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H - INFINITY FULL-STATE FEEDBACK CONTROL FOR A BALL-BALANCING ROBOT

ĐIỀU KHIỂN H - INFINITY PHẢN HỒI TOÀN TRẠNG THÁI CHO ROBOT CÂN BẰNG BÓNG

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ABSTRACT

The main purpose of this paper is to present the H - infinity controller proposed to control the ball - balancing robot. In this paper, a method of building state-space models is presented. The state - space model is built on three separate planes xy, yz and zx by applying Lagrange equations for each plane. Lagrange equation is the relationship between kinetic, potential, and non - potential force, the model of this system is found by solving this equation. The H - infinity controller is designed for two planes yz, zx. Because the state - space model in yz and zx are the same, the H - infinity controller is only designed for one plane. The H - infinity controller is approached by Linear matrix inequalities (LMIs) to optimize the cost function. For the xy plane, the state - space model is different, therefore the PID controller is used. The simulation results performed on MATLAB/Simulink are fully presented in this paper. With the PID controller for the xy plane, a minor test is done to examine the response of this controller.

Keywords: *Ball balancing robot, H - infinity, Linear matrix inequalities, PID controller.*

TÓM TẮT

Mục đích chính của bài báo này trình bày về bộ điều khiển H - infinity sử dụng trong robot cân bằng bóng. Bài báo này đã chỉ ra một phương pháp để xây dựng mô hình không gian trạng thái. Mô hình không gian trạng thái này được xây dựng bằng cách áp dụng phương trình Lagrange cho mỗi mặt phẳng xy, yz và zx. Phương trình Lagrange là phương trình thể hiện mối liên hệ giữa động năng, thế năng và các lực phi thế, mô hình của hệ thống được tìm ra bằng cách giải phương trình này. Bộ điều khiển H - infinity được thiết kế cho hai mặt phẳng yz và zx. Bởi vì mô hình không gian trạng thái của hệ thống trong hai mặt phẳng này là giống nhau, do đó chỉ cần thiết kế một bộ điều khiển H - infinity. Bộ điều khiển H - infinity được tiếp cận bằng các bất đẳng thức ma trận tuyến tính (LMIs) để tối ưu hóa hàm chi phí. Đối với mặt phẳng xy, mô hình không gian trạng thái của có sự khác biệt, nên bộ điều khiển PID được sử dụng. Các kết quả tính toán và mô phỏng được thực hiện bằng MATLAB/Simulink, đối với bộ điều khiển PID, một thí nghiệm nhỏ được đưa ra để kiểm tra đáp ứng của nó.

Keywords: *Robot cân bằng bóng, H - infinity, bất đẳng thức ma trận tuyến tính, bộ điều khiển PID.*

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1. INTRODUCTION

The ball - balancing robot (also known as the Ballbot), includes the main ball, three omniwheels, three motors, and a body, as described in Figure 1. In the body are control circuits, batteries, etc. Three motors and three omniwheels provide torques to the ball, directing the movement of the ball to keep the body upright. This model works almost like an inverted pendulum.

Ballbot can self-balance on a ball with a small footprint. Therefore, the object can easily navigate in difficult terrains. Ballbot has the ability to move on inclined surfaces as well as vintage in narrow spaces. Ballbot is a new model, successfully built in 2005 by Ralph Hollis [1]. The maximum tilt angle of the first ballbot is 1 degree and used LQR/PID to control it. Another ballbot was developed by Tohoku Gakuin University from 2006 to 2008 [2, 3], and has been designed to be better than CMU because although only using the PD controller, this ballbot's deflection angle could be up to 5 degrees. In addition, Amirkabir University of Technology [4], and the University of Adelaide [5] 2009 have a ballbot that is at a height of 1.6m and uses LQRI control, however, this ballbot cannot rotate around its own axis.

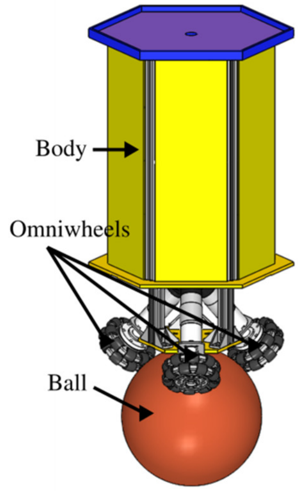

Figure 1. A ball balancing robot

The main contribution of this study is the successful application of the H - infinity controller to the ballbot system. That is the basis for applying this controller to another system, or it is possible to develop the H - infinity controller for the ballbot with external disturbance. Because the H - infinity controller is the controller to limit the exogenous. The dynamic model and some control methods are presented in detail [6]. In this study, a dynamic model of the ballbot was built by Euler - Lagrange method. The state-space equations are found by solving the Lagrange equation. The Lagrange equation is the binding between energy (including kinetic and potential energy) and non-potential forces. This model is separated into three subsystems, in that two subsystems describe the motion of the ballbot in yz and zx plane, and the rest of the subsystem describes the motion of the ballbot in xy plane. Furthermore, in this study, a linearization method is shown by the partial derivative of the equation of motion of the ballbot. Thus, the system is approximated around the balance position. With the H - infinity controller, there are many ways to approach, such as the transfer function approach. In this paper, the H - infinity controller is approached by linear matrix inequalitises. The H - infinity controller used is a full-state feedback controller and ignores all noise from the environment. Simulations are given to demonstrate the stability of the control theory.

This paper has a total of 6 sections, Section 2 presents the model of ballbot, Section 3 illustrates H - infinity control with the LMIs approach, Section 4 demonstrates a design of a PID controller for the direction of the body. Section 5 and Section 6 are simulations and conclusions.

2. MODELING AND LINEARIZATION OF BALLBOT

Table 1. Parameters of the planar model of ballbot

Parameter	Symbol	Value
Mass of the ball	m_K	7.13kg
Moment of inertia of the ball	J_K	$0.041kgm^2$
Radius of the ball	r_K	0.12m
Mass of the body	m_B	4.59kg
Moment of inertia of the body in yz/xz plane	J_B	$0.2kgm^2$
Moment of inertia of the body in xy plane	J_{Bxy}	$0.06kgm^2$
Distance from ball's COM to body's COM	l	0.5m
Mass of the omniwheel	m_W	0.19kg
Radius of the omniwheel	r_W	0.05m
Moment of inertia of the omniwheel	J_W	$0.0002375kgm^2$
Angle of the omniwheel relative to the ground	α	45^0
Angle between motors in xy plane	β	120^0
Gravity acceleration	g	$9.81\ m/s^2$
Torque limit	T_{max}	5 Nm

Assumptions: There is no slip, contact between the omniwheel and the ball as well as between the ball and ground. No external disturbances included assume the disturbances are negligible, and the ball moves only in the xy plane.

A detailed description of each step is written in [7], and it will be plainly presented in this section. First, to make it easier to build a ballbot model, the ballbot is simplified by separating into two rigid bodies, a ball, and a body, the latter contains the motor, omniwheel, and internal electronic components. The model parameters are given in Table 1. There are two ways to model the ballbot [6] either as a planar model and a 3D model. In the planar model, the dynamics of rotation around the individual inertia axes are assumed to be completely decoupled. In 3D modeling, the system is modeled with coupled dynamics. In this paper, for simplification, the planar model is used.

Binding equations:

$$x_K = \varphi_x r_K \quad (1)$$

$$y_K = \varphi_y r_K \quad (2)$$

$$x_B = x_K = \varphi_x r_K + \sin\theta_x l \quad (3)$$

$$y_B = x_K = \varphi_y r_K + \sin\theta_y l \quad (4)$$

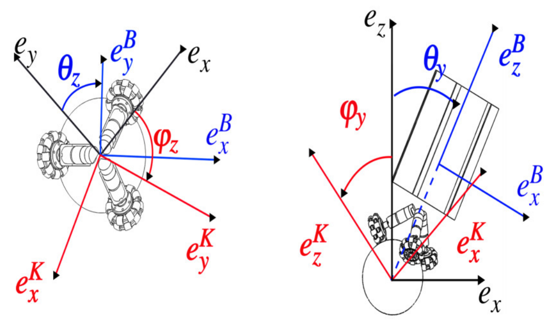

Figure 2. Planar model of Ballbot, showing the coordinates and variables. On the left: xy plane; on the right: zx plane.

where, φ_x, φ_y, φ_z are the angles of the ball in yz, zx, xy planes. θ_x, θ_y, θ_z are the angles of the ball in yz, zx, xy planes. For ease of system modeling, the process moments generated by the motors T_1, T_2, T_3 are converted to the moments of inertia of the ball in yz, zx, xy planes, T_x, T_y, T_z. The planar model in yz plane is the same as the planar model in zx plane. Thus, the equation of motion in yz plane is not different from the equation of motion in zx plane. The equation of motion can be written in matrix form as follow:

$$\begin{bmatrix} m_t r_K^2 + J_K + \frac{r_K^2}{r_W^2}J_W & -\frac{-r_K}{r_W^2} r_t J_W + \gamma r_K \cos\theta \\ -\frac{-r_K}{r_W^2} r_t J_W + \gamma r_K \cos\theta & \frac{r_t^2}{r_W^2}J_W + J_B + m_B l^2 + m_W r_t^2 \end{bmatrix}\ddot{q} + \begin{bmatrix} -r_K \gamma \sin\theta\dot{\theta} \\ -g\gamma\sin\theta \end{bmatrix} = \vec{f}_{NP} \quad (5)$$

where, $m_t = m_k + m_B + m_W$, $r_t = r_K + r_W$ and $\gamma = lm_B + (r_K + r_W)m_W$

In xy plane, the equation of motion in xy plane is written as follows:

$$\begin{aligned}\ddot{\varphi}_z &= -\frac{(r_W^2 J_{Bxy} + r_K^2 J_W \sin^2\alpha)T_f + r_K r_W J_{Bxy} \sin\alpha T_z}{r_W^2 J_{Bxy} J_K + r_K^2 (J_{Bxy} + J_K) J_W \sin^2\alpha} \\ \ddot{\theta}_z &= -\frac{r_K \sin\alpha (r_K J_W \sin\alpha T_f + r_W J_K T_z)}{r_W^2 J_{Bxy} J_K + r_K^2 (J_{Bxy} + J_K) J_W \sin^2\alpha}\end{aligned} \tag{6}$$

where, $T_f = \dfrac{r_K r_W J_{Bxy} \sin\alpha T_z}{r_W^2 J_{Bxy} + r_K^2 J_W \sin^2\alpha}$

The equation of motion in xz/yz plane in state - space form:

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = Ax + Bu = f \tag{7}$$

Linearization the system by first-order Taylor expansion at equilibrium position with parameters of ballbot in Table 1, the matrices of system are determined:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -29.709 & 0 & 0 \\ 0 & 22.918 & 0 & 0 \end{bmatrix};$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 18.39 \\ -5.53 \end{bmatrix}; \quad C = I_4; \quad D = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{8}$$

where, I_4 is the unit matrix. The linearization of the system only approximates the system around the equilibrium position. With the above equation, it can be recognized that this model is like an inverted pendulum. Do the same with *xy* plane, from equation (6), the matrices of system is:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 66.984 \\ -23.302 \end{bmatrix} \tag{9}$$

Matrixes C and D in xy plane are the same as matrixes C and D in zx/yz plane.

Since the inputs in the above models are virtual torque, T_1, T_2, T_3, a transformation is needed to find the torques required for each motor T_1, T_2, T_3. The relationships between the virtual torques and the torques of the motors are:

$$T_1 = \frac{1}{3}\left(T_z + \frac{2}{\cos\alpha}(T_z\cos\beta) - T_y\sin\beta\right) \tag{10}$$

$$T_2 = \frac{1}{3}\left(T_z + \frac{1}{\cos\alpha}(\sin\beta(-\sqrt{3}T_z + T_y) - \cos\beta(T_x + \sqrt{3}T_y))\right) \tag{11}$$

$$T_3 = \frac{1}{3}\left(T_z + \frac{1}{\cos\alpha}(\sin\beta(\sqrt{3}T_z + T_y) + \cos\beta(-T_x + \sqrt{3}T_y))\right) \tag{12}$$

3. LINEAR MATRIX INEQUALITIES FOR H$_\infty$ FULL - STATE FEEDBACK CONTROL

There are various approaches to the control of H - infinity, one of which is quite popular using the transfer function. This method is presented in [8]. The approach by LMIs is also present in the papers [9-12] are also presented. However, their presentation is incomplete and confounding. Furthermore, the controller mentioned in [13] has been applied without disturbances, the result is the same as the LQR controller.

For the planar model in xy plane, $\ddot{\theta}$ and $\ddot{\varphi}$ depend only on the input, T_z. Hence, a PID controller will be used, and this will be explained in Section 4. As for the zx and yz planes, these two planes have the same equation, therefore a H - infinity controller for the yz plane will be designed and copied to the zx plane.

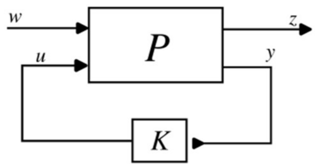

Figure 3. H - infinity problem

Equations of motion of system include H - infinity controller [14]:

$$\begin{aligned}\dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{11} w + D_{12} u \\ y &= C_2 x + D_{21} w + D_{22} u \\ u &= K y\end{aligned}$$

Figure 3 shows a model of a closed loop of the controller, where w is the exogenous, z is the control error, u is the control signal, y is the output of the system, the signals that can be measured by sensor, $P = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]$ is plant to be controlled, and $K = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$ is the controller. Follow [14], the controller is designed by choosing a matrix K to minimize:

$$\| P_{11} + P_{12}(I - KP_{22})^{-1} KP_{21} \| \tag{13}$$

To simplify the controller, remove the A_K, B_K, C_K matrices, so, it is only necessary to find the D_K matrix. For full-state feedback, $y = x$, $C_x = I$, $D_{21} = 0$ and $D_{22} = 0$. With the assumption of no external disturbances, we get B_1, D_{11} and D_{12} are equal to zero. Thus, the closed - loop state - space representation is:

$$\underline{S}(\hat{P}, \hat{K}) = \left[\begin{array}{c|c} A + B_2 K & 0 \\ \hline C_1 & 0 \end{array}\right] \tag{14}$$

In [14], the following are equivalent:

1. There exists D_K such that,

$$\left\| \underline{S}\left(\left[\begin{array}{c|cc} A & 0 & B_2 \\ \hline C_1 & 0 & 0 \\ I & 0 & 0 \end{array} \right], \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & D_K \end{array} \right] \right) \right\|_\infty \leq \gamma \tag{15}$$

2. There exists a positive definite matrix X and Z such that,

$$\begin{bmatrix} A^T X + XA + Z^T B_2^T + B_2 Z & 0 & XC_1^T \\ 0 & -\gamma I & 0 \\ C_1 X & 0 & -\gamma I \end{bmatrix} < 0$$

and $D_K = ZX^{-1}$ (16)

Apply to the planar model in yz/zx plane,

$$K = \begin{bmatrix} 6.22 & 117.740 & 7.67 & 31.26 \end{bmatrix} \tag{17}$$

4. PID CONTROLLER FOR XY PLANE

The control of the system in the xy plane is extremely simple. Because the deflection angle of the ballbot in the xy plane depends only on T_z. The system model in the xy plane is a linear system. However, it is very difficult to design an LQR or H - infinity controller here, because the A matrix in state - space equation of the system is a special matrix. Therefore, a classic controller is chosen, which is PID. The PID controller is described as below:

$$u = K_p e + K_i \int_0^t e dt + K_d \frac{de}{dt} \tag{18}$$

where, e is error.

By checking several sets of parameters, chose $K_p = 0.5$, $K_i = 0$, $K_d = 0.3$.

5. SIMULATION

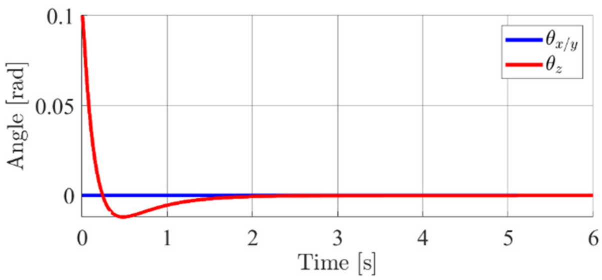

Figure 4. Angles of ballbot while rotating yaw axis (PID)

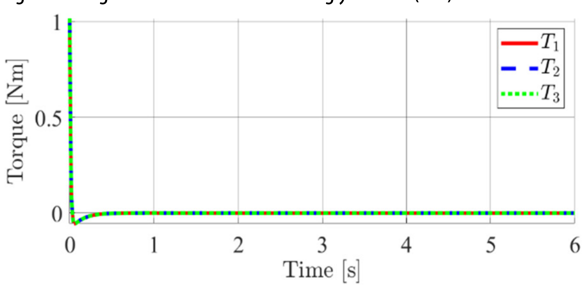

Figure 5. Torques input while rotate the yaw axis (PID)

Because the system is modeled on discrete planes, similar responses of the system in the yz and zx planes are recorded. In this study, two tests are concluded to examine the stability of the H infinity control. For the xy plane, the PID controller (shown in (18)) controls yaw axis of the ballbot in Figure 4 and torques for each omniwheels shown in Figure 5.

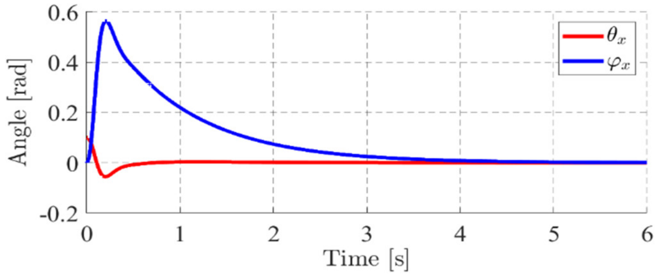

Figure 6. Angles of ballbot while balancing ballbot (H-inf)

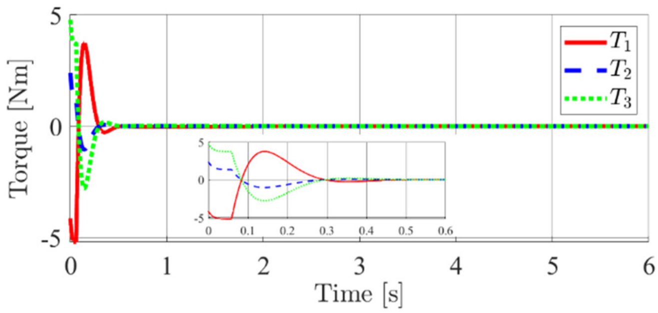

Figure 7. Torques input while balancing ballbot (H-inf)

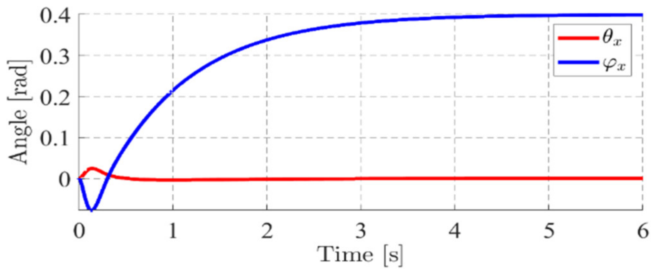

Figure 8. Angles of ballbot while move ballbot (H-inf)

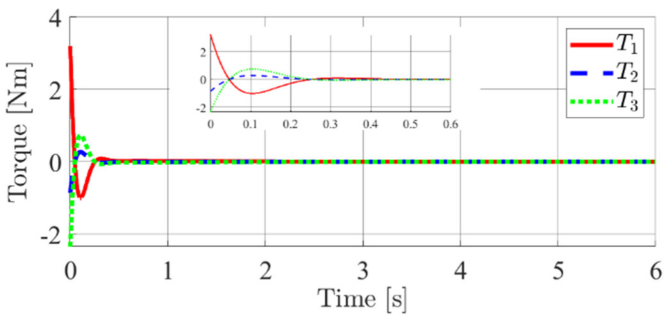

Figure 9. Torques input while moving ballbot (H-inf)

With the H - infinity controller, balancing the ballbot with initial angles of ballbot is 0.1 (rad) for all plane, after 4 seconds the ballbot has completely stabilized (Figure 6). And the overshoot of the body tilt angle as above does not cause the ballbot to fall. Besides, the torque of each omniwheel (in Figure 7) is limited to less than T_{max} (in Table 1). And similarly, when moving the ballbot to the reference position (in Figure 8), the ballbot remains stable and the torque is still tightly controlled.

6. CONCLUSIONS

In summary, this study has presented the planar model of the ballbot and successfully applied the H - infinity control to it. This controller is approached by linear matrix inequalities, with the purpose of controlling the tilt angle of the ballbot. Besides, also successfully used the PID controller to control the yaw axis. With the results obtained, both controllers are working well. In the modeling section, it is difficult to fully describe the system due to the system splitted into 3 planes and remove the noises from the environment. Therefore, the simulation results only approximate the system response. In the future, these control theories will be applied to the 3D model for better results.

REFERENCES

[1]. Lauwers T., Kantor G., Hollis R., 2007. *One Is Enough!*. In: Thrun, S., Brooks, R., Durrant-Whyte, H. (eds) Robotics Research. Springer Tracts in Advanced Robotics, vol 28. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-540-48113-3_30

[2]. M. Kumagai, R. L. Hollis, 2011. *Development of a three-dimensional ball rotation sensing system using optical mouse sensors.* 2011 IEEE International Conference on Robotics and Automation, pp. 5038-5043, doi: 10.1109/ICRA.2011.5979899.

[3]. M. Kumagai, T. Ochiai, 2009. *Development of a robot balanced on a ball - Application of passive motion to transport.* 2009 IEEE International Conference on Robotics and Automation, pp. 4106-4111, doi: 10.1109/ROBOT.2009.5152324.

[4]. S. kordbacheh, K. baghestan, S. s. gheidary, 2018. *Modeling and Robust Control of Ballbot Robot with Improved Power Transfer Mechanism.* 2018 6th RSI International Conference on Robotics and Mechatronics (IcRoM), pp. 353-358, doi: 10.1109/ICRoM.2018.8657623.

[5]. J. Fong, S. Uppill, B. Cazzolato, 2009. *899: Ballbot.* Preliminary Report, University of Adelaide.

[6]. Jespersen Thomas, 2019. *Kugle - Modelling and Control of a Ball-balancing Robot*. Thesis for: Master of Science (MSc) in Engineering, Control and Automation, doi: 10.13140/RG.2.2.31490.73928.

[7]. Asgari Pouya, Zarafshan Payam, Moosavian S.A.A., 2013. *Dynamics modelling and stable motion control of a Ballbot equipped with a manipulator.* International Conference on Robotics and Mechatronics, ICRoM 2013. 259-264. doi: 10.1109/ICRoM.2013.6510115.

[8]. N. Dey, U. Mondal, D. Mondal, 2016. *Design of a H-infinity robust controller for a DC servo motor system*. 2016 International Conference on Intelligent Control Power and Instrumentation (ICICPI), pp. 27-31, doi: 10.1109/ICICPI.2016.7859667.

[9]. M. R. Fikri, D. W. Djamari, S. M. Levy, 2021. *H-Infinity Controller Design of Two-Wheeled Mobile Robot Under Disturbance.* 2021 International Seminar on Intelligent Technology and Its Applications (ISITIA), pp. 147-152, doi: 10.1109/ISITIA52817.2021.9502218.

[10]. Ashok Kumar M, Kanthalakshmi S., 2018. H∞ *tracking control for an inverted pendulum.* Journal of Vibration and Control. Journal of Vibration and Control, 24(16), pp. 3515-3524. doi: 10.1177/1077546317750977

[11]. Gadewadikar J., Lewis F.L., Subbarao K., et al., 2009. *H-Infinity Static Output-feedback Control for Rotorcraft*. J Intell Robot Syst 54, 629–646. doi: 10.1007/s10846-008-9279-5

[12]. Z. Gao, X. Wu, Y. Liu, 2018. *Input and Output Strictly Passive H-infinity Control of Switched Linear Systems.* 2018 Chinese Control And Decision Conference (CCDC), pp. 4705-4708, doi: 10.1109/CCDC.2018.8407945.

[13]. P. T. Thanh, D. P. Nam, V. Van Tu, T. Q. Huy, N. Van Huong, 2019. *Robust control law using H-infinity for wheeled inverted pendulum systems*. International Journal of Mechanical Engineering and Robotics Research, vol. 8, no. 3, p. 483–48, doi: 10.18178/ijmerr

[14]. Gahinet Pascal, Apkarian Pierre, 1994. *A Linear Matrix Inequality Approach to H∞ Control.* Int. J. Robust & Nonlinear Control. pp. 421-448. doi: 10.1002/rnc.4590040403.

THÔNG TIN TÁC GIẢ

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