7. U. Von Barth and L. Hedin (1972), J. Phys. C Solid State Phys. 5, 1629.
8. M. Weinert, G. Schneider, R. Podloucky, and J. Redinger (2009), J. Phys. Condens. Matter 21, 084201.
9. E. Wimmer, H. Krakauer, M. Weinert, and A. J. Freeman (1981), Phys. Rev. B 24, 864.
10. S. Youn and A. J. Freeman (2001), Phys. Rev. B 63, 085112.
11. M. Kim, A. J. Freeman, and C. B. Geller (2005), Phys. Rev. B 72, 035205.
12. T. Van Quang and K. Miyoung (2016), J. Korean Phys. Soc. 68, 393.
13. U. Von Barth and L. Hedin (1972), J. Phys. C Solid State Phys. 5, 1629.
14. M. Weinert, G. Schneider, R. Podloucky, and J. Redinger (2009), J. Phys. Condens. Matter 21, 084201.
15. S. Blügel and G. Bihlmayer (2006), Comput. Nanosci. 31, 85.
16. T. Van Quang and M. Kim (2017), J. Appl. Phys. 122, 245104.
17. P. Kurz, G. Bihlmayer, and S. Blügel (2002), J. Phys. Condens. Matter 14, 6353.
18. S. Abdelouahed, N. Baadji, and M. Alouani (2007), Phys. Rev. B 75, 094428.
19. A. B. Shick, A. I. Liechtenstein, andW. E. Pickett (1999), Phys. Rev. B 60, 10763.
20. C. Santos, W. Nolting, and V. Eyert (2004), Phys. Rev. B 69, 214412
21. T. Van Quang and M. Kim (2013), J. Appl. Phys. 113, 17A934.
22. T. Van Quang and M. Kim (2014), IEEE Trans. Magn. 50, 1000904.
23. T. Van Quang and M. Kim (2016), J. Korean Magn. Soc. 26, 39.
24. J. O. Dimmock and A. J. Freeman(1964), Phys. Rev. Lett. 13, 750.
25. L. Oroszlány, A. Deák, E. Simon, S. Khmelevskyi, and L. Szunyogh (2015), Phys. Rev. Lett. 115, 096402.
26. B. N. Harmon, V. P. Antropov, A. I. Liechtenstein, I. V. Solovyev, and V. I. Anisimov (1995), J. Phys. Chem. Solids 56, 1521.
27. A. C. Jenkins and W. M. Temmerman (1999), J. Magn. Magn. Mater. 198, 567.
28. A. C. Jenkins, W. M. Temmerman, R. Ahuja, O. Eriksson, B. Johansson, and J. Wills (2000), J. Phys. Condens. Matter 12, 10441.
29. M. Petersen, J. Hafner, and M. Marsman (2006), J. Phys. Condens. Matter 18, 7021.

PHƯƠNG PHÁP FLAPW VÀ ỨNG DỤNG NGHIÊN CỨU TINH THỂ Gd: CẤU TRÚC ĐIỆN TỬ VÀ TÍNH BỀN VỮNG CỦA CÁC PHA TỪ FM VÀ AFM

Tóm tắt: *Mô tả và hiểu biết về cấu trúc điện tử và thuộc tính từ tính của gadolinium là một vấn đề đầy thử thách. Đặc biệt, cấu trúc từ bền vững nhất của Gd đã gây nhiều tranh luận trong một thời gian dài. Trong báo cáo này, phương pháp thế toàn phần sóng phẳng gia tăng được giới thiệu để nghiên cứu các tính chất của Gd. Do các trạng thái f định xứ rất mạnh, phép tính có thể cho kết quả rất lạ lùng phụ thuộc vào tham số định nghĩa khác nhau. Việc tính toán bao gồm các mô hình 4f-core và 4f-band được thực hiện. Phân tích cấu trúc điện tử và độ ổn định pha từ được trình bày và thảo luận. Tất cả các kết quả là phù hợp tuyệt vời với các kết quả thực nghiệm và báo cáo lý thuyết trước đó.*

Từ khoá: *Cấu trúc Gd bền vững, cấu trúc, mô hình f-lõi, mô hình f-band, mô hình f hóa trị, phương pháp FLAPW.*

HIGH ENERGY SCATTERING AMPLITUDE IN THE LINEARIZED GRAVITATIONAL THEORY

Do Thu Ha
Hanoi University of Science

Abstract: *The asymptotic behavior of the elastic scattering amplitude by the exchange of graviton between two scalar particles at high energies and fixed momentum transfers is reconsidered in the Logunov-Tavkhelidze equation in the linearized gravitational theory. The corrections to the eikonal approximation in the quasi-potential approach of relative order 1/p is developed with the principal contributions at high energy. The eikonal expression of scattering amplitude and the formal first correction are derived. The Yukawa potential is applied to discuss the results.*

Keywords: *Field theory, scattering amplitude, eikonal approximation.*

Received: 11 March 2020

Accepted for publication: 20 April 2020

Email: thuhahunre@gmail.com

1. INTRODUCTION

The eikonal approximation (which is also called straight-line path approximation) is an effective method of calculating the scattering amplitude at high energies and is studied by many authors in quantum field theory [1-4], and in quantum gravity theory recently [5-13]. However, in different approaches, only the main term of amplitude was considered, while the first correction does not have an explicit solution. Researches [9,10] in which the path-integral method with a modified perturbation theory and Logunov-Tavkhelidze quasi-potential are used to give the analytic expression of the first correction. Thus, the advantage of the quasi-potential approach is affirmed and need to be studied more deeply.

The aim of this paper is to make a more detailed investigation of the quasi-potential approach by solving the quasi-potential equation [9-10] to find the eikonal scattering amplitude and the first correction at high energies and minor momentum transfers.

The paper is organized as follows. In section 2.1, an eikonal approximation for the scattering amplitude and the first correction are derived by using a quasi-potential approach in the coordinate representation. This result is applied to the Yukawa potential in section 2.2.

The last section, we draw our conclusion.

2. CONTENT

2.1. Correction terms of scattering amplitude

First, we will derive homogenous equation for one-time wave function of an interactive two scalar particle system. To do this, we start from 4-time Green function $G_{ab}(x, y; x', y')$ which must be satisfied the Bethe-Salpeter equation [14], and can be written down in a symbolic form [14,15].

$$G = \alpha G^0 + \alpha^{-1} G^0 K G, \qquad \alpha = (2\pi)^4, \tag{1}$$

where G^0 is the Green function of free particles, and the kernel K can be found by the perturbative method.

Solving (1) by using the reduction technique with a relation between the 2-time Green function $\tilde{G}_{ab}$ and 4-time Green function G_{ab}, following the procedure of ref. [16], we have an explicit equation for the 2-time Green function in momentum representation.

$$F(p'^2; E^2)\tilde{G}(p', p, E) - \int dq^3 V(p', q, E)\tilde{G}(q, p, E) = \delta(p - p') \tag{2}$$

where $F_{a,b}(p^2; E^2) = (p^2 + m_{a,b}^2 - E^2)\sqrt{p^2 + m_{a,b}^2}$, and $V(p', q, E)$ is a potential matrix.

From (2) and the relation $\tilde{G}_{ab}(t, \vec{x}, \vec{y}) = \sum_n \varphi_n(t; \vec{x}, \vec{y})\, \varphi_n^+(t; \vec{x}, \vec{y})$ between the 1-time wave function and the 2-time Green function, the homogenous equation of 1-time wave function will be

$$(\vec{p}^2 - E^2 + m^2)\psi(\vec{p}) = \int d\vec{q}\, \frac{V[(\vec{p} - \vec{q})^2; E]\psi(\vec{q})}{\sqrt{m^2 + \vec{q}^2}} \tag{3}$$

Considering Eq. (3) in the coordinate representation with a purely imaginary local quasi-potential $V(\vec{r}; E) = ipEv(\vec{r})$ in which $v(\vec{r})$ is a smooth positive function and $p = |\vec{p}|$. At high energies and small scattering angles, wave function $\psi_p(\vec{r})$ can be written in the form $\psi_p(\vec{r}) = e^{ipz} F_p(\vec{r})$, $F_p(\vec{r})\big|_{z\to-\infty} = 1$. By the way of expanding terms in inverse powers of momentum, and keeping only terms of the order 1/p takes the form, the solution of Eq. (3) will be [18,19]

$$F_p(\vec{r}) = \exp\left[-\frac{z\theta(z)\gamma(\rho)}{2ip} - \int_{-\infty}^{z} v(\rho, z')\, dz' - \frac{1}{2ip}\int_{-\infty}^{z} \tilde{\chi}^{(1)}(\rho, z')\, dz'\right], \tag{4}$$

where

$$\tilde{\chi}^{(1)}(\rho,z)=\chi^{(1)}(\rho,z)-\theta(z)\gamma(\rho),\ \int_{-\infty}^{z}\chi^{(1)}(\rho,z')dz'=\int_{-\infty}^{z}\tilde{\chi}^{(1)}(\rho,z')dz'+z\theta(z)\gamma(\rho)$$

$$\chi^{(0)}=v(r),\ \chi^{(1)}=-3\left[\partial_z v(r)-v(r)^2-\eta_\perp(\vec{r},v)\right],\ \eta_\perp(\vec{r},\chi)=-\int_{-\infty}^{z}\nabla_\perp^2\chi(\rho,z')dz'+\left(\int_{-\infty}^{z}\nabla_\perp\chi(\rho,z')dz'\right)^2.$$

The scattering amplitude is related to the wave function as follows

$$T(\Delta^2;E)=\frac{1}{(2\pi)^3}\int dr\,\psi^{*(0)}_{k}V(E;r)\,\psi_p(r), \tag{5}$$

where, $\Delta^2=(p-k)^2=\Delta_\perp^2+\Delta_z^2\ =-t$ and$\Delta_z=\frac{\Delta_\perp^2}{2p}+O\left(\frac{1}{p^2}\right)$.

Substituting (4) into (5) and integrating by part, we obtain

$$T(\Delta^2;E)=T^{(0)}(\Delta^2;E)+\frac{1}{2ip}T^{(1)}(\Delta^2;E)+\ldots, \tag{6}$$

where the eikonal approximation for the amplitude is

$$T^{(0)}(\Delta^2;E)=-2ipE\,\frac{1}{(2\pi)^3}\int d^2\rho e^{i\rho\Delta_\perp}(e^{-\int_{-\infty}^{\infty}v(\rho,z')\,dz'}-1), \tag{7}$$

and the first correction in this approximation

$$\begin{aligned}&T^{(1)}(\Delta^2;E)\\&=2ipE\,\frac{1}{(2\pi)^3}\Bigg\{\int d^2\rho\, e^{i\rho\Delta_\perp}e^{-\int_{-\infty}^{\infty}v(\rho,z')\,dz'}3\int_{-\infty}^{\infty}v^2(\rho,z)\,dz\int_{-\infty}^{\infty}dz\,z\,v(\rho,v)\\&-\int d^2\rho dz\ e^{i\rho\Delta_\perp}\Delta_\perp^2 e^{-\int_{-\infty}^{z}v(\rho,z')\,dz'}\\&+\int d^2\rho\, e^{i\rho\ \perp}\int_{-\infty}^{\infty}dz\,\eta_\perp(\rho,v)\left(e^{-\int_{-\infty}^{z}v(\rho,z')\,dz}\ -\ e^{\int_{-\infty}^{\infty}v(\rho,z')\,dz}\right)\Bigg\}.\end{aligned} \tag{7}$$

Using method of integration by part [9] and quasi-potential approach in the momentum representation [10] these results (7), (8) can also be found.

Now, let us consider the case where momentum transfers t = 0 and the quasi-potential has the Gaussian form $V(E;\Delta^2)=i\mathrm{sge}^{\mathrm{at}},\ t=-\Delta^2$which the corresponding form in the coordinate representation is

$$V(E;r)=isg\sqrt{\pi/a}\,e^{-r^2/4a}. \tag{9}$$

Since$t=\Delta_\perp^2+\Delta_z^2$= 0, it follows$\Delta_\perp=0$. Substituting (9) into (8), and noticing that on the mass shell$p^2=E^2-m^2\quad\Rightarrow p\propto E\propto\sqrt{s}$, the first correction term will be

$$T^{(1)}(\Delta^2 = 0; E) \propto 3isg\frac{g}{\pi\sqrt{8\pi a}}\int d^2\rho e^{-\rho^2/2a}e^{2i\chi_0} + isg\frac{1}{8\pi^2 a}\int d^2\rho e^{-\rho^2/4a}(1 -)$$

$$\times \int_{-\infty}^{\infty} dz \int_{-\infty}^{z} V(z')dz'\left(\exp\left[2i\chi\int_{-\infty}^{z} V(z')\ dz'\right] - \exp\left[2i\chi\int_{-\infty}^{\infty} V(z')\ dz'\right]\right), \quad (10)$$

Where $2i\chi = -4\pi g e^{-\rho^2/4a}$, $V(z) = \frac{1}{\sqrt{4\pi a}}e^{-z^2/4a}$.

The similar result Eq. (10) is also found by the Born approximation in momentum representation [7].

2.2. Asymptotic behavior of the scattering amplitude at high energies

In the previous section, the general form of the scattering amplitude of two scalar particles is found in the potential $V(\vec{r}; E)$.

Now, let us consider a particular example in which the graviton exchange[1] [9] the quasi-potential increases with energy $V(r, s) = (\kappa^2 s e^{-\mu r}/2\pi r)$. Substituting this Yukawa potential into (7), (8) and noticing that at high energies, $p \propto E \propto \sqrt{s}$, we find the leading term of the scattering amplitude

$$T^{(0)}(\Delta^2; E) \propto \frac{\kappa^2 s}{(2\pi)^4}\left(\frac{1}{\mu^2 - t} - \frac{\kappa^4}{2(2\pi)^2}F_1(t) + \frac{\kappa^4}{3(2\pi)^5}F_2(t)\right) \quad (11)$$

and the first correction term

$$T^{(1)}(\Delta^2; E) = \frac{3i\kappa^6}{(2\pi)^6}\left(F_1(t) - \frac{2\kappa^3}{(2\pi)^3}F_2(t)\right) \quad (12)$$

Where

$$F_1(t) = \frac{1}{t\sqrt{1 - \frac{4\mu^2}{t}}}\ln\left|\frac{1 - \sqrt{1 - 4\mu^2/t}}{1 + \sqrt{1 - 4\mu^2/t}}\right| \quad (13)$$

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[1] The model of interaction of a scalar "nucleons" with a gravitational field in the linear approximation to $h_{\mu\nu}(x)$ $L(x) = L_{0,\varphi}(x) + L_{0.grav}(x) + L_{\text{int}}(x)$ where

$L_{0,\varphi}(x) = \frac{1}{2}\left[\partial^\mu\varphi(x)\partial_\mu\varphi(x) - m^2\varphi^2(x)\right]$; $L_{\text{int}}(x) = -\frac{\kappa}{2}h^{\mu\nu}(x)T_{\mu\nu}(x)$;

$T_{\mu\nu}(x) = \partial_\mu\varphi(x)\partial_\nu\varphi(x) - \frac{1}{2}\eta_{\mu\nu}\left[\partial^\mu\varphi(x)\partial_\mu\varphi(x) - m^2\varphi^2(x)\right]$; and $T_{\mu\nu}(x)$ is the energy momentum tensor of the scalar field. The coupling constant κ is related to the Newton constant of gravitation G by$\kappa^2 = 32\pi G = 32\pi l_{PL}^2$. $l_{PL} = 1,6.10^{-3}\ cm$ is the Planck length.

$$F_2(t) = \int_0^1 dy \frac{1}{(ty+\mu^2)(y-1)} \ln\left|\frac{\mu^2}{y(ty+\mu^2-t)}\right| \tag{14}$$

These results (11), (12) have similar forms in [9]. Moreover, from these equations we see that the first correction term of the eikonal expression of the scattering amplitude at high energies and fixed momentum transfers increases rapidly in the linearized gravitational theory. Comparison of these above potentials has made it possible to draw the following conclusions: in the model with the scalar exchange, the total cross section σ_t decreases as $(1/s)$, and only the Born term predominates in the entire eikonal equation; the vector model leads to a total cross section σ_t approaching a constant value as $s \to \infty, (t/s) \to 0$. In both cases, the eikonal phases are purely real and consequently the influence of inelastic scattering is disregarded in this approximation, $\sigma^{in} = 0$. In the case of graviton exchange the Froissart limit is violated. A similar result is also obtained in Ref. [20] with the eikonal series for reggeized graviton exchange.

3. CONCLUSION

The asymptotic behavior of the scattering amplitude at high energies and fixed momentum transfers has been studied within a quasi-potential approach in the coordinate representation in the linearized gravitational theory. The obtained results of eikonal expression of the scattering amplitude and the corresponding first correction term coincide with the results found by other authors [9-10]. The Yukawa potential has been used to concretize the results.

Acknowledgements

The author is grateful to Prof. Nguyen Suan Han for his suggestions of the problem and many useful comments. This work is supported in part by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 103.01-2018.42 and the project 911 of Hanoi University of Science – VNUHN.

REFERENCES

1. M. Barbashov, S. P. Kuleshov, V. A. Matveev, V. N. Pervushin, A. N. Sissakian and A. N. Tavkhelidze (1970), *Phys.Lett. 33B 484.*
2. E.Eichen and R. Jackiw (1971), Phys. Rev. D4 439.
3. G.'tHooft (1987), Phys. Lett. 198B 61.
4. L. N. Lipatov(1992, 1991), Phys. Lett. B116 411; Nucl. Phys.B365 614.