THE STUDY OF HIGHER-ORDER NONCLASSICAL PROPERTIES OF THE SQUEEZING-ENHANCED COHERENT STATE

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Abstract: In this paper, we study the nonclassical properties such as Hillery's higher-order squeezing effect, higher-order sub-Poissonian statistic, and higher-order antibunching of the squeezing-enhanced coherent state. The results show that the squeezing-enhanced coherent state is stronger squeezing at higher-order, comparing the values of squeezing effect at higher-order to choose the suitable value to enhanced accurate measurements in quantum information, quantum optic, and show characteristic of higher-order sub-Poissonian statistic and higher-order antibunching. All results indicate that higher-order nonclassical properties of the squeezing-enhanced coherent state have stronger than the normal order.

Keyword: Higher order, antibunching, Hillery's squeezing effect, sub-Poissonian statistic.

1. INTRODUCTION

Currently, technology is more and more developing, we need higher exactly transfer information, including speed, security, and data process. In quantum information and quantum entanglement, based on the effect of the nonclassical state, scientists have investigated the states which limit errors in processing formation, giant data, and security. Investigating the nonclassical properties of nonclassical state play a crucial in developing science and technology. In 1963, a coherent state was introduced by Glauber [1] and Sudarshan [2], after that squeezing coherent state was introduced [3]. In 2012, the squeezing-enhanced coherent state was introduced [4], in which the state shows that the degree of squeezing increases not only with squeezing parameter λ, but also with squeezing parameter r. This state also indicates sub-Poissonian statistic normal order [4]. Comparing with the usual squeezing coherent state, all results indicate that this state indeed has stronger squeezing and shows some new statistical properties [4]. According to Ref. [4], the squeezing-enhanced state was introduced as

$$|\psi\rangle = V(\lambda, r)|\alpha\rangle, \quad (1)$$

where $V(\lambda, r) = exp\left[-\frac{i\lambda}{2}(Q^2e^2 - e^{-r}P^2)\right]$ is an enhanced squeezing operator, and the operators $\hat{Q} = \frac{\hat{a}+\hat{a}^\dagger}{\sqrt{2}}$, $\hat{P} = \frac{\hat{a}-\hat{a}^\dagger}{\sqrt{2}i}$, where $\hat{a}$ is bosonic annihilation operator and $\hat{a}^\dagger$ is bosonic creation operator. When parameter $r = 0$, the enhanced squeezing operator

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becomes $V(\lambda,0) = exp\left[\frac{-i\lambda(a^{\dagger 2}+a^2)}{2}\right]$ is the normally squeezing operator [4]. The $|\alpha\rangle$ is a coherent state which was defined as

$$|\alpha\rangle = \exp(-\tfrac{1}{2}|\alpha|^2)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \quad (2)$$

where α is a complex number, and $|n\rangle$ is a Fock state. Recently, we have proposed a simple experimental scheme to implement the squeezing operation of a special case of $\hat{Q}$ and $\hat{P}$ operators in the form as

$$V(\lambda,r) = exp\left[-\frac{i\lambda}{2}(\hat{a}^2\cosh r + (1+2\hat{a}^\dagger\hat{a})\sinh r + \hat{a}^{\dagger 2}\cosh r\right], \quad (3)$$

where λ and r are two real parameters. Studying the low-order non-classical properties of the squeezing-enhanced coherent state was shown in detail [4]. In this paper, we focus on studying the higher-order nonclassical properties that have not yet been performed. We study Hillery's higher-order squeezing effect, higher-order sub-Poissonian statistic, and higher-order antibunching of the squeezing-enhanced coherent state.

2. HILLERYS HIGHER-ORDER SQUEEZING EFFECT OF THE SQUEEZING-ENHANCED COHERENT STATE

Hillery's higher-order criterion was introduced by Hillery [5] and this squeezing effect was studied by many scientists later [6], [7], [8]. A state is called Hillery's higher-order squeezing effect if this state satisfies the inequality as

$$\langle(\Delta\hat{Q}_l(\psi))^2\rangle < \frac{1}{4}\langle F_l\rangle, \ (4)$$

where $\langle F_l\rangle$ can also be normally ordered and its explicit form reads [7]

with $l^{(\tau)} = l(l-1)\ldots(l-\tau+1)$. To be convenient, we give Hillery's higher-order squeezing coefficient as

$$S_l = \frac{4\langle:(\Delta\hat{Q}_l(\psi))^2:\rangle}{\langle F_l\rangle} = \frac{2\{\langle a^{\dagger l}\hat{a}^l\rangle + \Re\left[e^{-2il\psi}\langle\hat{a}^{2l}\rangle\right] - 2(\Re\left[e^{-il\psi}\langle\hat{a}^l\rangle\right])^2}{\sum_{\tau=1}^{l}\frac{l!\,l^\tau}{(l-\tau)!\,\tau!}\langle(\hat{a}^\dagger)^{l-\tau}\hat{a}^{l-\tau}\rangle}, \quad (6)$$

in which the condition of Hillery's higher-order squeezing effect of any state is squeezing if coefficient S_l lies into a range from $-1 \le S_l < 0$ and a state is ideally squeezing state if $S_l = -1$. We study Hillery's first order and Hillery's higher-order squeezing effect of the squeezing-enhanced coherent state at the orders $l = 1, 2, 3, 4$.

2.1. The first-order squeezing effect

We examine the first-order squeezing effect at $l = 1$, which is the low-order squeezing effect. Substituting $l = 1$ into Eq. (5) and Eq. (6), we have Hillery's first-order squeezing coefficient of the squeezing-enhanced coherent state as

$$S_1 = \frac{2\left\{\langle\hat{a}^\dagger\hat{a}\rangle + \Re\left[e^{-2i\varphi}\langle\hat{a}^2\rangle\right] - 2\left(\Re\left[e^{-i\varphi}\langle\hat{a}\rangle\right]\right)^2\right\}}{\langle F_1\rangle}. \quad (7)$$

For $l = 1$, we calculate values of $\langle\hat{a}^\dagger\hat{a}\rangle, \Re\left[e^{-2i\varphi}\langle\hat{a}^2\rangle\right], \Re\left[e^{-i\varphi}\langle\hat{a}\rangle\right], \langle F_1\rangle$ as

$$\langle\hat{a}^\dagger\hat{a}\rangle = [|\alpha|^2(|\mu|^2 + |\nu|^2) + T_1 + |\nu|^2], \quad (8)$$

$$\langle\hat{a}\rangle = (\mu\alpha + \nu\alpha^*), \quad (9)$$

$$\langle\hat{a}^2\rangle = (\mu^2\alpha^2 + \mu\nu(2|\alpha|^2 + 1) + \nu^2\alpha^{*2}), \quad (10)$$

$$\langle F_1\rangle = \langle[\hat{a}, \hat{a}^\dagger]\rangle = \sum_{\tau=1}^{1} \frac{1!\,1^{(1)}}{(1-1)!\,1!}\langle\hat{a}^{\dagger(1-1)}\hat{a}^{(1-1)}\rangle = 1, \quad (11)$$

where $T_1 = \mu^*\nu\alpha^{*2} + \nu^*\mu\alpha^2$.

Substituting Eqs. from (8) to (11) into Eq. (7), we plot a graph of Hillery's first-order squeezing coefficient S_1 as in Figure 1.

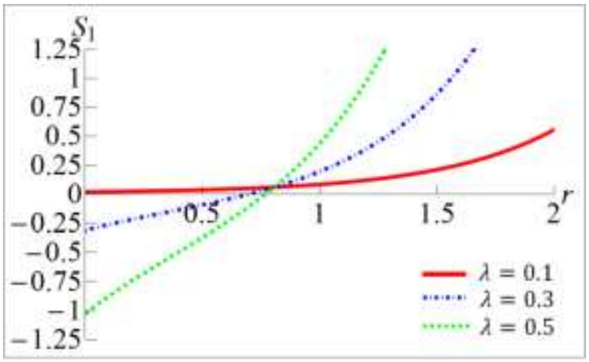

Figure 1. *The squeezing coefficient S_1 is a function of r with different squeezing parameters* $\lambda = 0.1$ *(the solid line),* $\lambda = 0.3$ *(the dot-dashed line), and* $\lambda = 0.5$ *(the dashed line)*

We study the squeezing coefficient S_1 which is a function of r that has a range from 0 to 2, Figure 1 shows that the squeezing coefficient S_1 is more negative as we increase the squeezing parameters $\lambda = 0.1$, 0.3, and 0.5. Therefore, the degree of squeezing is stronger as squeezing parameter λ increases, which is the same as the result [4].

2.2. The second-order squeezing effect

We examine the second-order squeezing effect at $l = 2$, which is the high-order squeezing effect. Substituting $l = 2$ into Eq. (5) and Eq. (6), we have Hillery's second-order squeezing coefficient of the squeezing-enhanced coherent state as

$$S_2 = \frac{2\{\langle\hat{a}^{\dagger 2}\hat{a}^2\rangle + \Re\left[e^{-4il\psi}\langle\hat{a}^4\rangle\right] - 2\left(\Re\left[e^{-2il\psi}\langle\hat{a}^2\rangle\right]\right)^2\}}{\langle F_2\rangle}. \quad (12)$$

For $l = 2$, we calculate values of $\langle\hat{a}^{\dagger 2}\hat{a}^2\rangle, \Re\left[e^{-4il\psi}\langle\hat{a}^4\rangle\right], \Re\left[e^{-2il\psi}\langle\hat{a}^2\rangle\right], \langle \mathrm{F}_2\rangle$ as

$$\langle \hat{a}^{\dagger 2}\hat{a}^2\rangle = (|\alpha|^4(|\mu|^4 + |\alpha|^4) + T_2 + |\mu\nu|^2(1 + 8|\alpha|^2 + 4|\alpha|^4) + |\mu|^2 T_1(2|\alpha|^2 + 1) + |\nu|^2 T_1(5 + 2|\alpha|^2) + |\nu|^4(2 + 4|\alpha|^2)), \quad (13)$$

$$\langle \hat{a}^2\rangle = (\mu^2\alpha^2 + \mu\nu(2|\alpha|^2 + 1) + \nu^2\alpha^{*2}), \quad (14)$$

$$\langle \hat{a}^4\rangle = (\mu^4\alpha^4 + \nu^4\alpha^{*4} + 3\mu^2\nu^2(1 + 4|\alpha|^2 + 2|\alpha|^4) + 2(\mu^3\nu\alpha^2 + \mu\nu^3\alpha^{*2})(2|\alpha|^2 + 3)), \quad (15)$$

$$\langle F_2\rangle = 4[|\alpha|^2(|\mu|^2 + |\nu|^2) + T_1 + |\nu|^2 + 1], \quad (16)$$

where $T_2 = \mu^{*2}\nu^2\alpha^{*4} + \mu^2\nu^{*2}\alpha^4$.

Substituting Eqs. from (13) to (16) into Eq. (12), we plot a graph of Hillery's second-order squeezing coefficient S_2 in Figure 2.

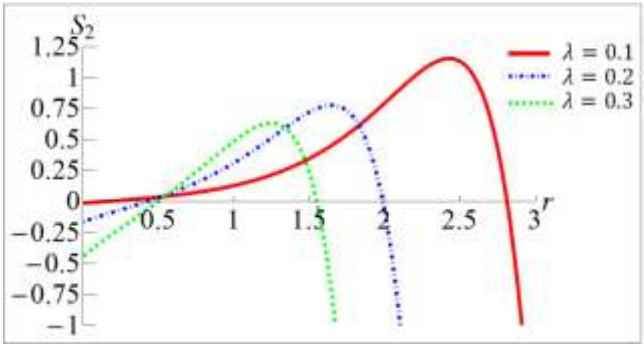

Figure 2. *The squeezing coefficient S_2 is a function of r with different squeezing parameters $\lambda =$ 0.1 (the solid line), $\lambda = 0.2$ (the dot-dashed line), and $\lambda = 0.3$ (the dashed line)*

We study the squeezing coefficient S_2 which is a function of r that has a range from 0 to 3, Figure 2 shows that the squeezing coefficient S_2 is more negative as we increase the squeezing parameters $\lambda = 0.1,\ 0.2,$ and 0.3. Therefore, the degree of squeezing is stronger as squeezing parameter λ increases, which is the same as the result of the first-order squeezing effect.

2.3. The third-order squeezing effect

We examine the third-order squeezing effect at $l = 3$, which is also the high-order squeezing effect. Substituting $l = 3$ into Eq. (5) and Eq. (6), we have Hillery's third-order squeezing coefficient of the squeezing-enhanced coherent state as

$$S_3 = \frac{2\{\langle \hat{a}^{\dagger 3}\hat{a}^3\rangle + \Re\left[e^{-6il\psi}\langle \hat{a}^6\rangle\right] - 2\left(\Re\left[e^{-3il\psi}\langle \hat{a}^3\rangle\right]\right)^2\}}{\langle F_3\rangle}. \quad (17)$$

For $l = 3$, we calculate values of $\langle \hat{a}^{\dagger 3}\hat{a}^3\rangle, \Re\left[e^{-6il\psi}\langle \hat{a}^6\rangle\right], \Re\left[e^{-3il\psi}\langle \hat{a}^3\rangle\right], \langle F_3\rangle$ as

$$\langle \hat{a}^{\dagger 3}\hat{a}^3\rangle = (|\alpha|^6(|\mu|^6 + |\nu|^6) + 3|\mu|^4 T_1(|\alpha|^4 + |\alpha|^2) + 3|\mu|^2 T_2(|\alpha|^2 + 1) + T_3 + 9|\mu|^4|\nu|^2(|\alpha|^6 + 3|\alpha|^4 + |\alpha|^2) + 9|\mu\nu|^2 T_1(|\alpha|^4 + 4|\alpha|^2 + 2)$$

$$+3|\nu|^2T_2(|\alpha|^2+\ 4)+9|\mu|^2|\nu|^4(|\alpha|^6+\ 6|\alpha|^4+7|\alpha|^2+1)$$
$$+3|\nu|^4T_1(|\alpha|^4+\ 7|\alpha|^2+9)+|\nu|^6(9|\alpha|^4+18|\alpha|^2+6)), \tag{18}$$
$$\langle\hat{a}^3\rangle=(\mu^3\alpha^3+3\mu^2\nu(|\alpha|^2\alpha+\alpha)+3\mu\nu^2(|\alpha|^2\alpha^*+\alpha^*)+\nu^3\alpha^{*3}), \tag{19}$$
$$\langle\hat{a}^6\rangle=(\mu^6\alpha^6+\nu^6\alpha^{*6}+15(\mu^2\nu^4\alpha^{*2}+\mu^4\nu^2\alpha^2)(|\alpha|^4+4|\alpha|^2+3)$$
$$+(\mu^5\nu\alpha^4+\mu\nu^5\alpha^{*4})(6|\alpha|^2+15)+5\mu^3\nu^3(4|\alpha|^6+18|\alpha|^4+18|\alpha|^2+3)), \tag{20}$$
$$\langle F_3\rangle=9[|\alpha|^4(|\mu|^4+|\alpha|^4)+T_2+|\mu\nu|^2(1+8|\alpha|^2+4|\alpha|^4)$$
$$+|\mu|^2T_1(2|\alpha|^2+1)+|\nu|^2T_2(5+2|\alpha|^2)+|\nu|^4(2+4|\alpha|^2)$$
$$+3[|\alpha|^2(|\mu|^2+|\nu|^2)+T_1+|\nu|^2]+3], \tag{21}$$

where $T_3=\mu^{*3}\nu^3\alpha^{*6}\ +\ \mu^3\nu^{*3}\alpha^6$.

Substituting Eqs. from (18) to (21) into Eq. (17), we plot a graph of Hillery's third-order squeezing coefficient S_3 in Figure 3.

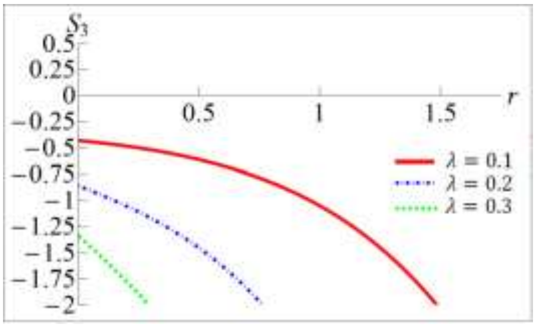

Figure 3. *The squeezing coefficient S_3 is a function of r with different squeezing parameters $\lambda =$* 0.1 *(the solid line), $\lambda = 0.2$ (the dot-dashed line), and $\lambda = 0.3$ (the dashed line)*

We study the squeezing coefficient S_3 which is a function of r that has a range from 0 to 1.5, Figure 3 shows that the squeezing coefficient S_3 is more negative as we increase the squeezing parameters $\lambda = 0.1,\ 0.2,$ and 0.3. Therefore, the degree of squeezing is stronger as squeezing parameter λ increases, which is the same as the result of the first-order and the second-order.

2.4. The fourth-order squeezing effect

Similarly we substitute $l = 4$ into Eqs (5), Eqs (6). We have Hillery's fourth-order squeezing coefficient of the squeezing-enhanced coherent state as

$$S_4=\frac{2\{\langle\hat{a}^{\dagger4}\hat{a}^4\rangle+\Re\left[e^{-8il\psi}\langle\hat{a}^8\rangle\right]-2(\Re\left[e^{-4il\psi}\langle\hat{a}^4\rangle\right])^2}{\langle F_4\rangle}. \tag{22}$$

For $l = 4$, we calculate values of $\langle\hat{a}^{\dagger4}\hat{a}^4\rangle, \Re\left[e^{-8il\psi}\langle\hat{a}^8\rangle\right], \Re\left[e^{-4il\psi}\langle\hat{a}^4\rangle\right], \langle F_4\rangle$ as

$$\langle a^{\dagger4}\hat{a}^4\rangle=(2|\mu|^6T_1(2|\alpha|^6+3|\alpha|^4)+3|\mu|^4T_2(1+4|\alpha|^2+2|\alpha|^4)$$

$$+2|\mu|^2T_3(2|\alpha|^2+3)+T_4+4|\mu|^6|\nu|^2(4|\alpha|^8+16|\alpha|^6+9|\alpha|^4)$$
$$+6|\mu|^4|\nu|^2T_1(4|\alpha|^6+22|\alpha|^4+22|\alpha|^2+3)+4|\nu\mu|^2T_2(4|\alpha|^4+24|\alpha|^2+21)$$
$$+2|\nu|^2T_3(11+2|\alpha|^2)+9|\mu\nu|^4(1+24|\alpha|^2+60|\alpha|^4+32|\alpha|^6+4|\alpha|^8)$$
$$+6|\mu|^2|\nu|^4T_1(4|\alpha|^6+38|\alpha|^4+86|\alpha|^2+39)+3|\nu|^4T_2(41+20|\alpha|^2+2|\alpha|^4)$$
$$+4|\mu|^2|\nu|^6(4|\alpha|^8+48|\alpha|^6+153|\alpha|^4+132|\alpha|^2+18)+2|\nu|^6T_1(84+96|\alpha|^2$$
$$+27|\alpha|^4+2|\alpha|^6)+(|\mu|^8+|\nu|^8)|\alpha|^8+|\nu|^8(4+32|\alpha|^2+38|\alpha|^4+12|\alpha|^6), \quad (23)$$

$$\langle\hat{a}^4\rangle=(\mu^4\alpha^4+\nu^4\alpha^{*4}+3\mu^2\nu^2(1+4|\alpha|^2+2|\alpha|^4)$$
$$+2(\mu^3\nu\alpha^2+\mu\nu^3\alpha^{*2})(2|\alpha|^2+3)), \quad (24)$$

$$\langle\hat{a}^8\rangle=(\mu^8\alpha^8+\nu^8\alpha^{*8}+2(\mu^7\nu\alpha^6+\mu\nu^7\alpha^{*6})(14+4|\alpha|^2)$$
$$+(\mu^6\nu^2\alpha^4+\mu^2\nu^6\alpha^{*4})(210+168|\alpha|^2+28|\alpha|^4)$$
$$+2(\mu^5\nu^3\alpha^2+\mu^3\nu^5\alpha^{*2})(210+420|\alpha|^2+210|\alpha|^4+28|\alpha|^6)$$
$$+5\mu^4\nu^4(21+168|\alpha|^2+252|\alpha|^4+112|\alpha|^6+14|\alpha|^8), \quad (25)$$

$$\langle F_4\rangle=16(|\alpha|^6(|\mu|^6+|\nu|^6)+3|\mu|^4T_1(|\alpha|^4+|\alpha|^2)+3|\mu|^2T_2(|\alpha|^2+1)$$
$$+T_3+9|\mu|^4|\nu|^2(|\alpha|^6+3|\alpha|^4+|\alpha|^2)+9|\mu\nu|^2T_1(|\alpha|^4+4|\alpha|^2+2)$$
$$+3|\nu|^2T_2(|\alpha|^2+4)+9|\mu|^2|\nu|^4(|\alpha|^6+6|\alpha|^4+7|\alpha|^2+1)$$
$$+3|\nu|^4T_1(|\alpha|^4+7|\alpha|^2+9)+|\nu|^6(9|\alpha|^4+18|\alpha|^2+6)$$
$$+6[|\alpha|^4(|\mu|^4+|\alpha|^4)+T_2+|\mu\nu|^2(1+8|\alpha|^2+4|\alpha|^4)+|\mu|^2T_1(2|\alpha|^2+1)+|\nu|^2T_1(5+2|\alpha|^2)+|\nu|^4(2+4|\alpha|^2)]$$
$$+16[|\alpha|^2(|\mu|^2+|\nu|^2)+T_1+|\nu|^2]+16) \quad (26)$$

where $T_4=\mu^{*4}\nu^4\alpha^{*8}+\mu^4\nu^{*4}\alpha^8$.

Substituting Eqs. from (23) to (26) into Eq. (22), we plot a graph of Hillery's fourth-order squeezing coefficient S_4 in Figure 4.

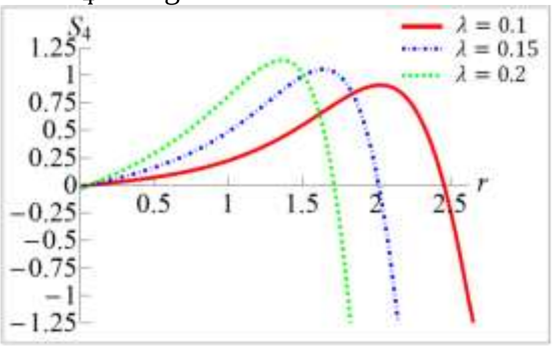

Figure 4. *The squeezing coefficient* S_4 *is a function of* r *with different squeezing parameters* $\lambda=0.1$ *(the solid line),* $\lambda=0.15$ *(the dot-dashed line), and* $\lambda=0.2$ *(the dashed line)*

We study the squeezing coefficient S_4 which is a function of r that has a range from 0 to 2.5, Figure 4 shows that the squeezing coefficient S_4 is more negative as we increase the squeezing parameters $\lambda = 0.1,\ 0.15,$ and 0.2. Therefore, the degree of squeezing effect is also stronger as squeezing parameter λ increases, which is the same as the result of the first-order, the second-order, and the third-order.

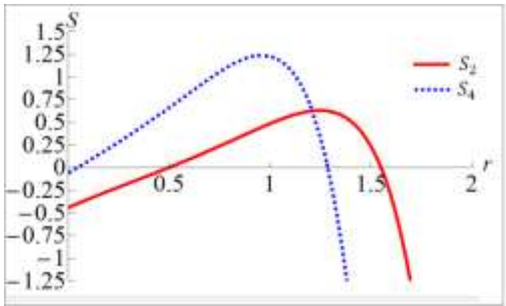

Figure 5. *The difference between the squeezing coefficient S_2 (the solid line) and the squeezing coefficient S_4 (the dashed line) at squeezing parameter $\lambda = 0.3$*

In Figure 5, we observe the graph of the second-order squeezing coefficient S_2 and the graph of the fourth-order squeezing coefficient S_4 at $\lambda = 0.3$ with r from 0 to 2, which show that the degree of squeezing of the second-order squeezing effect is stronger than the degree of squeezing of the fourth-order squeezing effect. This means that at even order, the low-order squeezing effect show to be stronger than the high-order squeezing effect.

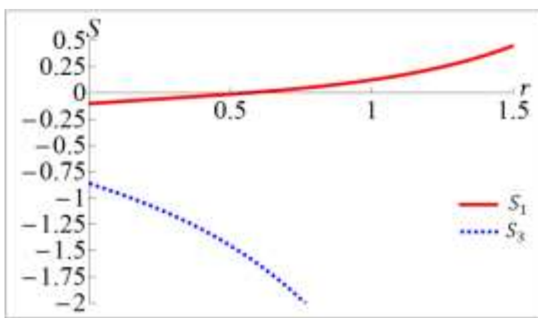

Figure 6. *The difference between the squeezing coefficient S_1 (the solid line) and the squeezing coefficient S_3 (the dashed line) at squeezing parameter $\lambda = 0.2$*

In Figure 6, we observe the graph of the first-order squeezing coefficient S_1 and the graph of the third-order squeezing coefficient S_3 at $\lambda = 0.2$ with r from 0 to 1.5, which show that the degree of squeezing of the third-order squeezing effect is stronger than the degree of squeezing of the first-order squeezing effect. This means that at odd order, the high-order squeezing effect show to be stronger than the low-order squeezing effect.

In conclusion, we conclude that the degree of Hillery's squeezing effect shows to be stronger as we increase squeezing parameter λ. For even order, the degree of squeezing is weaker at high order, and the degree of squeezing is stronger at high order for odd order.

3. HIGHER-ORDER SUB-POISSONIAN STATISTIC AND HIGHER-ORDER ANTIBUNCHING OF THE SQUEEZING-ENHANCED COHERENT STATE

Concept of higher-order sub-Poissonian statistic is recommended in [9]. Using formula $\langle \hat{n}^{(l)} \rangle = \langle \hat{n}(\hat{n}-1) \ldots . (\hat{n}-l+1) \rangle = \langle \hat{a}^{\dagger l} \hat{a}^{l} \rangle$ with $\langle \hat{n} \rangle = \langle \hat{a}^{\dagger} \hat{a} \rangle$, coefficient P_l is defined as [8]

$$P_l = \frac{\langle a^{\dagger l} \hat{a}^{l} \rangle}{(\langle \hat{a}^{\dagger} \hat{a} \rangle)^{l}} - 1, \tag{27}$$

with l is a positive number. For all values of $l \geq 2,\ P_l = 0$, this is the Poisson statistic. For $l = 2, P_l > 0$ show that the first-order super-Poissionian statistic and $P_l < 0$ show that the first-order sub-Poissonian statistic. For $l > 2, P_l > 0$ show that higher-order super-Poissionian statistic and $P_l < 0$ show that higher-order sub-Poissonian statistic. Because sub-Poissonian statistic and antibunching are relevant to the mathematical side, which is function depends on a parameter such as l, α, r, λ and from the concept of sub-Poissonian statistic and antibunching, if the state shows sub-Poissonian statistic which will show antibunching. Now, we will study higher-order sub-Poissonian statistic and antibunching of squeezing-enhanced coherent state at the orders $l = 2, 3,$ and 4.

3.1. The first-order sub-Poissonian statistic and the first-order antibunching

We examine the first-order sub-Poissonian statistic and the first-order antibunching at $l = 2$. Substituting $l = 2$ into Eq. (27), we have coefficient P_2 of the squeezing-enhanced coherent state as

$$P_2 = \frac{\langle a^{\dagger 2} \hat{a}^{2} \rangle}{(\langle \hat{a}^{\dagger} \hat{a} \rangle)^{2}} - 1. \tag{28}$$

For $l = 2$, we calculate values of $\langle \hat{a}^{\dagger} \hat{a} \rangle, \langle a^{\dagger 2} \hat{a}^{2} \rangle$ as

$$\langle \hat{a}^{\dagger} \hat{a} \rangle \quad = [|\alpha|^2(|\mu|^2 + |\nu|^2) + T_1 + |\nu|^2], \tag{29}$$

$$\begin{aligned} \langle a^{\dagger 2} \hat{a}^{2} \rangle &= (|\alpha|^4(|\mu|^4 + |\alpha|^4) + T_2 + |\mu\nu|^2(1 + 8|\alpha|^2 + 4|\alpha|^4) \\ &+|\mu|^2 T_1(2|\alpha|^2 + 1) + |\nu|^2 T_1(5 + 2|\alpha|^2) + |\nu|^4(2 + 4|\alpha|^2). \end{aligned} \tag{30}$$

Substituting Eq. (29) and Eq. (30) into Eq. (28), we have coefficient P_2 as

$$\begin{aligned} P_2 \ &= \{(|\alpha|^4(|\mu|^4 + |\alpha|^4) + T_2 + |\mu\nu|^2(1 + 8|\alpha|^2 + 4|\alpha|^4) \\ &\quad +|\mu|^2 T_1(2|\alpha|^2 + 1) + |\nu|^2 T_1(5 + 2|\alpha|^2) + |\nu|^4(2 + 4|\alpha|^2)) \\ &\quad \times [|\alpha|^2(|\mu|^2 + |\nu|^2) + T_1 + |\nu|^2]^{-2}\} - 1. \end{aligned} \tag{31}$$

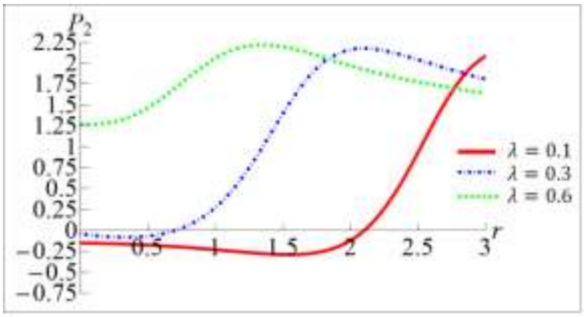

Figure 7. *The sub-**Poissonian** statistic P_2 is a function of r with different squeezing parameters $\lambda = 0.1$ (the solid line), $\lambda = 0.3$ (the dot-dashed line), and $\lambda = 0.6$ (the dashed line)*

We plot a graph of the first-order sub-Poissonian statistic P_2 in Figure 7. It is showed that the first-order sub-Poissonian statistic of the squeezing-enhanced coherent state shows to be weaker as we increase the squeezing parameters $\lambda =$ 0.1, 0.3, and 0.6. This means that the graph is larger than zero as squeezing parameter λ increases. Therefore, the degree of the first-order antibunching is also weaker as we increase squeezing parameter λ because sub- Poissonian statistic and antibunching are relevant to the mathematical side, which is the same as the result [4].

3.2. The second-order sub-Poissonian statistic and the second-order antibunching

We examine the second-order sub-Poissonian statistic and the second-order antibunching at $l = 3$. Substituting $l = 3$ into Eq. (27), we have coefficient P_3 of the squeezing-enhanced coherent state as

$$P_3 = \frac{\langle a^{\dagger 3}\hat{a}^3\rangle}{(\langle\hat{a}^\dagger\hat{a}\rangle)^3} - 1. \tag{32}$$

For $l = 3$, we calculate value of $\langle a^{\dagger 3}\hat{a}^3\rangle$ as

$$\begin{aligned}\langle a^{\dagger 3}\hat{a}^3\rangle = (&|\alpha|^6(|\mu|^6 + |\nu|^6) + 3|\mu|^4T_1(|\alpha|^4 + |\alpha|^2) + 3|\mu|^2T_2(|\alpha|^2 + 1)\\ &+ T_3 + 9|\mu|^4|\nu|^2(|\alpha|^6 + 3|\alpha|^4 + |\alpha|^2) + 9|\mu\nu|^2T_1(|\alpha|^4 + 4|\alpha|^2 + 2)\\ &+3|\nu|^2T_2(|\alpha|^2 + 4) + 9|\mu|^2|\nu|^4(|\alpha|^6 + 6|\alpha|^4 + 7|\alpha|^2 + 1)\\ &+3|\nu|^4T_1(|\alpha|^4 + 7|\alpha|^2 + 9) + |\nu|^6(9|\alpha|^4 + 18|\alpha|^2 + 6)).\end{aligned} \tag{33}$$

Substituting Eq. (33) and Eq. (29) into Eq. (32), we have coefficient P_3 of the squeezing-enhanced coherent state as

$$\begin{aligned}P_3 = [&(|\alpha|^6(|\mu|^6 + |\nu|^6) + 3|\mu|^4T_1(|\alpha|^4 + |\alpha|^2) + 3|\mu|^2T_2(|\alpha|^2 + 1)\\ &+T_3 + 9|\mu|^4|\nu|^2(|\alpha|^6 + 3|\alpha|^4 + |\alpha|^2) + 9|\mu\nu|^2T_1(|\alpha|^4 + 4|\alpha|^2 + 2)\\ &+3|\nu|^2T_2(|\alpha|^2 + 4) + 9|\mu|^2|\nu|^4(|\alpha|^6 + 6|\alpha|^4 + 7|\alpha|^2 + 1)\\ &+3|\nu|^4T_1(|\alpha|^4 + 7|\alpha|^2 + 9) + |\nu|^6(9|\alpha|^4 + 18|\alpha|^2 + 6)]\\ &\times [|\alpha|^2(|\mu|^2 + |\nu|^2) + T_1 + |\nu|^2]^{-3} - 1.\end{aligned} \tag{34}$$

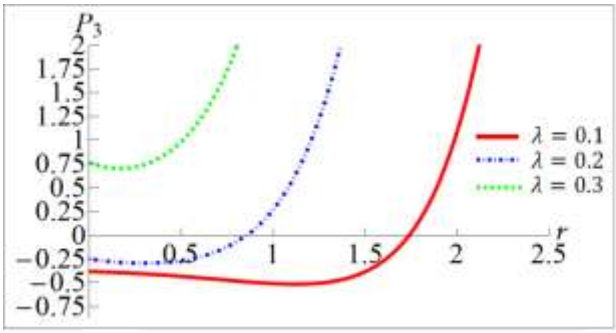

Figure 8. *The sub-**Poissonian** statistic P_3 is a function of r with different squeezing parameters $\lambda = 0.1$ (the solid line), $\lambda = 0.2$ (the dot-dashed line), and $\lambda = 0.3$ (the dashed line)*

We plot a graph of the second-order sub-Poissonian statistic of P_3 in Figure 8. It is showed that the second-order sub-Poissonian statistic of the squeezing-enhanced coherent state shows to be weaker as we increase the squeezing parameters $\lambda = 0.1$, 0.2 and 0.3. This means that the graph is larger than zero as squeezing parameter λ increases. Therefore, the degree of the second-order antibunching is also weaker as we increase squeezing parameter λ. Likewise, the results of the second-order are the same as the first-order, but the second-order is expressed clearly more than the first-order.

3.3. The third-order sub-Poissonian statistic and the third-order antibunching

For the third-order, we study at $l = 4$. Substituting $l = 4$ into Eq. (27), we have coefficient P_4 of the squeezing-enhanced coherent state

$$P_4 = \frac{\langle a^{\dagger 4}\hat{a}^4\rangle}{(\langle\hat{a}^\dagger\hat{a}\rangle)^4} - 1. \tag{35}$$

For $l = 4$, we calculate value of $\langle a^{\dagger 4}\hat{a}^4\rangle$ as

$$\begin{aligned}\langle a^{\dagger 4}\hat{a}^4\rangle = [&(2|\mu|^6T_1(2|\alpha|^6 + 3|\alpha|^4) + 3|\mu|^4T_2(1 + 4|\alpha|^2 + 2|\alpha|^4)\\ &+2|\mu|^2T_3(2|\alpha|^2 + 3) + T_4 + 4|\mu|^6|\nu|^2(4|\alpha|^8 + 16|\alpha|^6 + 9|\alpha|^4)\\ &+6|\mu|^4|\nu|^2T_1(4|\alpha|^6 + 22|\alpha|^4 + 22|\alpha|^2 + 3) + 4|\nu\mu|^2T_2(4|\alpha|^4 + 24|\alpha|^2 + 21)\\ &+2|\nu|^2T_3(11 + 2|\alpha|^2) + 9|\mu\nu|^4(1 + 24|\alpha|^2 + 60|\alpha|^4 + 32|\alpha|^6 + 4|\alpha|^8)\\ &+6|\mu|^2|\nu|^4T_1(4|\alpha|^6 + 38|\alpha|^4 + 86|\alpha|^2 + 39) + 3|\nu|^4T_2(41 + 20|\alpha|^2 + 2|\alpha|^4)\\ &+4|\mu|^2|\nu|^6(4|\alpha|^8 + 48|\alpha|^6 + 153|\alpha|^4 + 132|\alpha|^2 + 18) + 2|\nu|^6T_1(84 + 96|\alpha|^2\\ &+27|\alpha|^4 + 2|\alpha|^6) + (|\mu|^8 + |\nu|^8)|\alpha|^8 + |\nu|^8(4 + 32|\alpha|^2 + 38|\alpha|^4 + 12|\alpha|^6)].\end{aligned} \tag{36}$$

Substituting Eq. (36) and Eq. (29) into Eq. (35), we have coefficient P_4 of the squeezing-enhanced coherent state

$$\begin{aligned}P_4 = [&(2|\mu|^6T_1(2|\alpha|^6 + 3|\alpha|^4) + 3|\mu|^4T_2(1 + 4|\alpha|^2 + 2|\alpha|^4)\\ &+2|\mu|^2T_3(2|\alpha|^2 + 3) + T_4 + 4|\mu|^6|\nu|^2(4|\alpha|^8 + 16|\alpha|^6 + 9|\alpha|^4)\end{aligned}$$

$$+6|\mu|^4|\nu|^2T_1(4|\alpha|^6+22|\alpha|^4+22|\alpha|^2+3)+4|\nu\mu|^2T_2(4|\alpha|^4+24|\alpha|^2+21)$$
$$+2|\nu|^2T_3(11+2|\alpha|^2)+9|\mu\nu|^4(1+24|\alpha|^2+60|\alpha|^4+32|\alpha|^6+4|\alpha|^8)$$
$$+6|\mu|^2|\nu|^4T_1(4|\alpha|^6+38|\alpha|^4+86|\alpha|^2+39)+3|\nu|^4T_2(41+20|\alpha|^2+2|\alpha|^4)$$
$$+4|\mu|^2|\nu|^6(4|\alpha|^8+48|\alpha|^6+153|\alpha|^4+132|\alpha|^2+18)+2|\nu|^6T_1(84+96|\alpha|^2$$
$$+27|\alpha|^4+2|\alpha|^6)+(|\mu|^8+|\nu|^8)|\alpha|^8+|\nu|^8(4+32|\alpha|^2+38|\alpha|^4+12|\alpha|^6)]$$
$$\times[|\alpha|^2(|\mu|^2+|\nu|^2)+T_1+|\nu|^2]^{-4}-1. \tag{37}$$

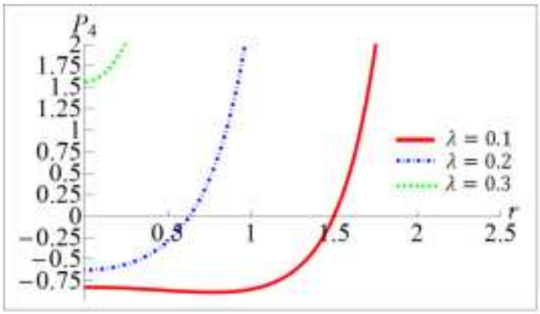

Figure 9. *The sub-**Poissonian** statistic P_4 is a function of r with different squeezing parameters $\lambda = 0.1$ (the solid line), $\lambda = 0.2$ (the dot-dashed line), and $\lambda = 0.3$ (the dashed line)*

We plot a graph of the third-order sub-Poissonian statistic of P_4 in Figure 9. It is showed that the third-order sub-Poissonian statistic of the squeezing-enhanced coherent state shows to be weaker as we increase the squeezing parameters $\lambda = 0.1, 0.2,$ and 0.3. This means that the graph is larger than zero as squeezing parameter λ increases. Therefore, the degree of the third-order antibunching is also weaker as we increase squeezing parameter λ. We observe that the results of the third-order are the same as the first-order and the second-order. However, the third-order shows to be clear more than the first-order and the second-order via the value of P_3 which is larger than the values of P_2 and P_1. From the results of the first-order, the second-order, and the third-order were shown in alternate Figure 7, 8, and 9 of sub-Poissonian statistic and antibunching of the squeezing-enhanced coherent state, we conclude that these effects are relevant together, which means that sub-Poissonian statistic and antibunching show to be weaker as squeezing parameter λ increases at higher order.

4. CONCLUSION

In this paper, we study the higher-order non-classical properties of the squeezing-enhanced coherent state, including Hillery's higher-order squeezing effect, higher-order sub-Poissonian statistic, and higher-order antibunching of the squeezing-enhanced coherent state. We observe that Hillery's higher-order squeezing effect shows to be stronger as squeezing parameter λ increases, but even order shows that the high-order squeezing effect shows to be weaker than the low-order squeezing effect. However, the high-order squeezing effect show to be stronger than the low-order squeezing effect for odd order. Besides, we realize that higher-order sub-Poissonian statistic is weaker as we increase values l and λ. Because sub-Poissonian statistic and antibunching are relevant to the mathematical side, which also shows that two effects are also the same as a graph, and this shows that antibunching is also weaker as we increase values l and λ. Therefore, we conclude that the squeezing-enhanced coherent state shows the higher-order non-classical properties.

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