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TỔNG HỢP, CẤU TRÚC MỘT SỐ KETONE A, Β- KHÔNG NO TỪ ACETONE VÀ ALDEHYDE

Tóm tắt: *Các ketone α,β- không no có hoạt tính rất đa dạng, như kháng khuẩn, chống nấm, diệt cỏ dại và trừ sâu, chống ung thư gan, phổi,... đã được đề cập trong nhiều công trình nghiên cứu [1,2,3,4,5,6,7]. Ngoài ra, các ketone α,β- không no còn là loại dẫn xuất có vai trò rất quan trọng trong việc tổng hợp các hợp chất chứa dị vòng có hoạt tính rất phong phú. Đã có nhiều nghiên cứu tổng hợp [1,2,3,4,5,6,7,...] và chuyển hóa chúng thành các hợp chất chứa dị vòng [8, 9, 10,..] cho thấy vai trò quan trọng của các ketone α,β- không no. Bài viết này giới thiệu kết quả nghiên cứu tổng hợp, cấu trúc một số ketone α,β- không no từ acetone và aldehyde.*

Từ khóa: *Xeton α, β- không no, phổ IR và phổ NMR, cấu trúc phân tử.*

CONSIDER SOME PRIMARY PROBLEMS FROM PERSPECTIVE OF ALGEBRAIC STRUCTURES

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Abstract: *This paper considers the nature of some primary problems from the perspective of the algebraic structure of groups, rings, fields, etc. Thereby explaining the rationale for the solution to those problems.*

Keywords: *Algebraic structure, primary problem, field extension, number set.*

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1. INTRODUCTION

In high school, we often study primary problems with basic solutions on arithmetic properties, equivalent transformations, or acknowledged methods. For example, when solving a product equation on the real number set $A(x).B(x) = 0$, we give the equivalent of solving two equations $A\ (x) = 0$ or $B\ (x) = 0$; or solving a first-degree equation with an unknown $ax = b$ where $a, b \in \mathbb{R}, a \neq 0$, we divide both sides by a and get the solution $x = \frac{b}{a}$. Besides, the number sets $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ and $\mathbb{C}$ also are studied, but the construction of them is not paying attention. Here we will consider the nature of the solutions of the above problems by the properties of algebraic structures such as group, ring, integer, and field properties of the set considered respectively.

2. CONTENT

First we will review some algebraic structures.

2.1. Group, ring and field

A *group* is a set G together with a binary operation, denote by (*), such that:

1) Associativity: For any $x, y, z \in G$, we have $(x * y) * z = x * (y * z)$.

2) Identity: There exists an $e \in G$ such that $e * x = x * e = x$ for any $x \in G$. We say that e is an identity element of G.

3) Inverse: For any $x \in G$, there exists a $y \in G$ such that $x * y = e = y * x$. We

say that y is an inverse of x.

A *ring* A is a set with two binary operations (addition and multiplication) such that:

i) A is an abelian group with respect to addition (so that A has a zero element, denoted by 0, and every $x \in A$ has an inverse, $-x$).

ii) Multiplication is associative: $(xy)z = x(yz)$ for all $x, y, z \in A$.

iii) Multiplication distributive over addition:

$$x(y+z) = xy + xz, (y+z)x = yx + zx \text{ for all } x, y, z \in A.$$

A *zero-divisor* in a ring A is an element x which "divides 0", i.e., for which there exists $y \neq 0$ in A such that $xy = 0$. In other hand, if $xy = 0$ then $x = 0$ or $y = 0$.

A ring A, which is commutative ($xy = yx$), has an identity element (denoted by 1) and has no zero-divisor (and in which $1 \neq 0$), is called an *integral domain*.

A *field* is an integral domain A in which every non-zero element is a unit.

A given binary relation ~ on a set *X* is said to be an *equivalence relation* if and only if it is reflexive, symmetric and transitive. That is, for all *a*, *b* and *c* in *X*:

- $a \sim a$. (Reflexivity)
- $a \sim b$ if and only if $b \sim a$. (Symmetry)
- if $a \sim b$ and $b \sim c$ then $a \sim c$. (Transitivity)

The equivalence class of $a \in X$ under ~, denoted $\bar{a}$, is defined as $\bar{a} = \{b \in X | b \sim a\}$. The set

$$X/\sim = \{\bar{a} | a \in X\}$$

is called the quotient set of *X* by ~.

A field extension E/F is called a *simple extension* if there exists an element θ in E with $E = F(\theta)$. The element θ is called a primitive element, or generating element, for the extension; we also say that E is generated over F by θ. In other hand, E is a simple extension of F generated by θ then it is the smallest field which contains both F and θ.

2.2 Product equation

The rationale for explaining the solution of a product equation is the notion of zero-divisor. For the sets, $\mathbb{Z}, \mathbb{Q}, \mathbb{R},$ and $\mathbb{C}$ have no zero-divisor so that $ab = 0$ implies $a = 0$ or $b = 0$.

Consider the product equation $A(x).B(x) = 0$ on $\mathbb{Z}$ (or $\mathbb{Q}, \mathbb{R}, \mathbb{C}$). In fact, this is the problem of finding $x \in \mathbb{Z}$ (or $\mathbb{Q}, \mathbb{R}, \mathbb{C}$ respectively) such that $A(x).B(x) = 0$. For each $x \in \mathbb{Z}$ (or $\mathbb{Q}, \mathbb{R}, \mathbb{C}$ respectively) then $A(x), B(x)$ are elements of $\mathbb{Z}$ (or $\mathbb{Q}, \mathbb{R}, \mathbb{C}$ respectively). For the sets, $\mathbb{Z}, \mathbb{Q}, \mathbb{R},$ and $\mathbb{C}$ have no zero-divisor so that $A(x).B(x) = 0$ implies $A(x) = 0$ or $B(x) = 0$. Reserve, if $A(x) = 0$ or $B(x) = 0$ then $A(x).B(x) = 0$ is obviously. Therefore, $A(x).B(x) = 0$ is equivalent to $A(x) = 0$ or $B(x) = 0$.

The same explanation for the solution of a product equation in the case of two variables, three variables, etc.

Example 1. Consider the equation $(x+3)(2x+4)=0$ (1) on $\mathbb{R}$. For every $x \in \mathbb{R}$, we have $x+3 \in \mathbb{R}, 2x+3 \in \mathbb{R}$. Since $\mathbb{R}$ has no zero-divisor so that (1) be equivalent to $x+3=0$ or $2x+4=0$, lead to $x=-3$ and $x=-2$ satisfies equation (1).

We have another way to explain equation (1) as follows: The expressions $x+3$ and $2x+4$ are two elements of the polynomial ring $\mathbb{R}[x]$. Since $\mathbb{R}$ is a field, $\mathbb{R}[x]$ is an integral domain, so that $\mathbb{R}[x]$ has no zero-divisor. Thus (1) is equivalent to $x+3=0$ or $2x+4=0$.

2.3. First degree equation with an unknown and difference between two numbers

The nature of these problems is the existence of the symmetric element x' of an element x in group X. If the operation on X is a multiplication, we call x' as the inverse of x. If the operation on X is an addition, we call x' the opposite element of x.

If b has the symmetric element then we can define the division (dividing a by b is multiplying a by the inverse element of b) and subtract (a minus b equals a plus with the opposite element of b).

Consider $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$, and $\mathbb{C}$ together with addition, they are groups, so every element has an opposite element. Therefore, we always solve the problem of finding the difference of two numbers a and b on these sets. The result is the sum of a and the opposite element of b, that is, $a-b = a + (-b)$.

Consider the first-degree equation with an unknown $ax=b$ with the coefficients $a, b \in \mathbb{R}$, $a \neq 0$ (similar to $\mathbb{Q}$ or $\mathbb{C}$). Since $\mathbb{Q}, \mathbb{R}$, and $\mathbb{C}$ are fields so a has an inverse element a^{-1}. Now we multiply both sides by the inverse element of a, we get $x = a^{-1}b = \frac{b}{a}$.

2.4. Number sets

In this section, we consider the construction of number sets from the perspective of algebraic structures.

2.4.1. Integer set

In elementary school teaching, integers are often intuitively defined as the (positive) natural numbers, zero, and the negations of the natural numbers. However, this style of definition leads to many different cases (each arithmetic operation needs to be defined on each combination of types of integer) and makes it tedious to prove that these operations obey the laws of arithmetic. Therefore, in modern set-theoretic mathematics a more abstract construction, which allows one to define the arithmetical operations without any case distinction, is often used instead. The integers can thus be formally constructed as the equivalence classes of ordered pairs of natural numbers (a,b). Now we will construct the set of integers $\mathbb{Z}$:

Consider $\mathbb{N} \times \mathbb{N} = \{(a,b) | a, b \in \mathbb{N}\}$ and we define an a binary relation $\sim$ on these pairs with the following rule: : $(a,b) \sim (c,d)$ if and only if $a+d=b+c$. It is easy to check, reflexive property: $(a,b) \sim (a,b)$, symmetric property: if $(a,b) \sim (c,d)$ then $(c,d) \sim (a,b)$

and transitive property: if $(a,b)\sim(c,d)$ and $(c,d)\sim(e,f)$ then $(a,b)\sim(e,f)$. Therefore, $\sim$ is an equivalence relation on $\mathbb{N}\times\mathbb{N}$. Set $\mathbb{Z}=\ \mathbb{N}\times\mathbb{N}/\sim$ is a set of equivalence classes. With $x\in\mathbb{Z}$ and $(a,b)\in x$, we denote $x=\overline{(a,b)}$. Now, we equippe two binary operations on $\mathbb{Z}$, one called addition and the other called multiplication:

$$\overline{(a,b)}+\overline{(c,d)}=\overline{(a+c,b+d)}$$
$$\overline{(a,b)}.\overline{(c,d)}=\overline{(ac+bd,ad+bc)}.$$

It is easy to prove $\mathbb{Z}$ with two binary operations is a commutative ring whose the additive identity is $\overline{(0,0)}=\overline{(n,n)}$ and the multiplicative identity is $\overline{(1,0)}=\overline{(n',n)}$, forall n $\in\mathbb{N}$, the inverse of $\overline{(a,b)}$ is $\overline{(b,a)}$ which is denoted $-\overline{(a,b)}$. Now, we prove $\mathbb{Z}$ has no nonzero zero divisors: if $\overline{(a,b)}.\overline{(c,d)}=\overline{(ac+bd,ad+bc)}=\overline{(n,n)}$ then $ac+bd=ad+bc$ or $(a-b)(c-d)=0$. Without loss of generality we may assume that $-b\in\mathbb{N}$ and $c-d\ \in\mathbb{N}$. Then we have $a-b=0$ or $c-d=0$, lead to $a=b$ or $c=d$. In case of $a=b$ then $\overline{(a,b)}=\overline{(0,0)}$; and when $c=d$ then $\overline{(c,d)}=\overline{(0,0)}$. Thus $\mathbb{Z}$ is an integer domain. Consider a map

$$f:\mathbb{N}\to\mathbb{Z}, a\mapsto\overline{(a,0)}.$$

Since $\overline{(a,0)}=\overline{(b,0)}$ if and only if $a=b$. Hence f is an injection. Forall $a,b\in\mathbb{N}$, we have

$$f(a+b)=\overline{(a+b,0)}=\overline{(a,0)}+\overline{(b,0)}=f(a)+f(b),$$
$$f(ab)=\overline{(ab,0)}=\overline{(a,0)}.\overline{(b,0)}=f(a)f(b).$$

Consequently, f is both a homomorphism of the monoids with addition and both a homomorphism of the monoids with multiplication. Since f is an injection, it follows that f is both a monomorphism of the monoids with addition and both a monomorphism of the monoids with multiplication.

Since $\overline{(a,b)}=\overline{(a,0)}+\overline{(b,0)}=\overline{(a,0)}-\overline{(b,0)}=f(a)-f(b)$. Hence, forall elements in $\mathbb{Z}$ have the form $f(a)-f(b)$.

The pair $(\mathbb{Z},f)$ defined as above is unique, differing from one isomorphism, meaning that if there is a pair (P,g) where P is a ring and $g:\mathbb{N}\to P$ is both a monomorphism of monoids with addition and both a monomorphism of monoids with multiplication, the elements of P have the form $g\ (a)\ -g\ (b)$, there exists an isomorphism $\varphi:\mathbb{Z}\to P$ such that $\varphi.f=g$. This is evidenced by a map:

$$\varphi:\mathbb{Z}\to P, f(a)-f(b)\mapsto g(a)-g(b).$$

It is tobe noticed that, f is a monomorphism. Hence, we can identical element $n\in\mathbb{N}$ with element $\overline{(n,0)}\in\mathbb{Z}$. This leads to $\mathbb{N}\subset\ \mathbb{Z}$. Therefore, every $x\in\mathbb{Z}, x=\overline{(a,b)}$, we have: if $a\geq b$ then $x=\overline{(a-b,0)}=a-b$. And if $a<b$ then $x=-\overline{(b,a)}=-\overline{(b-a,0)}=-(b-a)$. It follows that every $x\in\mathbb{Z}$, either $x\in\mathbb{N}$ or $-\ x\ \in\mathbb{N}$.

The ring $\mathbb{Z}$ above is called *ring of integers*.

2.4.2. Rational set

In mathematics at high school, a rational number is a number that can be expressed as the quotient or fraction $\frac{a}{b}$ of two integers, a numerator a, and a non-zero denominator b. Since b may be equal to 1, every integer is a rational number. The set of all rational numbers often referred to as "the rationals", is usually denoted by $\mathbb{Q}$.

In algebraic structures, the set of all rational numbers is constructed as a field. Rational numbers can be formally defined as equivalence classes of pairs of integers (a,b) such that $b \neq 0$, for the equivalence relation defined by $(a,b) \sim (c,d)$ if, and only if $ad = bc$. With this formal definition, the fraction $\frac{a}{b}$ becomes the standard notation for the equivalence class of (a,b). Now we will construct a field of rational numbers from an integer ring:

Consider $\mathbb{Z} \times \mathbb{Z}^* = \{(a,b) \mid a,b \in \mathbb{Z}, b \neq 0\}$, where $\mathbb{Z}^*$ is the set of nonzero integer numbers. We define an a binary relation $\sim$ on these pairs with the following rule: $(a,b) \sim (c,d)$ if and only if $a.d = b.c$. It is easy to check that $\sim$ is an equivalence relation on $\mathbb{Z} \times \mathbb{Z}^*$. Set $\mathbb{Q} = \mathbb{Z} \times \mathbb{Z}^*/\sim$ is the set of equivalence classes.

With $x \in \mathbb{Q}$ and $(a,b) \in x$, we denote $x = \overline{(a,b)}$. Now, we equippe two binary operations on $\mathbb{Q}$, one called addition and the other called multiplication:

$$\overline{(a,b)} + \overline{(c,d)} = \overline{(ad+bc, bd)}, \overline{(a,b)}.\overline{(c,d)} = \overline{(ac,bd)}.$$

It is easy to prove $\mathbb{Q}$ with two binary operations above is a field whose the additive identity is $\overline{(0,1)}$ and the multiplicative identity is $\overline{(1,1)}$. Consider the map

$$f: \mathbb{Z} \to \mathbb{Q}, a \mapsto \overline{(a,1)}.$$

It is clear that f is a ring monomorphism. Hence, we can identical element $a \in \mathbb{Z}$ with element $\overline{(a,1)} \in \mathbb{Q}$. This leads to $\mathbb{Z} \subset \mathbb{Q}$. And every $x \in \mathbb{Q}$ can be written

$$x = \overline{(a,b)} = \overline{(a,1)}.\overline{(1,b)} = f(a)f(b)^{-1} = ab^{-1}.$$

The pair $(\mathbb{Q}, f)$ defined as above is unique, differing from one isomorphism, meaning that if there is a pair (P, g) where P is a field and $g: \mathbb{Z} \to P$ is a ring monomorphism, the elements of P have the form $x = g(a)g(b)^{-1}$, $a,b \in \mathbb{Z}, b \neq 0$, there exists an isomorphism

$$\varphi: \mathbb{Q} \to P, f(a)f(b)^{-1} \mapsto g(a)g(b)^{-1}$$

such that $\varphi.f = g$.

The fact that $\mathbb{Q}$ is the field of quotients of the integral domain of integers. The field $\mathbb{Q}$, which is constructed above, is called *field of rational numbers*. Every rational number may be expressed in the form $\frac{a}{b}$, which is called fraction, where a and b are integer numbers and $b \neq 0$. Then two fractions are equal, write

$$\frac{a}{b}=\frac{c}{d} \text{ if and only if } ad=bc;$$

two fractions are added as follows:

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd};$$

and the rule for multiplication is:

$$\frac{a}{b}.\frac{c}{d}=\frac{ac}{bd}.$$

2.4.3. Real set

In basis mathematics, a real number is a value of a continuous quantity that can represent a distance along a line. The real numbers include all the rational numbers, such as the integer -3 and the fraction 5/4, and all the irrational numbers, such as $\sqrt{3}, \sqrt[3]{4}, e, \pi$. There are also many ways to construct the real number system, for example, starting from natural numbers, then defining rational numbers algebraically, and finally defining real numbers as equivalence classes of their Cauchy sequences or as Dedekind cuts, which are certain subsets of rational numbers. Another possibility is to start from some rigorous axiomatization of Euclidean geometry (Hilbert, Tarski, etc.) and then define the real number system geometrically. All these constructions of the real numbers have been shown to be equivalent, that is the resulting number systems are isomorphic. In this section, we describe a constructive way set of real numbers from Cauchy sequences.

Let X be the set of all Cauchy sequences of rational numbers. That is, sequences $x_1, x_2, \ldots, x_n, \ldots$ of rational numbers such that for every rational $\mathcal{E}>0$, there exists an integer N such that for all natural numbers $m, n>N$ we have $|x_m-x_n|<\mathcal{E}$.

Cauchy sequences $\{x_n\}$ and $\{y_n\}$ can be added and multiplied as follows:

$$\{x_n\}+\{y_n\}=\{x_n+y_n\}, \{x_n\}.\{y_n\}=\{x_n.y_n\}.$$

It is easy to prove X with two binary operations above is a commutative ring whose the additive identity is $\{0\}_{n\in\mathbb{N}}$ and the multiplicative identity is $\{1\}_{n\in\mathbb{N}}$. And subset

$$I=\left\{\{x_n\}\in X| \lim_{n\to+\infty} x_n=0\right\}$$

is an ideal of X. Therefore, the quotient X/I is a field. The inverse of nonzero elements $\alpha=\overline{\{x_n\}}=\{x_n\}+I\in X/I$ is $\overline{\{y_n\}}\in X/I$ where

$$y_n=\begin{cases} 0 \; if \; n\leq n_1 \\ \frac{1}{x_n} \; if \; n>n_1 \end{cases}.$$

The field X/I, which is denoted by $\mathbb{R}$, is called *field of real numbers*.