HIGH PERFORMANCE CONCRETE MIXTURE PROPORTIONING: MULTI-OBJECTIVE OPTIMIZATION APPROACH

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ABSTRACT

This paper presents the application of multi-objective optimization approach to high performance concrete mixture proportioning. An integrated mathematical model was developed in order to optimize six criteria, which are the chlorine ion diffusion coefficient, per cubic meter cost, the amount of cement, fly ash, slag, chemical admixture. This model needs to satisfy with ten functional constraints and seven design variables. The Visual Interactive Analysis Method (VIAM) was used to solve the multicriteria task. Eventually, twelve solutions have been found for the different cases in terms of criteria during the process of proportioning high performance concrete mixture. They are all Pareto solutions, which allow experts to choose in the proposed cases.

Keywords: High performance concrete; mix proportion; multi-objective optimization; Pareto solution; Visual Interactive Analysis Method; VIAM.

1. Introduction

The parts of the world in which large-scale concrete construction takes place have extended enormously. Due to the recent trends in construction industries (i.e., increased number of heavily reinforced concrete structures), construction of large and taller structures, and developments of construction techniques (i.e., efficient concrete pumping techniques), the industries and companies in general strive to cast massive volume of concrete. When this large volume of concrete is used for construction, the safety and durability of cast concrete become fundamental issues. To ensure these issues, much effort has been focused on the developments of high-performance concrete (Neville and Aitcin, 1998).

High-performance concrete is designed to give optimized performance characteristics for a given set of materials, usage, and exposure conditions, consistent with strength, workability, service life, and durability. Engineers and constructors all over the world are finding that using high performance concrete allows them to build more serviceable structures at comparable cost. High-performance concrete is being used for structures in aggressive environments: marine structures, highway bridges and pavements, nuclear structures, tunnels, precast units, etc. (Aitcin, 2000).

Meanwhile, in Vietnam in recent years, high-performance concrete has played an important role in the engineering structures like bridges, roads, high-rise buildings in the big cities (Hanoi, Ho. Ho Chi Minh City, Da Nang). Especially, in the construction of reinforced concrete bridge and tunnel by new technology high-performance concrete was used properly, such as intersections at Chuong Duong Bridge in Hanoi, Hai Van tunnel in Da Nang or Thu Thiem tunnel in Ho Chi Minh (Pham, 2008).

The major difference between conventional concrete and high-performance concrete is essentially the use of chemical and mineral admixtures. The use of chemical admixtures reduces the water content, thereby at the same time reduces the porosity within the hydrated cement paste. The reduction in the water content to a very low value with high dosage of chemical admixtures is undesirable, and the effectiveness of chemical admixtures such as superplasticizer principally depends on the ambient temperature, cement chemistry, and fineness. Mineral admixtures, also called as cement replacement materials, act as pozzolanic materials as well as fine fillers; thereby, the microstructure of hardened cement matrix becomes denser and stronger. At ambient temperature, their chemical reaction with calcium hydroxide is generally slow. However, the finer and more vitreous the pozzolan is, the faster will be this reaction. If durability is of primary interest, then the slow rate of setting and hardening associated with the incorporation of fly ash or slag in concrete is advantageous. Also, the mineral admixtures are generally industrial by-products and their use can provide a major economic benefit. Therefore, the combined use of superplasticizer and cement replacement materials can lead to economical high-performance concrete with enhanced strength, workability, and durability. It is also reported that the concrete containing cement replacement materials typically provides lower permeability, reduced heat of hydration, reduced alkali–aggregate reaction, higher strength at later ages, and increased resistance to attack from sulfates. However, the effect of cement replacement materials on the performance of concrete varies markedly with their properties (Hassan et al. 2000). To obtain the special combinations of performance and uniformity requirements, a near-optimum mix proportion of high-performance concrete is very important.

In this paper, high-performance concrete of class 60 MPa is a selected object used for the multi-objective optimization. The constituent materials of this concrete are Portland cement, water, fly ash, fine slag, sand, stone and chemical admixture, as illustrated in Figure 1. The costly materials such as cement, slag, fly ash and admixture, cost of 1m3 concrete, and diffusion factor, which represents concrete durability are the objective functions. The optimal solution for mix proportion should be a concrete with low costly materials content, low diffusivity and low total cost of 1m^3 concrete.

Figure 1. Concrete constituent materials for high-performance concrete

2. Problem statement

The literature review has revealed that in Xie's work (Xie et al., 2011), a mathematical model for multi-objective optimization of concrete mix has been established. However, these authors only have considered two criteria such as the chlorine ion diffusion coefficient and cost of 1m3. In fact, the amounts of costly components like Portland cement, fly ash, slag and, chemical admixtures, which are also criteria in objective function, need to be minimized when designing a concrete mix. Therefore, in this paper, an integrated mathematical model was developed for multicriteria design of high performance concrete, which is better adapted to the production process in real conditions in Vietnam. Therefore, the cost of constitutent materials, which is considered in this paper, was taken at the current circumstance at the area of Ho Chi Minh City.

Mathematical model of the problem in this paper are presented in the diagram below (Figure 2).

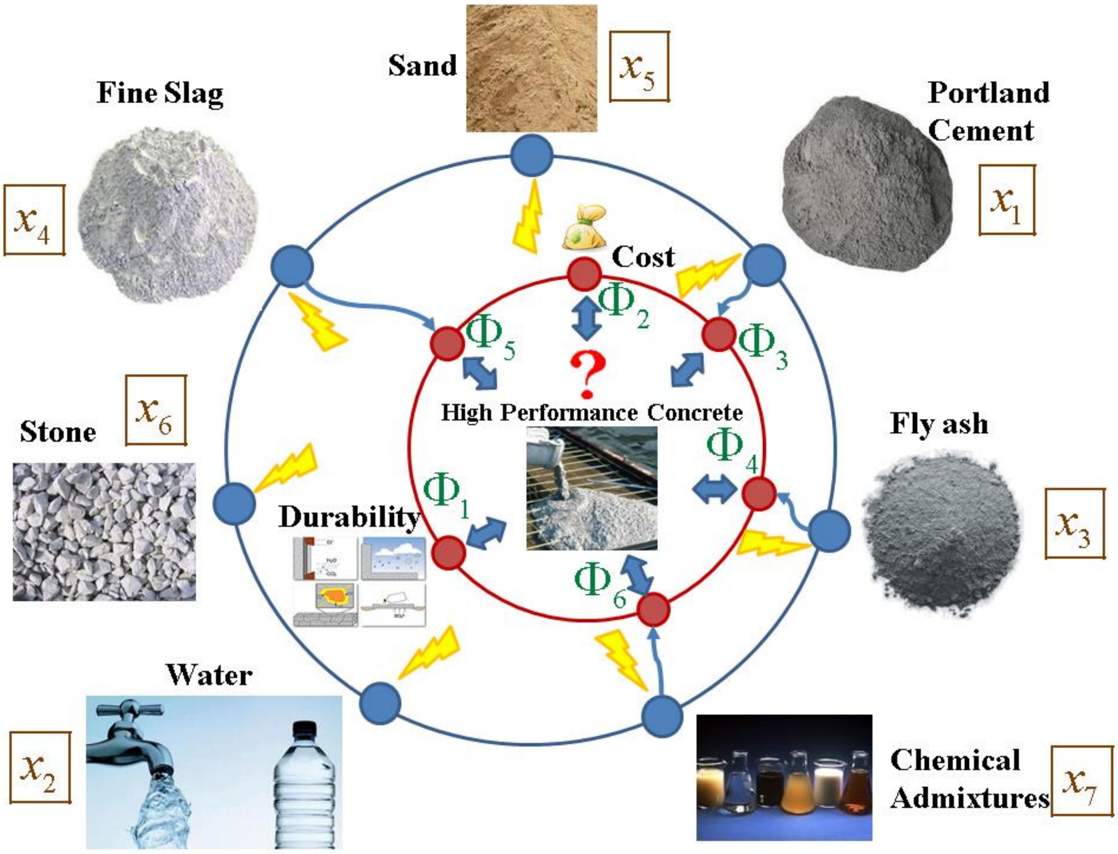

Figure 2. Model for multicriteria design of high performance concrete mix

In this model, three factors are variables, constraints and criteria, which are stated as follows:

Design variable

The control variables and their corresponding contraints in the mathematical model are included in Table 1.

Table 1

Design variables and their constraint

Design variable	*Meaning:* Amount of materials	*Units*	*Initial lower admissible value*	*Initial upper admissible value*
x_1	Portland cement	kg/m^3	300	500
x_2	Water	kg/m^3	130	210
x_3	Fly ash	kg/m^3	45	155
x_4	Fine slag	kg/m^3	60	200

Design variable	*Meaning:* Amount of materials	*Units*	*Initial lower admissible value*	*Initial upper admissible value*
x_5	Sand	kg/m^3	500	1000
x_6	Stone	kg/m^3	900	1400
x_7	Chemical Admixtures	kg/m^3	2.5	12

Functional constraints

The functional constraints are given by the following equality and inequalities (see Table 2).

Table 2
Functional constraints

Function	*Expression*	*Type of constraint*	*Meaning*
f_1	$-\dfrac{x_2}{x_1+x_3+x_4}+0.2$	≤ 0	The range of water to binder ratio
f_2	$\dfrac{x_2}{x_1+x_3+x_4}-0.4$	≤ 0	
f_3	$-\dfrac{x_5}{x_5+x_6}+0.35$	≤ 0	The range of sand ratio, which is the ratio of the amount of sand to the amount of overall aggregates
f_4	$\dfrac{x_5}{x_5+x_6}-0.4$	≤ 0	
f_5	$450-(x_1+x_3+x_4)$	≤ 0	The range of the amount of cementitious material including cement, fly ash and slag.
f_6	$x_1+x_3+x_4-600$	≤ 0	
f_7	$-\dfrac{x_7}{x_1+x_3+x_4}+0.01$	≤ 0	The High–Range Water–Reducing Admixture (HRWRA) is used to improve the workability and micro-structure of concrete. These are its ratio to cement
f_8	$\dfrac{x_7}{x_1+x_3+x_4}-0.02$	≤ 0	
f_9	$\sum_{i=1}^{7}\dfrac{x_i}{\rho_i}-990$	$= 0$	The volume of concrete mixture is made up of the absolute volume of each content and the volume of the air captured in the mixture. The following expression should be met for the amount of materials for each cubic meter of concrete mixture
f_{10}	$-0.304\lambda_c f_{ce,k}\left(\dfrac{x_1+x_3+x_4}{x_2}+0.62\right)$ $+f_{cu,k}-t\sigma$	≤ 0	The strength of concrete, which is affected by various factors, is the most important parameter in concrete design

where ρ_i (i = 1..7) represents the density of each ingredient (ton/m^3): ρ_1 = 3.11; ρ_2 = 1; ρ_3 = 2.11; ρ_4 = 2.45; ρ_5 = 2.61; ρ_6 = 2.76; ρ_7 = 1.08. λ_c is the affluence coefficient of the strength class of concrete. It should be determined according to statistics and in general cases it can be 1.13; $f_{ce,k}$ represents the grading strength of cement and $f_{ce,k}$ = 50.5; $f_{cu,k}$ is the standard value of compressive strength of concrete and $f_{cu,k}$ = 68; t is the degree of probability and t = –1.64; σ is the standard deviation of concrete strength. It is determined according to the national standard code for acceptance of constructional quality of concrete structure and σ = 5 (Pham, 2008).

Performance criteria

The performance criteria are shown in Table 3:

Table 3
Performance criteria

Criteria	*Expression*	*Meaning*
Φ$_1$ → MIN	$$\frac{\left\{\left[5.760+5.81\cdot\left(\frac{x_2}{x_1+x_3+x_4}-0.45\right)\Big/0.2 -0.567\cdot\left(x_1+x_3+x_4-425\right)/175+1.323 +0.74\cdot\left(\frac{x_3}{x_1+x_3+x_4}\cdot100-22.5\right)\Big/22.5 -2.117\cdot\left(\frac{x_4}{x_1+x_3+x_4}\cdot100-35\right)\Big/35 -\left(2.78\cdot0.472\right)-\left(0.254\cdot0.286\right)-\left(0.368\cdot1\right) +1.171\cdot\frac{\frac{x_2}{x_1+x_3+x_4}-0.45}{0.2}\cdot\frac{\frac{x_3}{x_1+x_3+x_4}\cdot100-22.5}{22.5} -2.891\cdot\frac{\frac{x_2}{x_1+x_3+x_4}-0.45}{0.2}\cdot0.472 -1.053\cdot\frac{\frac{x_3}{x_1+x_3+x_4}\cdot100-22.5}{22.5}\cdot0.472\right]^2\right\}}{365\cdot24\cdot3600}\cdot10^{-6}$$	The chlorine ion diffusion coefficient on the 28th day for concrete without microsilica under a molding temperature of 21 Celsius degree. (m^2/s)
Φ$_2$ → MIN	$\sum_{i=1}^{7}\left(y_i\cdot x_i\right)$	Per cubic meter cost (VND/m^3)
Φ$_3$ → MIN	x_1	Amount of Portland cement per cubic meter (kg/m^3)
Φ$_4$ → MIN	x_3	Amount of Fly ash per cubic meter (kg/m^3)
Φ$_5$ → MIN	x_4	Amount of Fine slag per cubic meter (kg/m^3)
Φ$_6$ → MIN	x_7	Amount of Chemical Admixtures per cubic meter (kg/m^3)

where y_i (i = 1..7) the unit price of each ingredient (VND/kg): y_1 = 1500; y_2 = 12; y_3 = 550; y_4 = 5050; y_5 = 118; y_6 = 135; y_7 = 21000.

In this mathematical model, we need to optimize 6 standard criteria $\mathbf{\Phi}_i$ (i = 1..6), which are necessary to satisfy with 10 functional constraints and 7 design variables x_k (k = 1..7).

3. Method of solution and calculation

In recent years, the single-objective and multi-objective optimization methods have been used commonly. However, most of the preceding studies have focused on the development of optimization algorithms for a single-objective function. The problem of a multicriteria task most of the time was converted into a representative single criteria by means of the methods, for instance, Weighted Minimax (Maximin), Compromise Programming, Weighted Sum, Bounded Objective Function, Modified Tchebycheff, Weighted Product, Exponential Weighted Sum, etc.

Xie and colluegues (Xie et al., 2011) have also chosen that option. After proposing an equivalent objective function, those authors used the method of Sequencial Quadratic Programming to find out the minimum. It is important to note that there are many methods to find the minimum of an equivalent function, such as algoritms Cooko, Fireflies, Hybrid, Genetic, Swarm, ect. Every algoritm gives the minimum with a small discrepancy. However, the problem is that the solution of the equivalent function does not represent the solution of the individual function. This means that one criteria reaches the optimum by using a certain algoritm, but another criteria does not reach the optimum by using another algoritm.

There are two questions that have not been reviewed in detail in the abovementioned work applied to a single-objective function:

- Will the equivalent criteria be able to actually substitute for the individual analysis of single criteria, when importance grade of every single criteria at certain moment and production circumstance is different from one expert to another?
- In the course of preparation and real production process, how will the experts be able to analyze directly, and opt for the priority consideration of criteria flexibly, which in turn make an appropriate desicion?

The significane of the optimization algorithm is enormous, however in practice when a flexible compromise needs to be made to find out the most feasible production option, the criteria should be analyzed individually and repeatly in comparative process. Then the “give and take” process should be done in order to achieve an aggrement among the criteria. Therefore, it is necessary to have a tool or an approach to solve a multicriteria task with high applicability. In this paper, an application of Visual Interactive Analysis Method (VIAM) is proposed to tackle with the issue of high performance concrete mixture proportioning.

The VIAM was described in details, elsewhere (Gavriushin and Dang, 2016). The main idea of this method includes: i) set up an interactive table, containing the range value of criteria, which satisfies with all contraints; ii) based on the current circumstance and determined production demand, the experts would give the threshold values of the criteria (the threshold is within the range value); iii) the final step is to find the variable vector, which satisfy with the threshold values.

There are many ways to find a valid variable vector. VIAM uses two main approaches; such as filling and spatial parameter survey, and space conversion variables - functional constraints - criteria. In this paper, the authors will take into account the second approach. The process to solve the mathematical task is presented below.

Determination of the range value of criteria and set it up in the interactive table.

Using an available single-objective optimization method, we can find the minimum of the objective function and the interactive table is presented as follows:

Table 4

The Interactive Table

minΦ_1 = 0	**minΦ_2 = 1.1×10^{6}**	**minΦ_3 = 300**	**minΦ_4 = 45**	**minΦ_5 = 60**	**minΦ_6 = 4.5**
…	…	…	…	…	…
[Φ_1]	**[Φ_2]**	**[Φ_3]**	**[Φ_4]**	**[Φ_5]**	**[Φ_6]**
…	…	…	…	…	…
maxΦ_1 = 5.78×10^{-13}	**maxΦ_2 = 2.04×10^{6}**	**maxΦ_3 = 495**	**maxΦ_4 = 155**	**maxΦ_5 = 200**	**maxΦ_5 = 12**
The chlorine ion diffusion coefficient (m^2/s)	Per cubic meter cost (VND/m^3)	Amount of Portland cement (kg/m^3)	Amount of Fly ash (kg/m^3)	Amount of Fine slag (kg/m^3)	Amount of Chemical Admixtures (kg/m^3)

When using the interactive table in the production process, there are many different cases and the corresponding production methods. In this paper, three production cases are solved by using VIAM.

Case 1: there is a hypothesis that the experts have discussed and indicated the required threshold value of criteria, as included in Table 5:

Table 5

Case 1

Φ_2 →	**Φ_3 →**	**Φ_1 →**	**Φ_5 →**	**Φ_4 →**	**Φ_6 ■**
1.3×10^{6}	400	4.5×10^{-13}	100	100	8

- First of all, we have minΦ_2, and it has been set before that $\Phi_2^{\oplus} = \min \Phi_2 = 1.3 \times 10^6$. Since this threshold is within the range valur of **Φ_2**, there exist definitely satisfied variable vectors. Three of those vectors are represented in the matrix form in Figure 3. In the first row, there are 7 variables, in the second row there are functional constraints and in the last row they are criteria values.

$$\begin{bmatrix} 363.416639110167 & 170.217943644146 & 77.0888438524496 & 64.4699760167256 & 649.887293902405 & 1055.44355292330 & 7.88993346107516 & NULL & NULL & NULL \\ -0.13708 & -0.06292 & -0.03110 & -0.01890 & -54.979 & -95.02 & -0.005624 & -0.004376 & 0.02 & -0.230 \\ 5.1300\ 10^{-13} & \boxed{1.3000\ 10^{6}} & 363.416639110167 & 77.0888438524496 & 64.4699760167256 & 7.88993346107516 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} 374.816976356103 & 174.317185191679 & 154.992402183237 & 61.8924516786018 & 559.002420565760 & 1035.20793240516 & 6.29355574116044 & NULL & NULL & NULL \\ -0.09461 & -0.10539 & -0.00065 & -0.04935 & -141.70 & -8.30 & -0.000636 & -0.009364 & 0.01 & -11.254 \\ 2.3526\ 10^{-13} & \boxed{1.3000\ 10^{6}} & 374.816976356103 & 154.992402183237 & 61.8924516786018 & 6.29355574116044 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (2)$$

$$\boxed{\begin{bmatrix} 398.241887940642 & 176.410623360612 & 78.2600293856075 & 64.7658412622898 & 605.294308698476 & 1058.69614362576 & 5.52671515505546 & NULL & NULL & NULL \\ -0.12592 & -0.07408 & -0.01376 & -0.03624 & -91.266 & -58.73 & -0.000211 & -0.009789 & -0.01 & -2.849 \\ 4.4340\ 10^{-13} & \boxed{1.3000\ 10^{6}} & 398.241887940642 & 78.2600293856075 & 64.7658412622898 & 5.52671515505546 & NULL & NULL & NULL & NULL \end{bmatrix}} \quad (3)$$

Figure 3. Obtained solution $\Phi_2^{\oplus} = \min \Phi_2 = 1.3 \times 10^6$

The solutions (1) – (3) satisfy the criteria 2, 3, 5, and 6. However, only the solution (2) satisfies the criteria 1, but does not for the criteria 4 from the expert's point of view. Although the solutions (1) and (3) do not satisfy the criteria 1, they excel for the criteria 4. Therefore, only the solution (3) satisfies all of criteria from the expert's standpoint. Nevertheless, the value of criteria 1 is $4.43\text{x}10^{-13}$, which is very close to $4.5\text{x}10^{-13}$ or it is not really optimized. Additionally, it is still unknown what the optimum value of criteria 2 can be reached, when compromising that the criteria 2 is the most important one. Thus, let's move to the next step.

- Adding to the constraints the condition $\left|\Phi_2 - \Phi_2^{\oplus}\right| \le \varepsilon_2 = 10^{-6}$ to find minΦ_3. We obtain the following three results, as shown in Figure 4:

$$\begin{bmatrix} 300. & 146.74 & 103.34 & 79.515 & 632.95 & 1143.4 & 7.6580 & NULL & NULL & NULL \\ -0.10390 & -0.09610 & -0.00631 & -0.04369 & -32.855 & -117.14 & -0.005860 & -0.004140 & 0.02 & -8.576 \\ 2.9355\ 10^{-13} & 1.3000\ 10^{6} & \boxed{300.} & 103.34 & 79.515 & 7.6580 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} 300. & 144.51 & 104.29 & 79.268 & 633.67 & 1147.8 & 7.6611 & NULL & NULL & NULL \\ -0.09885 & -0.10115 & -0.00569 & -0.04431 & -33.558 & -116.44 & -0.005843 & -0.004157 & 0.01 & -10.009 \\ 2.7219\ 10^{-13} & 1.3000\ 10^{6} & \boxed{300.} & 104.29 & 79.268 & 7.6611 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} 300. & 136.67 & 152.83 & 82.754 & 690.28 & 1047.0 & 5.8846 & NULL & NULL & NULL \\ -0.05518 & -0.14482 & -0.04733 & -0.00267 & -85.584 & -64.42 & -0.000987 & -0.009013 & 0. & -24.77 \\ 8.2135\ 10^{-14} & 1.3000\ 10^{6} & \boxed{300.} & 152.83 & 82.754 & 5.8846 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (6)$$

Figure 4. Obtained solutions $\Phi_2^{\oplus} = \min \Phi_2 = 1.3 \times 10^6$ và $\Phi_3^{\oplus} \le 400$

Three solutions (4) – (6) satisfy the criteria 1, 2, 3, 5, and 6. Particularly, the criteria 1, 3, 5, and 6 excel the purposes of the experts. However, these solutions do not satisfy the criteria 4, because all of them are out of allowable limits according to the experts. Besides, for the criteria 3 the minimum value $\Phi_3^{\oplus} = 305$ can be obtained. Nevertheless, there is still no solution satisfying all of requirements from the experts at this step.

- Adding to the constraints the condition $\left|\Phi_3 - \Phi_3^{\oplus}\right| \le \varepsilon_3 = 10^{-6}$ to find minΦ_1. We obtain the following three results, as shown in Figure 5:

$$\begin{bmatrix} 305.00 & 158.08 & 155.00 & 73.444 & 668.71 & 1008.5 & 8.0663 & NULL & NULL & NULL \\ -0.09634 & -0.10366 & -0.04871 & -0.00129 & -83.444 & -66.56 & -0.005121 & -0.004879 & 0. & -10.743 \\ 2.2755\ 10^{-13} & 1.3000\ 10^{6} & 305.00 & 155.00 & 73.444 & 8.0663 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} 305.00 & 170.38 & 144.49 & 86.341 & 634.96 & 1016.3 & 5.3725 & NULL & NULL & NULL \\ -0.11797 & -0.08203 & -0.03452 & -0.01548 & -85.831 & -64.17 & -0.000026 & -0.009974 & -0.02 & -4.825 \\ \boxed{3.0212\ 10^{-13}} & 1.3000\ 10^{6} & 305.00 & 144.49 & 86.341 & 5.3725 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (8)$$

$$\boxed{\begin{bmatrix} 305.00 & 158.90 & 95.878 & 91.394 & 691.73 & 1046.9 & 4.9227 & NULL & NULL & NULL \\ -0.12279 & -0.07721 & -0.04787 & -0.00213 & -42.272 & -107.73 & 0. & -0.010000 & -0.01 & -3.617 \\ 3.5325\ 10^{-13} & 1.3000\ 10^{6} & 305.00 & 95.878 & 91.394 & 4.9227 & NULL & NULL & NULL & NULL \end{bmatrix}} \quad (9)$$

Figure 5. Obtained solution $\Phi_2^{\oplus} = \min \Phi_2 = 1.3 \times 10^6$, $\Phi_3^{\oplus} \le 400$, $\Phi_1^{\oplus} \le 4.5 \times 10^{-13}$

Three solutions (7) – (9) satisfy the criteria 1, 2, 3, and 5. Looking at the criteria 5 and 6 for the solutions (7) – (9), they are opposite. At this moment, the solution (9) seems to be satisfied all of requirements from the experts. In principle, we can stop the work at this step. However, if more severely $\Phi_1^{\oplus} = 3.022\times10^{-13}$ is set for the criteria 1, we do not have any satisfied solution, because the solutions (7) and (8) do not satisfy the criteria 4. Thus, let's carry on the next step.

- Adding to the constraints the condition $\left|\Phi_1 - \Phi_1^{\oplus}\right| \le \varepsilon_1 = 10^{-6}$ to find minΦ_5. We obtain the following four results, as shown in Figure 5:

$$\begin{bmatrix} 305.01 & 161.36 & 155. & 63.232 & 648.60 & 1025.9 & 10.521 & NULL & NULL & NULL \\ -0.10839 & -0.09161 & -0.03734 & -0.01266 & -73.242 & -76.76 & -0.010107 & 0.000107 & 0. & -7.346 \\ 3.0220\ 10^{-13} & 1.3000\ 10^{6} & 305.01 & 155. & 63.232 & 10.521 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} 305.01 & 167.99 & 154.90 & 74.802 & 586.76 & 1066.8 & 7.8212 & NULL & NULL & NULL \\ -0.11417 & -0.08583 & -0.00484 & -0.04516 & -84.712 & -65.29 & -0.004627 & -0.005373 & -0.02 & -5.805 \\ 3.0224\ 10^{-13} & 1.3000\ 10^{6} & 305.01 & 154.90 & 74.802 & 7.8212 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} 304.99 & 170.98 & 154.96 & 79.922 & 575.88 & 1067.2 & 6.6479 & NULL & NULL & NULL \\ -0.11671 & -0.08329 & -0.00048 & -0.04952 & -89.872 & -60.13 & -0.002314 & -0.007686 & -0.02 & -5.149 \\ 3.0222\ 10^{-13} & 1.3000\ 10^{6} & 304.99 & 154.96 & \boxed{79.922} & 6.6479 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (12)$$

$$\begin{bmatrix} 304.98 & 175.35 & 154.99 & 87.397 & 600.66 & 1025.0 & 4.9639 & NULL & NULL & NULL \\ -0.12035 & -0.07965 & -0.01948 & -0.03052 & -97.367 & -52.63 & 0.0009314 & -0.010931 & 0. & -4.222 \\ 3.0210\ 10^{-13} & 1.2996\ 10^{6} & 304.98 & 154.99 & 87.397 & 4.9639 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (13)$$

Figure 6. Obtained solutions $\Phi_2^{\oplus} = \min\Phi_2 = 1.3\times10^6$, $\Phi_3^{\oplus} \le 400$, $\Phi_1^{\oplus} \le 4.5\times10^{-13}$, $\Phi_5^{\oplus} \le 100$

The minimum value of criteria 5, which can be reached after passing the system of 10 functional constraints, is 64 (at solution (10)). However, these solutions do not satisfy the criteria 4, thus we need to look into the criteria 4 at this step. At the moment, there is still no satisfied solution. Nevertheless, if select the threshold value of the criteria 4 according to the solutions (10) and (11), the criteria will be rarely satisfied. Thus, we opt for $\Phi_5^{\oplus} = 80$.

- Adding to the constraints the condition $\left|\Phi_5 - \Phi_5^{\oplus}\right| \le \varepsilon_5 = 10^{-6}$ to find minΦ_4. We obtain the following three results, as shown in Figure 7:

$$\begin{bmatrix} 305.00 & 148.14 & 99.948 & 79.999 & 695.50 & 1073.8 & 7.3666 & NULL & NULL & NULL \\ -0.10547 & -0.09453 & -0.04309 & -0.00691 & -34.947 & -115.05 & -0.005190 & -0.004810 & 0. & -8.139 \\ 3.0222\ 10^{-13} & 1.3000\ 10^{6} & 305.00 & 99.948 & 79.999 & 7.3666 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} 304.98 & 139.76 & 79.973 & 79.998 & 712.16 & 1104.7 & 7.6042 & NULL & NULL & NULL \\ -0.10059 & -0.09941 & -0.04196 & -0.00804 & -14.951 & -135.05 & -0.006355 & -0.003645 & -0.02 & -9.511 \\ 3.0226\ 10^{-13} & 1.3000\ 10^{6} & 304.98 & \boxed{79.973} & \boxed{79.998} & \boxed{7.6042} & NULL & NULL & NULL & NULL \end{bmatrix} \quad (15)$$

$$\begin{bmatrix} 305.03 & 138.04 & 75.864 & 80.002 & 676.56 & 1152.3 & 7.6026 & NULL & NULL & NULL \\ -0.09951 & -0.10049 & -0.01993 & -0.03007 & -10.896 & -139.11 & -0.006495 & -0.003505 & 0.01 & -9.819 \\ 3.0218\ 10^{-13} & 1.3000\ 10^{6} & 305.03 & \boxed{75.864} & \boxed{80.002} & \boxed{7.6026} & NULL & NULL & NULL & NULL \end{bmatrix} \quad (16)$$

Figure 7. Obtained solutions $\Phi_2^{\oplus} = \min\Phi_2 = 1.3\times10^6$, $\Phi_3^{\oplus} \le 400$, $\Phi_1^{\oplus} \le 4.5\times10^{-13}$, $\Phi_5^{\oplus} \le 100$, $\Phi_4^{\oplus} \le 100$

All of solutions (14), (15), (16) satisfy all of the criteria requirements, therefore they are satisfied solutions. However, we need to analyze whether the criteria 6 can be optimized more. Looking into the criteria (4), (5), (6) of the solutions (15) and (16), the minimum value of the criteria 4 does not worsen the value of criteria 6, and only influences on the value of criteria 5, besides it is within the allowable limits. Thus, we opt for $\Phi_4^{\oplus} = 76$.

- Adding to the constraints the condition $\left|\Phi_4 - \Phi_4^{\oplus}\right| \le \varepsilon_4 = 10^{-6}$ to find $\min\Phi_6$. We obtain the following two results, as shown in Figure 8:

$$\begin{bmatrix} 305.00 & 138.09 & 76.000 & 80.000 & 656.31 & 1173.4 & 7.5799 & NULL & NULL & NULL \\ -0.09954 & -0.10046 & -0.00870 & -0.04130 & -11.000 & -139.00 & -0.006442 & -0.003558 & 0. & -9.809 \\ 3.0220\ 10^{-13} & 1.3000\ 10^{6} & 305.00 & 76.000 & 80.000 & \boxed{7.5799} & NULL & NULL & NULL & NULL \end{bmatrix} \quad (17)$$

$$\begin{bmatrix} 305.00 & 138.09 & 76.000 & 80.002 & 640.70 & 1189.9 & 7.5615 & NULL & NULL & NULL \\ -0.09954 & -0.10046 & 0.00001 & -0.05001 & -11.002 & -139.00 & -0.006402 & -0.003598 & 0. & -9.809 \\ 3.0220\ 10^{-13} & 1.3000\ 10^{6} & 305.00 & 76.000 & 80.002 & \boxed{7.5615} & NULL & NULL & NULL & NULL \end{bmatrix} \quad (18)$$

Figure 8. Obtained solutions $\Phi_2^{\oplus} = \min\Phi_2 = 1.3\times10^6$, $\Phi_3^{\oplus} \le 400$, $\Phi_1^{\oplus} \le 4.5\times10^{-13}$, $\Phi_5^{\oplus} \le 100$, $\Phi_4^{\oplus} \le 100$, $\Phi_6^{\oplus} \le 8$

For the criteria 6, the solutions (17) and (18) do not turn out the significant optimization in comparison with the solution (14)-(16). However, they all satisfy the requirements from the experts included in Table 5. Therefore, for the case 1 we have 7 satisfied solution, those are solutions (3), (9), (14) – (18), all of them are Pareto solutions, which are not able to be optimized simultenously at all of criteria.

Case 2: the experts focus on the three criteria, which have a similar importance. The experts do not allow lowering the limit value of the criteria, as included in Table 6.

Table 6

Case 2

[**Φ**$_1$]	[**Φ**$_2$]	[**Φ**$_3$]
1.8 x 10^{-13}	1.3 x 10^{6}	390

We add to the constraints three conditions $\min\Phi_1 \le \Phi X_1 \le$ [**Φ**$_1$], $\min\Phi_2 \le \Phi X_2 \le$ [**Φ**$_2$], $\min\Phi_3 \le \Phi X_3 \le$ [**Φ**$_3$] to find the minimum value of the function

$$\min F = \min\left\{\left|\Phi_1 - \Phi X_1\right| + \left|\Phi_2 - \Phi X_2\right| + \left|\Phi_3 - \Phi X_3\right|\right\} \to 0 .$$

We obtained the following two results, as shown in Figure 9.

$$\begin{bmatrix} 347.65 & 160.72 & 150.53 & 71.101 & 615.56 & 1033.7 & 5.8363 & NULL & NULL & NULL \\ -0.08232 & -0.11768 & -0.02322 & -0.02678 & -119.28 & -30.72 & -0.000252 & -0.009748 & 0. & -15.058 \\ 1.7997\ 10^{-13} & 1.3000\ 10^{6} & 347.65 & 150.53 & \boxed{71.101} & 5.8363 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (19)$$

$$\begin{bmatrix} 328.86 & 148.37 & 142.93 & 77.319 & 603.76 & 1100.4 & 5.5216 & NULL & NULL & NULL \\ -0.07020 & -0.12980 & -0.00428 & -0.04572 & -99.109 & -50.89 & -0.000056 & -0.009944 & 0. & -19.149 \\ 1.3776\ 10^{-13} & 1.3000\ 10^{6} & 328.86 & 142.93 & \boxed{77.319} & 5.5216 & NULL & NULL & NULL & NULL \end{bmatrix} \quad (20)$$

Figure 9. Obtained solution in accordance with Table 6

The solution (20) is more optimized than the solution (19) at most of criteria, but it is only less at the criteria 5. However, the experts can estimate the importance of criteria 5 in comparison with the other 5 criteria to choose the solution (19) or (20). These two solutions excel the purpose of the experts at the criteria 3.

Case 3: the experts estimate the threshold value of all of criteria, as present in Table 7. The requirement is to the vector suitable for all of the criteria simultenously.

Table 7
Case 3

$[\mathbf{\Phi}_1]$	$[\mathbf{\Phi}_2]$	$[\mathbf{\Phi}_3]$	$[\mathbf{\Phi}_4]$	$[\mathbf{\Phi}_5]$	$[\mathbf{\Phi}_6]$
2×10^{-13}	1.4×10^{6}	400	110	140	8

Similarly to the case 2, we add to the constraints six conditions $\min\Phi_1 \le \Phi X_1 \le [\mathbf{\Phi_1}]$, $\min\Phi_2 \le \Phi X_2 \le [\mathbf{\Phi_2}]$, $\min\Phi_3 \le \Phi X_3 \le [\mathbf{\Phi_3}]$, $\min\Phi_4 \le \Phi X_4 \le [\mathbf{\Phi_4}]$, $\min\Phi_5 \le \Phi X_5 \le [\mathbf{\Phi_5}]$, $\min\Phi_6 \le \Phi X_6 \le [\mathbf{\Phi_6}]$ to find the minimum vale of function $\min F = \min\left\{\sum_{i=1}^{6}\left|\Phi_i - \Phi X_i\right|\right\} \to 0$. We obtain the following three results, as shown in Figure 10.

$$\begin{bmatrix} 300.05 & 135.81 & 100.00 & 100.29 & 637.82 & 1156.0 & 5.0245 & NULL & NULL & NULL \\ -0.07144 & -0.12856 & -0.00557 & -0.04443 & -50.34 & -99.66 & -0.000042 & -0.009958 & 0.01 & -18.719 \\ 1.4005\ 10^{-13} & 1.3501\ 10^{6} & 300.05 & 100.00 & 100.29 & 5.0245 & NULL & NULL & NULL & NULL \end{bmatrix} \tag{21}$$

$$\begin{bmatrix} 300.78 & 138.03 & 100.00 & 97.876 & 627.92 & 1162.5 & 4.9861 & NULL & NULL & NULL \\ -0.07680 & -0.12320 & -0.00071 & -0.04929 & -48.656 & -101.34 & 0.0000010 & -0.010001 & 0.01 & -16.874 \\ 1.5995\ 10^{-13} & 1.3378\ 10^{6} & 300.78 & 100.00 & 97.876 & 4.9861 & NULL & NULL & NULL & NULL \end{bmatrix} \tag{22}$$

$$\begin{bmatrix} 333.66 & 143.07 & 99.998 & 84.891 & 690.76 & 1067.4 & 5.1848 & NULL & NULL & NULL \\ -0.07590 & -0.12410 & -0.04288 & -0.00712 & -68.549 & -81.45 & 0.0000014 & -0.010001 & 0.01 & -17.181 \\ 1.8003\ 10^{-13} & 1.3204\ 10^{6} & 333.66 & 99.998 & 84.891 & 5.1848 & NULL & NULL & NULL & NULL \end{bmatrix} \tag{23}$$

Figure 10. Obtained solution in accordance with Table 7

These solutions are Pareto solutions, because there are superior and inferior criteria when comparing one to another. The values of criteria at these solutions are much better than the requirements of the experts included in Table 7.

4. Concluding remarks

It is important to note that the solution for multi-objective optimization task applied to high performance concrete mixture proportioning is not unique. Because, the solution is a set of criteria values, but every criteria has a different importance from one expert's standpoint to another at the certain production circumstance. Therefore, the evaluation of one solution or another based on an equivalent function for all of criteria is not comprehensive.

Above all, 12 solutions have been found for the different cases in terms of criteria during the process of proportioning high performance concrete mixture. They are all Pareto solutions, which allow experts to choose in the proposed cases. The task can also be extended with more variables, constraints, criteria when varying the amount, as well as the constituent material to make high performance concrete. Last but not least, the multi-objective optimization would definitely provide an optimum solution for high performance concrete mix propotioning with high durability and reasonable cost■

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