A LIOUVILLE TYPE THEOREM FOR KIRCHHOFF ELLIPTIC EQUATIONS INVOLVING Δ_λ OPERATORS

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Abstract: *In this paper, we study the Kirchhoff elliptic equations*

$$-M\left(\left\|\nabla_\lambda u\right\|^2\right)\Delta_\lambda u = -u^{-p}w(x) \text{ in } \mathbb{R}^N,$$

where Δ_λ *is the degenerate* Δ_λ *operator in* $\mathbb{R}^N, N \geq 3$, *the exponent* $p > 0$, *and* $w(x)$ *is a weight function. We establish a Liouville type theorem for the class of continuous positive stable solutions. In particular, our results improve existing results in [11] and [13].*

Keywords: *Liouville type theorem,* Δ_λ *operator, Stable solutions, Kirchhoff equations.*

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1. INTRODUCTION

In this paper, we are interested in the classification of stable classical solutions of the following elliptic equations of Kirchhoff type

$$-M\left(\left\|\nabla_\lambda u\right\|^2\right)\Delta_\lambda u = -u^{-p}w(x) \text{ in } \mathbb{R}^N, \tag{1.1}$$

where the exponent $p > 0$, $N \geq 3$, $\left\|\nabla_\lambda u\right\|^2 = \int_{\mathbb{R}^N}|\nabla_\lambda u|^2 dx$, and the function $M(t): \mathbb{R}^+ \to \mathbb{R}$ is a smooth monotone function such that

$$\gamma := \sup_{t\geq 0}\frac{tM'(t)}{M(t)} < \infty. \tag{1.2}$$

The weight $w(x) = O(|x|^\alpha)$ as $x \to \infty$ for some real number α. Here Δ_λ is the degenerate elliptic operator of the form

$$\Delta_\lambda = \sum_{j=1}^{N}\frac{\partial}{\partial x_j}\left(\lambda_j^2(x)\frac{\partial}{\partial x_j}\right) \tag{1.3}$$

where $x = (x_1, \dots, x_N) \in \mathbb{R}^N$ and the functions $\lambda_j: \mathbb{R}^N \to \mathbb{R}$ satisfies certain further conditions, see Section 2. We denote by Q the homogeneous dimension of $\mathbb{R}^N$with respect to the group of dilation $\{\delta_t\}_{t>0}$, $Q = \epsilon_1 + \cdots + \epsilon_N$, where ϵ_i is homogeneous degree of λ_j with respect to the group of dilation δ_t.

Let us first recall the existing results concerning this class of equations. In the case $\lambda_j \equiv 1$ for all $1 \leq j \leq N$, the operator Δ_λ is the classical Laplace operator Δ. Particularly, when the Kirchhoff function $M \equiv 1$, Problem (1.1) reduces to

$$-\Delta u(x) = -\,u(x)^{-p} w(x), x \in \mathbb{R}^N. \tag{1.4}$$

The equation (1.4) arises in many branches of applied sciences and has been studied in a number of recent works, see [11], [4] and the references therein. The nonexistence of positive stable classical solutions of the problem (1.4) was obtained in [11] with the weight $w \equiv 1$. This result was then generalized in [4] for positive stable weak solutions of a weighted equation. More precisely, the authors in [4] figured out the critical exponent and established an optimal Liouville type theorem for this class of solutions.

When $M \not\equiv 1$, the Kirchhoff equation containing the term $M\left(\int |\nabla u|^2 dx\right)$ has importantly meaningful applications. In the one dimensional case, the nonstationary equation

$$\rho u_{tt} - \left(\frac{P_0}{h} + \frac{E}{2L}\int_0^L |u_x|^2\, dx\right) u_{xx} = 0$$

describes the nonlinear vibration of a string when considering the effects of the changes in the length of the string. Moreover, Kirchhoff type equations also appear in other fields as, for example, biological systems where u describes a process depending on the average of itself (for instance, population density).

When studying the Kirchhoff equation with a nonlinearity with weights, based on Farina's approach in [7] and the technique in [2], Y. Wei and his collaborators in [10] assume a power lower bound near infinity for the weight $w(x)$ as follows

(**W**) The function $w \in C(\mathbb{R}^N)$ is non-negative and there exists constant $\alpha > -2$; $R_0, c_0 > 0$ such that

$$w(x) \geq\; c_0 |x|^\alpha \text{ for all } x \in\; \mathbb{R}^N, |x| \geq\; R_0.$$

Consider a special case of Kirchhoff operator:

$$M\colon [0,\infty) \to\; \mathbb{R}\,, t \mapsto M(t) = a + bt, a > 0, b \geq\; 0.$$

Wei and his collaborators proved the nonexistence of stable solutions as stated below.

Theorem 1. [13, Theorem 1.2] Assume that the weight w satisfies **(W)** and one of the following conditions

(A_1) $a > 0, b = 0$ and $2 \leq\; N < 10 + 4\alpha,\; p > 1$;

(A_2) $a > 0, b = 0$ and $N > 10 + 4d, 1 < p < \alpha_0$;

(A_3) $a > 0, b > 0$ and $2 \leq\; N < 4 + \alpha, p > 3$;

(A_4) $a > 0, b > 0$ and $4 + \alpha < N < 2 + \frac{4(2+\alpha)}{3}, p > \beta_0$;

(A_5) $a > 0, b > 0$ and $N = 2 + \frac{4(2+\alpha)}{3}, p > 4$;

(A_6) $a > 0, b > 0$ and $2 + \frac{4(2+\alpha)}{3} < \alpha < 2 + \frac{(1+\sqrt{3})(2+\alpha)}{2}, \beta_1 < p < \beta_2$;

where

$$\alpha_0 = 1 + \frac{2(2+\alpha)\left[N - 4 - \alpha + \sqrt{(2+\alpha)(2(N-1)+\alpha)}\right]}{(N-2)(N-10-4\alpha)}$$

$$\beta_0 = 1 + \frac{2(2+\alpha)\left[N - 4 - \alpha + \sqrt{(N-4-\alpha)^2 - (N-2)(3N-14-4\alpha)}\right]}{(N-2)(3N-14-4\alpha)}$$

$$\beta_{1,2} = 1 + \frac{2(2+\alpha)\left[N - 4 - \alpha \pm \sqrt{(N-4-\alpha)^2 - (N-2)(3N-14-4\alpha)}\right]}{(N-2)(3N-14-4\alpha)}$$

Then the equation

$$\left(-a - b\left||\nabla_\lambda u|\right|^2\right)\Delta_\lambda u = |u|^{p-1}uw(x) \text{ in } \mathbb{R}^N,$$

has no nontrivial stable solution.

Let us remark that, Theorem 1 describes the critical phenomenon, namely, the dimension is bounded by some constant determined by the weights.

To the best of our knowledge, the problem (1.1) involving more general Kirchhoff function corresponding the degenerate ∇_λ structure has not been investigated in the literature. Therefore, in this paper we aim at generalizing the known results in [2], [3], [11] to the problem (1.1).

The main result if the present work is the nonexistence of continuous stable positive solutions of the equation (1.1).

Theorem 2. Assume that $Q > 2, p > 0$ and the conditions (**W**), (1.2) hold. If

$$Q < 2 + \frac{2(2+\alpha)}{(1+2\gamma)(p+1)}\left(p + \sqrt{p^2 + (1+2\gamma)p}\right), \tag{1.5}$$

then Equation (1.1) has no nontrivial continuous positive stable weak solutions.

Our result generalizes the case $\lambda_j(x) = 1, 1 \leq j \leq N$, and the Kirchhoff function $M(t) = a + bt, a > 0, b \geq 0$ in Theorem 1.

As a special case, when $M(t) \equiv 1$ $(\gamma = 0)$and $w\,(x) \equiv 1$ $(\alpha = 0)$, we obtain the following corollary of Theorem 2.

Corollary 3. Assume that $Q > 2, p > 0$. If

$$Q < 2 + \frac{4}{p+1}\left(p + \sqrt{p^2 + p}\right), \tag{1.6}$$

then Equation (1.1) has no nontrivial continuous positive stable weak solutions.

We recover the results in [5, Corollary 1.3] and [11, Theorem 6].

2. PRELIMINARIES

2.1. The Δ_λ-operators and related functional setting

Following [9], Δ_λ is the second order differential operators associated with the continuous functions $\lambda_j: \mathbb{R}^N \to \mathbb{R}$ which satisfies

(H_1) There exists a group of dilation $\{\delta_t\}_{t>0}$, such that

$$\delta_t: \mathbb{R}^N \to \mathbb{R}^N, \delta_t(x_1, \dots, x_N) = (t^{\epsilon_1}x_1, \dots, t^{\epsilon_N}x_N).$$

where $1 \le \epsilon_1 \le \cdots \le \epsilon_N$.

The dilation induces the pullback on functions $\delta_t^* \varphi(x) = \varphi(\delta_t x), \forall \varphi$. The function λ_i is δ_t-homogeneous of degree $\epsilon_i - 1$, i.e.,

$$\lambda_i(\delta_t x) = t^{\epsilon_i - 1}\lambda_i(x), \text{for all } x \in \mathbb{R}^N, t > 0, i = 1,2, \dots, N.$$

(H_2) The functions λ_i satisfy $\lambda_1(x) = 1$ and $\lambda_j(x) = \lambda_j(x_1, \dots, x_{j-1}), j = 2, \dots, N$. Moreover, the functions λ_i are continuous on $\mathbb{R}^N$, strictly positive and of class C^2 on $\mathbb{R}^N \setminus \Pi$ with

$$\Pi = \left\{ (x_1, x_2, \dots, x_N) \in \mathbb{R}^N; \prod_{i=1}^{N} x_i = 0 \right\}.$$

(H_3) There is a constant $r > 0$ such that

$$0 \le x_k \frac{\partial \lambda_j(x)}{\partial x_k} \le r\lambda_j(x)$$

for all $1 \le k \le j - 1, i = 2,3, \dots, N$ and $x \in (\mathbb{R}_+)^N$.

These assumptions allow us to write

$$-\Delta_\lambda = \nabla_\lambda^* \nabla_\lambda,$$

where the λ-gradient which consists of N $-$vector fields

$$\nabla_\lambda = \left(\lambda_1(x)\partial_{x_1}, \lambda_2(x)\partial_{x_2}, \dots, \lambda_N(x)\partial_{x_N}\right).$$

Consequently, Δ_λ is homogeneous of degree 2 with respect to δ_t in the sense that

$$\Delta_\lambda(\delta_t^* u) = t^2\delta_t^* (\Delta_\lambda u), \forall u \in C^\infty(\mathbb{R}^N).$$

We define $W_\lambda^{1,2}(\mathbb{R}^N)$ the closure of $C_c^\infty(\mathbb{R}^N)$ with respect to the norm

$$||u||_{W_\lambda^{1,2}}^2 = \int_{\mathbb{R}^N} (|\nabla_\lambda u|^2 + u^2) dx.$$

The rest of this paper is devoted to the proof of our main result.

2.2. Stable solutions

We recall the concept of stable solutions of an equation with general nonlinearity $f(x,u)$.

Definition 4. A function $u \in W_\lambda^{1,2}(\mathbb{R}^N)$ is called a weak solution to the equation

$$-M\left(||\nabla_\lambda u||^2\right)\Delta_\lambda u = -u^{-p}w(x) \text{ in } \mathbb{R}^N, \tag{2.1}$$

if it satisfies

$$M\left(||\nabla_\lambda u||^2\right)\int_{\mathbb{R}^N} \nabla_\lambda u \cdot \nabla_\lambda \varphi dx - \int_{\mathbb{R}^N} f(x,u)\varphi dx = 0, \tag{2.2}$$

for all $\varphi \in C_c^1(\mathbb{R}^N)$.

Definition 5. ([2], [10]) A weak solution u of the equation (2.1) is said to be *stable* if the following inequality holds

$$M\left(||\nabla_\lambda u||^2\right)\int_{\mathbb{R}^N} |\nabla_\lambda \varphi|^2 dx + 2M'\left(||\nabla_\lambda u||^2\right)\left(\int_{\mathbb{R}^N} \nabla_\lambda u \cdot \nabla_\lambda \varphi dx\right)^2 \int_{\mathbb{R}^N} f_u'(x,u)\varphi dx \geq 0 \tag{2.3}$$

for all $\varphi \in C_c^1(\mathbb{R}^N)$.

Because $C_c^1(\mathbb{R}^N)$ is dense in $W_\lambda^{1,2}(\mathbb{R}^N)$, the equality and inequality above are also satisfied for all $\varphi \in W_\lambda^{1,2}(\mathbb{R}^N)$.

Our equation has a natural variational structure, therefore, we can introduce the energy functional

$$E(u) = \frac{1}{2}\mathcal{M}\left(||\nabla_\lambda u||^2\right) - \int_{\mathbb{R}^N} F(x,u)\varphi dx, \tag{2.4}$$

where $F(x,u) = \int_0^u f(x,s)ds$ and $\mathcal{M}(t) = \int_0^t M(s)ds$.

It is easy to check, $E(u) \in C^2\left(W_\lambda^{1,2}(\mathbb{R}^N)\right)$ and

$$E'(u)(\varphi) = M\left(||\nabla_\lambda u||^2\right)\int_{\mathbb{R}^N} \nabla_\lambda u \cdot \nabla_\lambda \varphi dx - \int_{\mathbb{R}^N} f(x,u)\varphi dx.$$

$$(E''(u)\varphi,\varphi) = M\left(||\nabla_\lambda u||^2\right)\int_{\mathbb{R}^N} |\nabla_\lambda \varphi|^2 dx + 2M'^{\left(||\nabla_\lambda u||^2\right)\left(\int_{\mathbb{R}^N} \nabla_\lambda u\cdot\nabla_\lambda \varphi dx\right)^2}$$

$$- \int_{\mathbb{R}^N} f_u'(x,u)\varphi dx.$$

Hence, a weak solution u is a critical point of $E(u)$ and the stability of u means the energy functional is definitely semi-positive.

3. PROOF OF THE MAIN RESULT

For simplicity, we denote by $\int$ the integral $\int_{\mathbb{R}^N} dx$ and M the term $M\left(\left||\nabla_\lambda u|\right|^2\right)$. Let us begin by establishing a key estimate.

3.1. Proof of Theorem 2

Lemma 6. If u is a stable solution of Equation (2.1) then one has

$$(1+2\gamma)M\left(\left||\nabla_\lambda u|\right|^2\right)\int|\nabla_\lambda\varphi|^2 \geq \int f_u'(x,u)\varphi, \forall\, \varphi \in C_c^1(\mathbb{R}^N). \tag{3.1}$$

Since u is a weak stable solution, for every $\varphi \in C_c^1(\mathbb{R}^N)$ one has

$$\int f_u'(x,u)\varphi dx \leq M\left(\left||\nabla_\lambda u|\right|^2\right)\int|\nabla_\lambda\varphi|^2 + 2M^{'\left(\left||\nabla_\lambda u|\right|^2\right)(\int \nabla_\lambda u\cdot\nabla_\lambda\varphi)^2} \tag{3.2}$$

By Cauchy-Schwarz inequality, it implies

$$M'\left(\left||\nabla_\lambda u|\right|^2\right)(\nabla_\lambda u,\nabla_\lambda\varphi)^2 \leq M'\left(\left||\nabla_\lambda u|\right|^2\right)\left||\nabla_\lambda u|\right|^2\left||\nabla_\lambda\varphi|\right|^2 \leq$$
$$M\left(\left||\nabla_\lambda u|\right|^2\right)\left||\nabla_\lambda\varphi|\right|^2,$$

where the last inequalities follows from the assumption (1.2).

Substituting in (3.2) we obtain (3.1).

Lemma 7. Let $N \geq 3, p > 0$ and assume that (1.2) holds. If u is a continuous positive stable solution of (1.1), then for $t < \frac{1}{2}$ there exists a positive constant C depending on t such that

$$\left(\frac{p}{1+2\gamma} - \frac{t^2}{1-2t}\right)\int u^{2t-p-1}\psi^2 w(x) \leq C\int u^{2t}(|\Delta_\lambda(\psi^2)| + |\nabla_\lambda\,\psi|^2) \tag{3.3}$$

for all $\psi \in C_c^\infty(\mathbb{R}^N)$.

Proof. The proof of this proposition is based on that of [2, Proposition 1] where the author exploited the generalized Hardy inequality, see [1,3]. Let us also mention that the inequality of type (3.3) for the nonvariational case was first established in [3]. Let us now prove Lemma 7.

Suppose that u is a positive stable solution of (1.1). We first use the stability condition (3.1) with $f(u) = -u^{-p}w(x)$ and $\varphi = u^t\psi$.

$$p\int u^{2t-p-1}\psi^2 w \leq (1+2\gamma)M\left(\left||\nabla_\lambda u|\right|^2\right)\int|\nabla_\lambda\,(u^t\psi)|^2. \tag{3.4}$$

By direct computation, one has

$$\int |\nabla_\lambda (u^t\psi)|^2 = \int (t^2u^{2t-2}|\nabla_\lambda u|^2\psi^2 + 2tu^{2t-1}\nabla_\lambda u \cdot \nabla_\lambda \psi + u^{2t}|\nabla_\lambda \psi|^2). \qquad (3.5)$$

To estimate the first term on the right hand side of (3.5), we use the weak form of (1.1) with the test function $u^{2t-1}\psi^2$ to get

$$M\left(\left\||\nabla_\lambda u|\right\|^2\right)\int \nabla_\lambda u \cdot \nabla_\lambda\left(u^{2t-1}\psi^2\right) = -\int u^{-p+2t-1}\psi^2 w$$

$$\Rightarrow (2t-1)M\int |\nabla_\lambda u|^2 u^{2t-2}\psi^2 = -\int u^{2t-p-1}\psi^2 w - 2M\int u^{2t-1}\psi\nabla_\lambda u \cdot \nabla_\lambda \psi. \qquad (3.6)$$

Combining (3.5) and (3.6), one gains

$$M\int|\nabla_\lambda (u^t\psi)|^2 = \frac{t^2}{1-2t}\int u^{2t-p-1}\psi^2 w - M\frac{2t(t-1)}{1-2t}\int u^{2t-1}\psi\nabla_\lambda u \cdot \nabla_\lambda\psi + M\int u^{2t}|\nabla_\lambda\psi|^2. \qquad (3.7)$$

Combining (3.4), (3.7) and integrating the first term on the right hand side below, one gets

$$\begin{aligned}\left(\frac{p}{1+2\gamma} - \frac{t^2}{1-2t}\right)&\int u^{2t-p-1}\psi^2 w \\ &\le \frac{2t(1-t)M}{1-2t}\int u^{2t-1}\psi\nabla_\lambda u \cdot \nabla_\lambda\psi + M\int u^{2t}|\nabla_\lambda\psi|^2 \qquad (3.8)\\ &\le \frac{M(t-1)}{2(1-2t)}\int u^{2t}\Delta_\lambda(\psi^2) + M\int u^{2t}|\nabla_\lambda\psi|^2\end{aligned}$$

Once u is known, the right hand side of (3.8) is bounded from above by

$$C\int u^{2t}(|\Delta_\lambda(\psi^2)| + |\nabla_\lambda \psi|^2),$$

we obtain the estimate (3.3).

End of the proof of Theorem 2. Suppose that u is a positive stable solution of (1.1). Recall that

$$\gamma = \sup_{t\ge 0}\left\{\frac{tM'(t)}{M(t)}\right\} \ge 0.$$

It is easy to see that, if

$$-\frac{p+\sqrt{p^2+(1+2\gamma)p}}{1+2\gamma} < \mathrm{t} < 0, \qquad (3.9)$$

then

$$\frac{p}{1+2\gamma} - \frac{t^2}{1-2t} > 0.$$

Thus, for any t in the range (3.9), we obtain from (3.3) that there exists a positive constant C depending on t, p, N and γ which satisfies

$$\int u^{2t-p-1}\psi^2 w \leq C \int u^{2t}(|\Delta_\lambda(\psi^2)| + |\nabla_\lambda \psi|^2)\,. \tag{3.10}$$

Let us consider a function $\chi \in C_c^\infty(\mathbb{R};[0,1])$ such that $\chi = 1$ on $[-1,1]$ and $\chi = 0$ outside $[-2,2]$. For $\epsilon > 0$ small enough, we set $\varphi(x) = \chi(\delta_\epsilon x)$. By homogeneity, we have

$$\operatorname*{Sup}_{x\in\mathbb{R}^N}(|\Delta_\lambda(\varphi^2)| + |\nabla_\lambda \varphi|^2) \leq C\epsilon^2, \tag{3.11}$$

here and in what follows C denotes a generic positive constant which may change from line to line and is independent of ϵ.

We now replace ψ in (3.10) by φ^m, where $m > 1$ is chosen later, we induce that

$$\int u^{2t-p-1}\varphi^{2m}w(x) \leq C \int u^{2t}(|\Delta_\lambda(\varphi^{2m})| + |\nabla_\lambda(\varphi^m)|^2) \tag{3.12}$$

By simple computation

$$|\nabla_\lambda(\varphi^m)|^2 = m^2\varphi^{2m-2}|\nabla_\lambda\varphi|^2 \tag{3.13}$$

And

$$|\Delta_\lambda(\varphi^{2m})| \leq 2m(2m-1)\varphi^{2m-2}|\nabla_\lambda\varphi|^2 + m\varphi^{2m-2}|\Delta_\lambda(\varphi^2)|. \tag{3.14}$$

From (3.12), one gets

$$\int u^{2t-p-1}\varphi^{2m}w \leq C\int u^{2t}\varphi^{2m-2}(|\Delta_\lambda(\varphi^2)| + |\nabla_\lambda\varphi|^2). \tag{3.15}$$

Apply

$$\begin{aligned} &\int u^{2t-p-1}\varphi^{2m}w \\ &\leq C\int u^{2t}\varphi^{2m-2}(|\Delta_\lambda(\varphi^2)| + |\nabla_\lambda\varphi|^2). \int u^{2t}\varphi^{2m-2}(|\Delta_\lambda(\varphi^2)| \\ &+ |\nabla_\lambda\varphi|^2) \\ &\leq \left(\int u^{2t-p-1}\varphi^{\frac{(m-1)(2t-p-1)}{t}}w(x)\right)^{\frac{2t}{2t-p-1}} \\ &\times\left(\int (|\Delta_\lambda(\varphi^2)| + |\nabla_\lambda\varphi|^2)^{\frac{2t-p-1}{-p-1}} w(x)^{\frac{2t}{p+1}}\right)^{\frac{-p-1}{2t-p-1}} \end{aligned} \tag{3.16}$$

Let us choose m sufficiently large such that $\frac{(m-1)(2t-p-1)}{t} > 2m$. Hence, from (3.16), (3.15) and (3.11), we obtain

$$\begin{aligned}\int u^{2t-p-1}\varphi^{2m}w(x) &\leq C\int (|\Delta_\lambda(\varphi^2)| + |\nabla_\lambda\varphi|^2)^{\frac{2t-p-1}{-p-1}} w(x)^{\frac{2t}{p+1}} \\ &\leq C\epsilon^{\frac{2(2t-p-1)}{-p-1}-Q-\frac{2t\alpha}{p+1}}.\end{aligned} \tag{3.17}$$

Now, we choose t in the interval (3.9) such that the exponent on the right hand side of (3.17) is positive. Indeed,

$$\frac{2(2t-p-1)}{-p-1} - Q - \frac{2t\alpha}{p+1} > 0 \Leftrightarrow Q - 2 < (-t)\frac{2(2+\alpha)}{p+1}.$$

We can choose t sufficiently close to $-\frac{p+\sqrt{p^2+(1+2\gamma)p}}{1+2\gamma}$ which satisfies such a condition, because

$$Q - 2 < \frac{p + \sqrt{p^2 + (1+2\gamma)p}}{1+2\gamma} . \frac{2(2+\alpha)}{p+1} \tag{3.18}$$

which is just the condition (1.5).

For such t, letting $\epsilon \to 0^+$ in (3.17), we get a contradiction. Hence, there does not exists a nontrivial continuous positive stable solutions of Equation (1.1).

4. CONCLUSION

In the present work, we prove the nonexistence of continuous positive stable weak solutions of a class of Kirchhoff equations involving the Δ_λ operator. Our main achievement improves the result in Ma and Wei [11] for degenerate elliptic equations involving Δ_λ and nonlinearities with weights. At the same time, our result concerning nonlinearity with negative exponents and more general Kirchhoff function complements the recent results of Wei and collaborators [13], which contribute to the classification of solutions of Kirchhoff type equations with power nonlinearities.

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ĐỊNH LÝ KIỂU LIOUVILLE CHO PHƯƠNG TRÌNH KIRCHHOFF LOẠI ELIP CHỨA TOÁN TỬ Δ_λ

Tóm tắt: *Trong bài báo, chúng tôi nghiên cứu phương trình Kirchhoff loại elip*

$$-M\left(\|\nabla_\lambda u\|^2\right)\Delta_\lambda u = -u^{-p}w(x) \text{ in } \mathbb{R}^N,$$

trong đó Δ_λ là toán tử suy biến trên $\mathbb{R}^N, N \geq 3$, số mũ $p > 0$, và $w(x)$ là hàm trọng. Chúng tôi chứng minh một định lý kiểu Liouville cho lớp nghiệm ổn định dương liên tục. Nói riêng, kết quả của chúng tôi mở rộng kết quả trong [11] và [13].

Từ khóa: *Định lý kiểu Liouville, toán tử Δ_λ, nghiệm ổn định, phương trình Kirchhoff.*