models with controllers produce signals representing for comfort performance and road holding that are superior to those of model with passive suspension system. Technically, the LQR case uses x(t) - ideal signal while the LQG case uses signals from Kalman Observer. Therefore, it is understandable and acceptable with the result that LQR is more efficient than LQG. However, when it comes to reality, the LQG controller requires fewer sensors needing to be equipped in vehicles than the LQR controller so it is considered being more suitable to apply on real vehicles.

Table 2. *Root-Mean-Square of* $\ddot{Z}_s, Z_s, Z_u, Z_s - Z_u$

	$\ddot{Z}_s$	Z_s	Z_u	$Z_s - Z_u$
Passive	0,8736	0,0225	0,0165	0,0151
LQR	0,6899	0,0190	0,0161	0,0130
LQG	0,6929	0,0192	0,0160	0.0131

According to the values in Table 2, the comparisons of RMS between the two controllers: LQR and LQG with the passive suspension system are shown in Figure 7. Here, the author considers the signal values in the case of the passive system as 100%. We can see that the difference of RMS in the case of LQG controller is insignificant, compared to LQR controller. Therefore, the use of the LQG controller by combining the LQR controller and Kalman-Bucy observer is suitable for satisfying the control objectives, as well as adapting for the application on real vehicles.

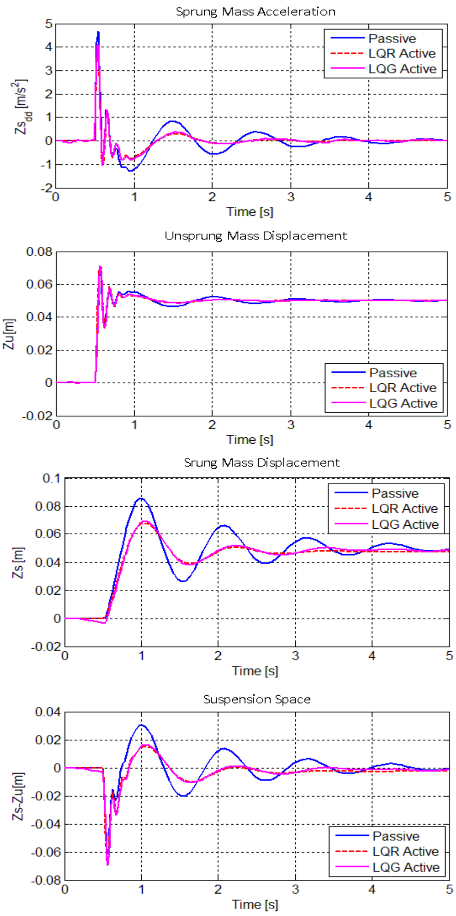

Figure 6. *Time response of* $\ddot{Z}_s, Z_s, Z_u, Z_s - Z_u$ *with the step road profile*

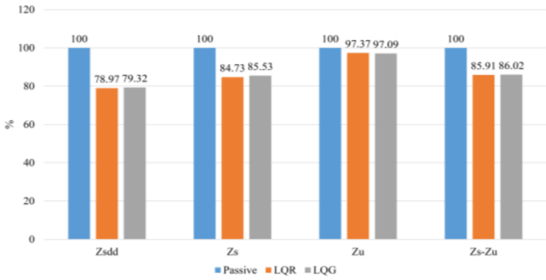

Figure 7. *RMS of* $\ddot{Z}_s, Z_s, Z_u, Z_s - Z_u$ *with the step road profile*

6. Conclusion

This investigation has demonstrated the effectiveness of using optimal controller designs in suspension system on vehicles by comparing vertical displacement, acceleration of sprung and unsprung masses in various road profile situations. The two optimal controllers LQR and LQG are synthesized to improve the quality of car vibrations through the ride comfort, the road safety and the suspension travel. The LQG controller is designed on the basis of the designed LQR controller and a Kalman-Bucy observer. The acceleration of sprung and unsprung masses are the only two control input signals. The obtained results have shown that optimal regulators are able to improve the performance of comfort and road holding criteria of the cars. Based on this simulation results, it is an interesting and practical idea to be able to perform the optimal control method for the active suspension with the full car model and actual cars.

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REFERENCES

[1]. R. Darus, and Y. Md Sam, "Modeling and control active suspension system for a full car model," IEEE 5th International Colloquium on Signal Processing & Its Applications, Kuala Lumpur, Malaysia, 2009.

[2]. X. Shen, and H. Peng, "Analysis of Active Suspension Systems with Hydraulic Actuators," *Vehicle System Dynamics*, vol. 41, pp. 143-152, 2014

[3]. M. S. Leegwater, "An active suspension system," PhD thesis, Eindhoven University of Technology, 2007.

[4]. S. Mouleeswaran, "Development of Active Suspension System for Automobiles using PID Controller," *Proceedings of the World Congress on Engineering*, London, UK, 2008, pp. 1-6.

[5]. W. Sun, and Z. Zhao, "Saturated Adaptive Robust Control for Active Suspension Systems," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 9, pp. 3889-3896, 2013.

[6]. A. J. Koshkouei, and K. J. Burnham, "Sliding Mode Controllers for Active Suspensions," *IFAC Proceedings Volumes*, vol. 41, no. 2, pp. 3416-3421, 2008.

[7]. P. Gaspar, I. Szaszi, and J. Bokor, "Active Suspension Design using LPV Control," *IFAC Proceedings Volumes*, vol. 37, no. 22, pp. 565-570, 2004.

[8]. M. Herrnberger, and B. Lohmann, "Nonlinear Control Design for an Active Suspension using Velocity-Based Linearisations," *IFAC Proceedings Volumes*, vol. 43, no. 7, pp. 330-335, 2010.

[9]. C. Q. Nguyen, "Design optimal controller for Active Suspension System on Automobiles," MA Thesis, University of Transport and Communications, 2019.

[10]. M. Kaleemullah, W. F. Faris, and F. Hasbullah, "Design of robust H_∞, fuzzy and LQR controller for active suspension of a quarter car model," 4th International Conference on Mechatronics (ICOM), Kuala Lumpur, Malaysia, 2011.

[11]. M. P. Nagarkar, G. J. Vikhe, K. R. Borole, and V. M. Nandedkar, "Active control of quarter car suspension system using linear quadratic regulator," *International Journal of Automotive and Mechanical Engineering*, vol. 3, pp. 364-372, 2011.

[12]. M S. Kumar, and S. Vijayarangan, "Design of LQR controller for active suspension system," *Indian journal of engineering & materials sciences*, vol. 13, pp. 173-179, 2006.

[13]. H. Pang, Y. Chen, J. Chen, and X. Liu, "Design of LQG Controller for Active Suspension without considering road input signals," *Shock and Vibration Journal*, vol. 2017, pp. 1-13, 2017.

[14]. Y. Taghirad, and E. Esmailzadeh, "Automobile passenger comfort assured through LQG/LQR active suspension," *Journal of Vibration and Control*, vol. 4, no. 5, pp. 603-618, 1997.

[15]. S. Chen, Y. Cai, J. Wang, and M. Yao, "A Novel LQG Controller of Active Suspension System for Vehicle Roll Safety," *International Journal of Control, Automation and Systems*, vol. 16, no. 5, pp. 2203-2213, 2018.

[16]. H. Li, C. yin Tang, and T. xia Zhang, "Controller of vehicle active suspension systems using LQG method," IEEE International Conference on Automation and Logistics, Qingdao, China, 2008.