DETERMINING OPTIMAL PARAMETERS OF THE TUNED MASS DAMPER TO REDUCE THE TORSIONAL VIBRATION OF THE MACHINE SHAFT BY USING THE FIXED-POINT THEORY

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ABSTRACT

This paper presents an analytical method to determine optimal parameters of tuned mass damper (TMD), such as the ratio between natural frequency of TMD and shaft, the ratio of the viscous coefficient of the TMD. Two novel findings of the present study are summarized as follows. First, the optimal parameters of the TMD for the shafts are given by using the fixed-point theory (FPT). Next, a numerical simulation is done for an example of the machine shaft to validate the effectiveness of the results obtained in this study. The simulation results indicate that the proposed method significantly increases the effectiveness in torsional vibration reduction of the machine shaft.

Keywords: *Tuned mass damper, torsional vibration, optimal parameters, machine shaft, fixed-point theory.*

TÓM TẮT

Bài báo trình bày kết quả nghiên cứu xác định các tham số tối ưu của bộ giảm chấn khối lượng TMD, chẳng hạn như tỷ số giữa tần số riêng của bộ TMD và tần số riêng của trục máy, tỉ số cản nhớt của bộ TMD. Hai phát hiện mới của nghiên cứu này được tóm tắt như sau: Đầu tiên, các tham số tối ưu của bộ TMD cho các trục được đưa ra bằng cách sử dụng lý thuyết điểm cố định FPT. Tiếp theo, một ví dụ về trục máy được mô phỏng để kiểm tra tính hiệu quả của các kết quả nghiên cứu thu được. Các kết quả mô phỏng đã chỉ ra rằng phương pháp đề xuất làm tăng đáng kể hiệu quả trong việc giảm dao động xoắn cho trục máy.

Từ khóa: *Giảm chấn khối lượng, dao động xoắn, tham số tối ưu, trục máy, lý thuyết điểm cố định.*

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1. INTRODUCTION

Research to reduce fluctuations in structure is a problem that many scientists studied [1-10]. The helical oscillation is determined by the relative torque between the ends of the shaft rarely being discussed. In fact, it is important to determine the spiral oscillation of the shaft as it allows the determination of stresses in the shaft, as well as evaluating the axial fatigue strength [8]. Optimal parameters of tuned mass damper (TMD) to reduce the torsional vibration of the shaft by using the principle of minimum kinetic energy has been investigated by Nguyen [9], the results were given by

$$\alpha_{opt}^{MKE} = \frac{1}{1+2\mu\gamma^2} \quad ; \ \xi_{opt}^{MKE} = \gamma\sqrt{\frac{\mu}{2(1+2\mu\gamma^2)}}$$

In order to develop and extend the research results in [9], In this paper, the fixed-point theory in Reference [1] is used for determining optimal parameters of the TMD.

2. SHAFT MODELLING AND EQUATIONS OF VIBRATION

As shows Fig. 1, the shaft has the torsion spring coefficient is k_t. The tuned mass damper (TMD) has a concentrated mass 2m at the top, spring constant k_m and damping constant c, the length of beam is 2L and the length mass $2m_t$. The TMD is installed in the shaft through a mass rotor, with radius ρ, mass M.

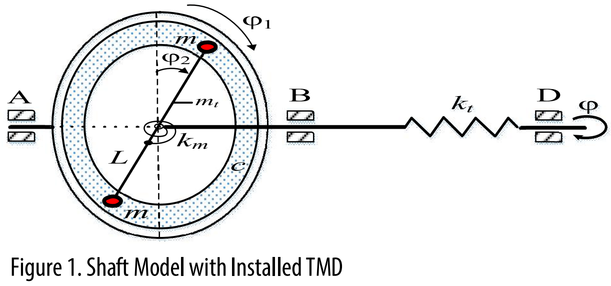

Figure 1. Shaft Model with Installed TMD

From [9], we have

$$(M\rho^2 + \frac{2}{3}m_tL^2 + 2mL^2)\ddot{\theta} + 2(\frac{1}{3}m_tL^2 + mL^2)\ddot{\varphi}_2 = M(t) - k_t\theta \quad (1)$$

$$2(\frac{1}{3}m_tL^2 + mL^2)\ddot{\theta} + 2(\frac{1}{3}m_tL^2 + mL^2)\ddot{\varphi}_2 = -k_m\varphi_2 - 2cL^2\dot{\varphi}_2 \quad (2)$$

where: $\varphi_1 - \varphi = \theta$ (3)

Eqs. (1, 2) can be used in the design of TMD.

3. DETERMINING OPTIMAL PARAMETERS OF THE TMD

For simplicity, following variables are introduced as [9]:

$$\mu=\frac{m+\frac{m_t}{3}}{M},\omega_D=\sqrt{\frac{k_t}{M\rho^2}},\omega_d=\sqrt{\frac{k_m}{2(m+\frac{m_t}{3})L^2}},$$
$$\xi=\frac{c}{2(m+\frac{m_t}{3})\omega_d},\alpha=\frac{\omega_d}{\omega_D},\ \gamma=\frac{L}{\rho},\ \beta=\frac{\omega}{\omega_D},\tag{4}$$

Substituting Eq.(4) into Eqs.(1,2). The matrix form of Eqs.(1, 2) are expressed as

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{C}\dot{\mathbf{q}}+\mathbf{K}\mathbf{q}=\mathbf{F}\tag{5}$$

where

$$\mathbf{q}=\{\theta\quad \varphi_2\}^T\tag{6}$$

The mass matrix, viscous matrix, stiffness matrix and excitation force vector can be derived as:

$$\mathbf{M}=\begin{bmatrix}1+2\mu\gamma^2 & 2\mu\gamma^2\\ 1 & 1\end{bmatrix};\mathbf{C}=\begin{bmatrix}0 & 0\\ 0 & 2\xi\alpha\omega_D\end{bmatrix};$$
$$\mathbf{K}=\begin{bmatrix}\omega_D^2 & 0\\ 0 & \omega_D^2\alpha^2\end{bmatrix};\ \mathbf{F}=\begin{Bmatrix}\frac{M(t)}{M\rho^2}\\ 0\end{Bmatrix}\tag{7}$$

The forced vibration of this system will be of the form

$$M(t)=\hat{M}e^{I\omega t}\tag{8}$$

Thus, the stationary response of this system which can be written as:

$$\theta(t)=\hat{\theta}e^{I\omega t},\quad \varphi_2(t)=\hat{\varphi}_2e^{I\omega t}\tag{9}$$

where

$\hat{\theta}$ and $\hat{\varphi}_2$ are complex amplitude vibration of the primary system and TMD, respectively.

Substituting Eqs.(7-9) into Eq.(5), this becomes

$$\left(-\beta^2\begin{bmatrix}1+2\mu\gamma^2 & 2\mu\gamma^2\\ 1 & 1\end{bmatrix}+2i\beta\begin{bmatrix}0 & 0\\ 0 & 2\xi\alpha\omega_D\end{bmatrix}+\begin{bmatrix}\omega_D^2 & 0\\ 0 & \omega_D^2\alpha^2\end{bmatrix}\right)\begin{bmatrix}\hat{\theta}\\ \hat{\varphi}_3\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}\left|\frac{\hat{M}}{k_s}\right|\tag{10}$$

Hence the stationary response of the primary system is expressed as:

$$\hat{\theta}=\frac{E_1+iE_2\xi}{E_3+iE_4\xi}\left|\frac{\hat{M}}{k_s}\right|\tag{11}$$

where $E_1=-\alpha^2+\beta^2;$ (12)

$$E_2=-2\alpha\beta;\tag{13}$$

$$E_3=2\alpha^2\beta^2\gamma^2\mu+\alpha^2\beta^2-\beta^4-\alpha^2+\beta^2\tag{14}$$

$$E_4=2\alpha\beta(2\beta^2\gamma^2\mu+\beta^2-1).\tag{15}$$

After short calculation the Eq.(11) we obtained the real amplitude of the vibration response, which can be written as:

$$\left|\hat{\theta}(t)\right|=\sqrt{\frac{E_1^2+E_2^2\xi^2}{E_3^2+E_4^2\xi^2}}\left|\frac{\hat{M}}{k_s}\right|=E\left|\frac{\hat{M}}{k_s}\right|\tag{16}$$

where E is called the amplifier function that is defined by

$$E=\sqrt{\frac{E_1^2+E_2^2\xi^2}{E_3^2+E_4^2\xi^2}}\tag{17}$$

Substituting Eqs. (12)-(15) into Eq.(17), The Ecan be determined as:

$$E=\sqrt{\frac{4\xi^2\alpha^2\beta^2+(-\alpha^2+\beta^2)^2}{4\xi^2\alpha^2\beta^2(2\beta^2\gamma^2\mu+\beta^2-1)^2+(2\alpha^2\beta^2\gamma^2\mu+\alpha^2\beta^2-\beta^4-\alpha^2+\beta^2)^2}}\tag{18}$$

Fig. 2 presents the graphs of the amplitude magnification factor E versus the frequency ratio β corresponding to some different values of the TMD's damping ratio ξ.

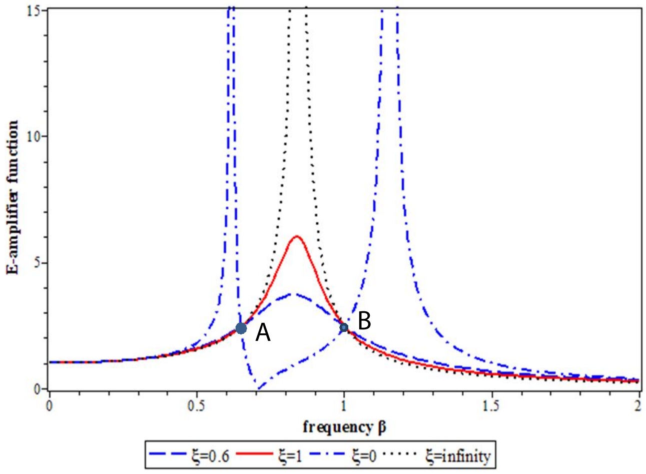

Figure 2. Graphs of the amplitude magnification factor versus the frequency ratio β

We observe from this graphs that there exist two fixed points A and B which are independent of ξ. The first step of this method is to specify two fixed points. Suppose that two points (A and B) with horizontal coordinates as a β_1, β_2. The conditions for E does not depend on the ξ is expressed as follows:

$$\left.\frac{\partial E}{\partial \xi}\right|=0\tag{19}$$

Substituting Eq.(18) into Eq.(19), this becomes:

$$-\frac{\xi(E_1^2E_4^2-E_2^2E_3^2)}{\left(E_4^2\xi^2+E_3^2\right)^2\sqrt{\frac{E_1^2+E_2^2\xi^2}{E_3^2+E_4^2\xi^2}}}=0,\tag{20}$$

$$\Rightarrow E_1^2E_4^2-E_2^2E_3^2=0\tag{21}$$

Therefore we have

$$\left|\frac{E_1}{E_3}\right|\Big|_{\beta=\beta_1}=\left|\frac{E_2}{E_4}\right|\Big|_{\beta=\beta_1} \tag{22}$$

$$\left|\frac{E_1}{E_3}\right|\Big|_{\beta=\beta_2}=\left|\frac{E_2}{E_4}\right|\Big|_{\beta=\beta_2} \tag{23}$$

We obtain the value of E at two points (A, B) these are expressed as follows:

$$E\big|_A=\left|\frac{E_2}{E_4}\right|\Big|_{\beta=\beta_1} \tag{24}$$

$$E\big|_B=\left|\frac{E_2}{E_4}\right|\Big|_{\beta=\beta_2} \tag{25}$$

Den Hartog [1] reported that the graph of amplifier function does not change in between the two peaks (A, B) when the vertical coordinates of the A and B must be equal. In this condition, we have

$$E\big|_A=E\big|_B \tag{26}$$

The optimal parameter of α and β are specified by solving Eqs.(22-26) which can be written as:

$$\alpha_{opt}^{FPT}=\frac{1}{2\mu\gamma^2+1} \tag{27}$$

$$\beta_1^2=\beta_1^{*2}=\frac{\sqrt{\mu}\gamma\sqrt{\mu\gamma^2+1}+\mu\gamma^2+1}{(\mu\gamma^2+1)(2\mu\gamma^2+1)} \tag{28}$$

$$\beta_2^2=\beta_2^{*2}=-\frac{\sqrt{\mu}\gamma\sqrt{\mu\gamma^2+1}-\mu\gamma^2-1}{(\mu\gamma^2+1)(2\mu\gamma^2+1)} \tag{29}$$

Then, the optimum absorber damping can be identified as follows:

$$\frac{\partial E}{\partial\beta}=0 \tag{30}$$

Eq. (17) gives

$$E^2\left(E_3^2+E_4^2\xi^2\right)=E_1^2+E_2^2\xi^2 \tag{31}$$

Taking derivative of Eq.(31) with respect to β, this becomes:

$$\xi^2=-\frac{-E^2E_3\dfrac{\partial E_3}{\partial\beta}-EE_3^2\dfrac{\partial E}{\partial\beta}+E_1\dfrac{\partial E_1}{\partial\beta}}{-E^2E_4\dfrac{\partial E_4}{\partial\beta}-EE_4^2\dfrac{\partial E}{\partial\beta}+E_2\dfrac{\partial E_2}{\partial\beta}} \tag{32}$$

Eliminating $\frac{\partial E}{\partial\beta}=0$ from Eq.(32) we obtain

$$\xi^2=-\frac{-E^2E_3\dfrac{\partial E_3}{\partial\beta}+E_1\dfrac{\partial E_1}{\partial\beta}}{-E^2E_4\dfrac{\partial E_4}{\partial\beta}+E_2\dfrac{\partial E_2}{\partial\beta}} \tag{33}$$

Substituting Eqs.(27-29) into Eq.(33), this becomes:

$$\xi_1^2=-\frac{E_1\dfrac{\partial E_1}{\partial\beta}-E^2E_3\dfrac{\partial E_3}{\partial\beta}}{E_2\dfrac{\partial E_2}{\partial\beta}-E^2E_4\dfrac{\partial E_4}{\partial\beta}}\Bigg|_{\beta=\beta_1} \tag{34}$$

and

$$\xi_2^2=-\frac{E_1\dfrac{\partial E_1}{\partial\beta}-E^2E_3\dfrac{\partial E_3}{\partial\beta}}{E_2\dfrac{\partial E_2}{\partial\beta}-E^2E_4\dfrac{\partial E_4}{\partial\beta}}\Bigg|_{\beta=\beta_2} \tag{35}$$

Brock [10] reported that the optimal value of ξ as follows

$$\xi_{opt}^{FPT}=\xi_{opt}=\sqrt{\frac{\xi_1^2+\xi_2^2}{2}} \tag{36}$$

Substituting Eqs.(34-35) into Eq. (36) we obtain the optimal value of ξ as following

$$\xi_{opt}^{FPT}=\frac{\gamma}{2}\sqrt{\frac{3\mu}{2(1+\mu\gamma^2)}} \tag{37}$$

4. NUMERICAL SIMULATION STUDY

In this section, numerical simulation is employed for the system by using the achieved optimal parameters of the TMD, as shown in Eq. (27) and Eq. (37). To demonstrate the above analysis, computations will be performed for a system with parameters given in Table 1 [9].

Table 1. The input parameters for shaft and TMD

Parameters	M	ρ	k_t	m_t	m	L
Value	500kg	1.0 m	10^5Nm/rad	15kg	10kg	0.9m

From the Eq. (4) and Table 1, the dimensionless parameters can be calculated and shown in Table 2.

Table 2. Value of the dimensionless parameters

Parameters	μ	γ
Value	0.03	0.9

From the Eqs. (27,37) and Table 2, the optimal parameters of the TMD are determined as Table 3.

Table 3. The optimal value of tuning and damping ratios

Optimal Parameters	α_{opt}^{FPT}	ξ_{opt}^{FPT}	C	k_m
Value	0.9537	0.0943	38.16 Ns/m	4419.94Nm/rad

*** Simulation Results**

Numerical simulations for torsional vibration of the machine shaft using the Maple are implemented in different operating conditions. Table 4 shows the different operating conditions of the machine shaft.

Table 4. The different operating conditions of the machine shaft

Cases	1	2	3
θ_0	5×10^{-2} (rad)	0.0(rad)	5×10^{-2} (rad)
$\dot{\theta}_0$	0.0(rad/s)	8×10^{-1}(rad/s)	8×10^{-1}(rad/s)

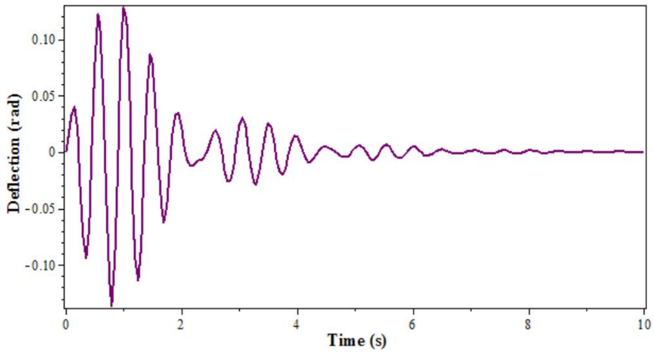

Figure 3. The vibration of the TMD with initial $\theta_0 = 5x10^{-2}$ (rad)

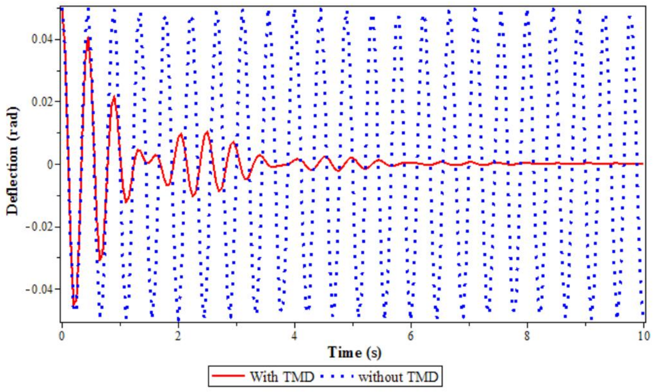

Figure 4. The vibration of the machine shaft with initial $\theta_0 = 5x10^{-2}$ (rad)

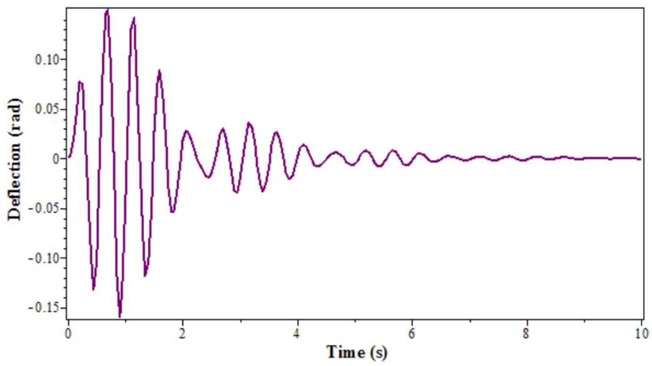

Figure 5. The vibration of the TMD with initial $\dot{\theta}_0 = 8\times10^{-1}$(rad/s)

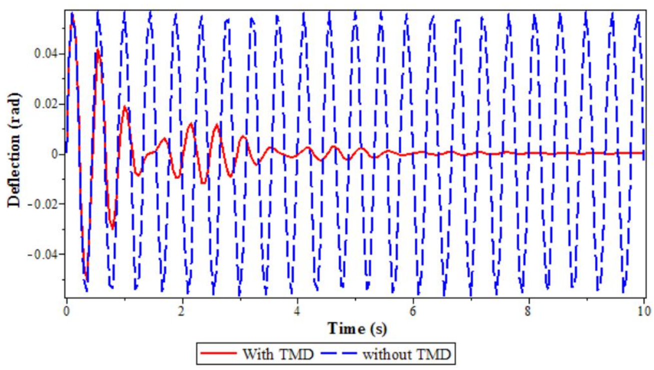

Figure 6. The vibration of the machine shaft with initial $\dot{\theta}_0 = 8\times10^{-1}$(rad/s)

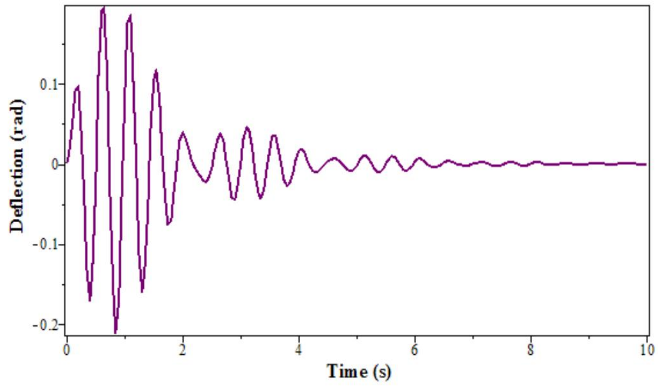

Figure 7. The vibration of the TMD with initials $\theta_0 = 5x10^{-2}$ (rad) and $\dot{\theta}_0 = 8\times10^{-1}$(rad/s)

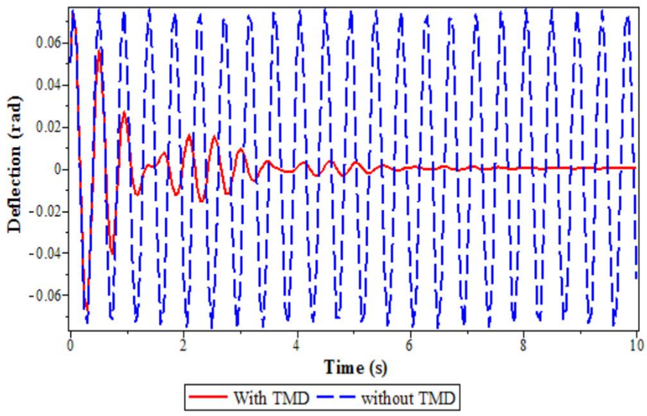

Figure 8. The vibration of the machine shaft with initials $\theta_0 = 5x10^{-2}$ (rad) and $\dot{\theta}_0 = 8\times10^{-1}$(rad/s)

Figs. 3, 5 and 7 show the time response of the TMD's deflection. The responses of the shart are shown in Figs 4, 6 and 8. The results show that the TMD can reduce the torsional vibration of the shaft in all case.

5. CONCLUSION AND DISCUSSION

This paper is concerned with an optimization problem of the tuned mass damper (TMD) for the shaft model. The novelty of this study can be summarized below.

- Optimal parameters of the TMD attached to the shaft using the fixed-point theory are found as in Eqs. (27) and (37).

- Numerical simulation studies are implemented by using the Maple software. Simulation results are shown to validate the reliability and feasibility of the proposed method.

- From the simulation of the vibration amplitude over time, in case the shaft is subject to harmonic excitation, it is found that the amplitude of the vibration of the shaft when designing the TMD according to the optimal parameters of the TMD look in this paper is very good. This meets the technical requirements set out.

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