FULL-DUPLEX RELAYING MULTI-USER NOMA SYSTEM UNDER IMPACTS OF REALISTIC CONDITIONS

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Abstract

In this article, we derive the exact performance expressions of the system performance in terms of outage probability and ergodic capacity for multi-user of a downlink non-orthogonal multiple access (NOMA) systems with a full-duplex decode-and-forward relay R over the Nakagami-m fading channels. For the considered FDR-NOMA system, we evaluate the impacts of realistic parameters of imperfect residual self-interference (RSI) cancellation, successive interference cancellation (SIC), and channel state information (CSI). The results of the Monte-Carlo simulation are provided to validate the accuracy of numerical analysis and show the potential to double the capacity of the FDR-NOMA system compared to the half-duplex NOMA (HDR-NOMA). Furthermore, we investigate the optimal position of the relay R to reach maximum capacity.

Index terms

Non-orthogonal multiple access (NOMA), successive interference cancellation (SIC), full-duplex (FD), residual self-interference (RSI), decode-and-forward (DF), relaying communication, channel state information (CSI).

1. Introduction

In this era of the Internet of Things, the explosion in the number of wireless devices and big data services causes the exhaustion of radio resources. Therefore, researchers ensuring service quality and improving spectral efficiency have attracted great attention. The non-orthogonal multiple access technique is one of the promising solutions for future wireless networks [1]–[5].

In addition, by allowing radio terminals to transmit and receive simultaneously on a single frequency channel, full-duplex (FD) communication potentially doubles the bandwidth compared to the traditional half-duplex (HD) systems when the interference is mostly eliminated [6]–[10].

The schemes combining NOMA and FD have significantly improved in terms of capacity, reliability and coverage for telecommunication systems [11]–[18]. In the work

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[13], the authors proposed FD-NOMA systems using the amplify and forward (AF) technique, then derived the expressions of the outage probability and the ergodic capacity of the system over the complex Gaussian channel. However, their results were limited to approximate mathematical expressions. In the study [15], the FD-NOMA relay system incorporates the EH energy harvesting technique. The article analyzed the performance of the system through the outage probability (OP) expressions and proposed the optimal power allocation factors to achieve the minimum OP while ensuring fairness between users. Similar to this model, in the article [17], the authors proposed to use FD/HD adaptive mode, which could activate FD or HD mode according to actual conditions, to improve the quality NOMA-FD system. On the other hand, NOMA systems combined with a cooperative communication scheme have also been proven effective as in many published articles [19]–[22]. Near users operating in FD mode act as relay nodes to support forwarding packets to far users. The schemes have achieved diversity gain when considering the direct transmission from the source node to the far user, which improves the coverage and system performance.

Although there have been many studies on the NOMA systems combined with FD full-duplex technique, with the author's knowledge, the researchers have not investigated the effects of realistic assumptions, such as residual noise RSI, imperfect SIC, and imperfect CSI over Nakagami-m general channels. The Nakagami-m channel can be equivalent to other wireless multipath fading channels with different parameter $m \in \left(\frac{1}{2}, \infty\right)$ which present that Nakagami-m channel can cover other fading channels generally [23], [24]. In other words, the analysis and evaluation of the performance of the FDR-NOMA system under the influence of actual parameters caused by the imperfect system and the conditions of the Nakagami-m channel allow for obtaining general results that can be. It is used to evaluate system performance for the ideal system case and other popular transmission channels such as Rayleigh, Rice, or Gauss. Moreover, the multi-user NOMA relay systems have not yet been given the exact closed-form expressions of outage probability and ergodic capacity.

To solve those remaining points, in this article, the authors study the FDR-NOMA system under the influence of the mentioned parameters. The main contributions of this article can be summarized as follows:

- Derive the exact closed-form expressions of OP and ergodic capacity of each user in the FDR-NOMA system using the decode-and-forward (DF) technique over the fading channel Nakagami-m.
- Investigate the impact of residual noise RSI, imperfect SIC, and channel estimation error on the system performance of the FDR-NOMA system and compare system capacity with that of the HDR-NOMA system.
- Survey the optimal position of the R node to maximize the capacity of the system.

The rest of this article is organized as follows. Section 2 presents the system model. Section 3 provides the detailed derivations of the closed-form expression of OP and ergodic capacity. Section 4 presents the simulation results and discusses performance evaluations. Finally, we conclude and propose further research directions in section 5.

2. System model

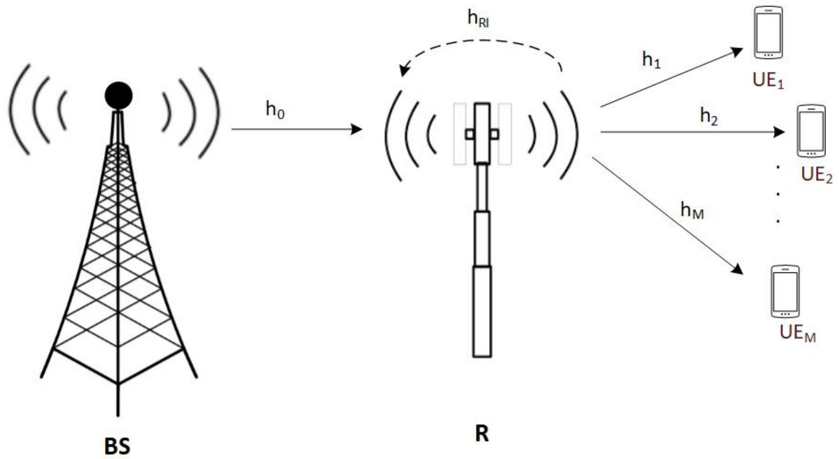

Fig. 1. System model of the FDR-NOMA network.

The system model of the FDR-NOMA network consisting of a base station BS, a relay R, and M users ($\mathrm{UE_1, UE_2 ... UE_m}$) is shown in Figure 1. There is no direct path between BS and the users because of extensive shadow-fading or the great distances. Hence, the user nodes receive signals from the BS node via the R node using the decode-and-forward (DF) relaying protocol. It is assumed that the channels between BS and R; R and $\mathrm{UE_1, UE_2 ... UE_m}$ are all considered flat, non-selective frequency fading, and independent and identically distributed (i.i.d) Nakagami-m fading with the correspondingly complex channel coefficients $h_0, h_1, h_2, ..., h_M$. Without losing generality, we assume that the channel gain from R to users is $|h_1| > |h_2| > ... > |h_M|$.

In the power domain NOMA technique, the source needs to know the channel gains through the CSI in order to allocate transmit power for users. However, in radio channels, it is extremely difficult to get the perfect CSI due to channel estimation errors. We solely discuss imperfect CSI due to channel estimate errors in this work. We assume that the channel estimation error is $\upsilon_i \sim CN\left(0, \sigma_{\upsilon_i}^2\right)$ modeled as Gaussian distributions, with $i \in \{0, 1, ..., M\}$, $\sigma_{\upsilon_i} = \tau_{\upsilon_i}\Omega_{\upsilon_i}$ where Ω_{υ_i} is the channel variance and $0 \leq \tau_{\upsilon_i} \leq 1$ is the correlation coefficient between the estimated channel and the actual channel. Then, the variance of the estimated channel is:

$$\hat{\Omega}_{\upsilon_i} = \left(1 - \tau_{\upsilon_i}\right)\Omega_{\upsilon_i} \tag{1}$$

In this article, R operates in full-duplex mode, receiving the signals from the BS and simultaneously forwarding them to the users. At the destination nodes, the $\mathrm{UE_i}$ performs SIC to get their own signal. The residual noise interference RSI at R is modeled as a Rayleigh fading variable with zero mean and variance Ω_{RI}. $n_i \sim CN\left(0, N_0\right), i \in \{1, 2\}$. $n_i \sim CN\left(0, N_0\right), i \in \{1, 2\}$ is the additive white Gaussian noise (AWGN), where N_0 is the noise power spectral density. The base station BS creates a superposition signal by combining M separate signals x_1,..., x_M of users, $s[k] = \sum_{i=1}^{M} \sqrt{a_i P_s} x_i[k]$, with a_i denote the power allocation coefficient of $\mathrm{UE_i}$ meeting the condition $a_1 < a_2 < ... < a_M$ and $\sum_{i=1}^{M} a_i = 1$. We also assume that the channels are affected by slow fading so the time index k for the channel gains can be ignored. The signal received at the relay node is represented as follows:

$$y_R = h_0 s + h_{RI} \sqrt{P_r} \tilde{s} + n_R \tag{2}$$

where $\tilde{s}$ is the signal to be decoded and forwarded by R in the previous phase. P_s, P_r are the transmit power at BS and relay R, respectively. The signal power to noise plus noise ratio SINR at R to detect the signal x_M is determined by the following formula:

$$\gamma_R^{x_M} = \frac{|h_0|^2 a_M \rho}{|h_0|^2 \sum\limits_{i=1}^{M-1} a_i \rho + |h_{RI}|^2 \rho + 1} \tag{3}$$

where $\rho = \frac{P_s}{N_0}$ is defined as the signal to noise ratio SNR. According to the SIC principle, R first decodes the signal x_M by treating the remaining users' signals as noise. After separating x_M from the superposition signal, R continues SIC to receive signals of $\mathrm{UE_{M-1}}$, $\mathrm{UE_{M-2}}$,..., $\mathrm{UE_1}$.

Considering the fact that it is not possible to completely remove the power of the previously separated signals, known as imperfect SIC, the SINR for detecting signal x_m of $\mathrm{UE_m}$ is determined:

$$\gamma_R^{x_m} = \frac{|h_0|^2 a_m \rho}{|h_0|^2 \sum\limits_{i=1}^{m-1} a_i \rho + |h_0|^2 \sum\limits_{k=m+1}^{M} a_k \rho \varepsilon + |h_{RI}|^2 \rho + 1} \tag{4}$$

where $0 \leq \varepsilon \leq 1$ is the residual power factor at R due to imperfect SIC. The received signals at the users are expressed as:

$$y_{UE_m} = |h_m|^2 \tilde{s} + n_m \tag{5}$$

It is assumed that the transmit powers at the BS and relay R are the same $P_s = P_r = P$. The $\mathrm{UE_m}$ must separate the stronger signals one by one to obtain their signals. Thus, the instantaneous ratio SINR to decode x_m at $\mathrm{UE_m}$ is given by:

$$\gamma_{UE_m}^{x_m} = \frac{|h_m|^2 a_m \rho}{|h_m|^2 \sum\limits_{i=1}^{m-1} a_i \rho + |h_m|^2 \sum\limits_{j=m+1}^{M} a_j \varepsilon \rho + 1} \tag{6}$$

The SINR to decode x_M at $\mathrm{UE_M}$ is written as:

$$\gamma_{UE_M}^{x_M} = \frac{|h_M|^2 a_M \rho}{|h_M|^2 \sum\limits_{i=1}^{M-1} a_i \rho + 1} \tag{7}$$

3. System performance

In this section, we derive the closed-form expressions for two important system performance parameters, i.e. outage probability and ergodic of the considered FDR-NOMA system.

3.1. Outage probability

The mathematical expression for the outage probability OP, which is defined as the probability that the received SNR falls below a specified threshold [25], is as follows:

$$OP = \Pr\left(\gamma_D < \gamma_{th}\right) = \int\limits_0^{\gamma_{th}} f_{\gamma_D}\left(\gamma\right) d\gamma \tag{8}$$

where $f_{\gamma_D}\left(\gamma\right)$ is the probability density function PDF of the SNR at a receiver. In other words, OP is the probability that the user cannot successfully decode the desired signal.

3.1.1. Outage probability of $\mathrm{UE_m}$*:* The outage event at the mth user occurs when the relay R and the mth user cannot successfully decode its signal and the jth users' signals, with $m < j \leq M$. Therefore, the outage probability of $\mathrm{UE_m}$ can be expressed as follows:

$$OP_m = 1 - \Pr\left(\begin{array}{c} \left(\gamma_R^{x_m} > \gamma_{th_m}, \gamma_R^{x_{m+1}} > \gamma_{th_{m+1}}, ..., \gamma_R^{x_M} > \gamma_{th_M}\right) \\ \cap \left(\gamma_{UE_m}^{x_m} > \gamma_{th_m}, \gamma_{UE_m}^{x_{m+1}} > \gamma_{th_{m+1}}, ..., \gamma_{UE_m}^{x_M} > \gamma_{th_M}\right) \end{array} \right) \tag{9}$$

Denote $\gamma_{th_m} = 2^{R_m} - 1$ with R_m (bit/s/Hz) is the target data rate of $\mathrm{UE_m}$.

Substituting (3), (4), (6), and (7) into (9), we obtaine OP at $\mathrm{UE_m}$ as follows:

$$OP_m = 1 - \Pr\left(X > \theta_{\max}\left(Z\rho + 1\right)\right) \Pr\left(Y_i > \theta_{\max}\right) \tag{10}$$

where $\left|h_0\right|^2 = X$, $\left|h_i\right|^2 = Y_i, i \in \{1, 2, .., M\}$, $\left|h_{RI}\right|^2 = Z$.

$$\theta_{\max} = \max\left(\theta_m, \theta_{m+1}, ..., \theta_M\right) \tag{11}$$

$$\theta_m = \frac{\gamma_{th_m}}{\rho\left(a_m - \sum\limits_{i=1}^{m-1} a_i \gamma_{th_m} - \sum\limits_{k=m+1}^{M} a_k \varepsilon \gamma_{th_m}\right)} \tag{12}$$

Note that the expression (10) needs to satisfy the condition:

$$a_m > \gamma_{th_m}\left(\sum_{i=1}^{m-1} a_i + \sum_{k=m+1}^{M} a_k \varepsilon\right) \tag{13}$$

In order to obtain OP_m, the subexpressions should be calculated.

At first, $\Pr\left(X > \theta_{\max}\left(Z\rho + 1\right)\right)$ can be rewritten as:

$$\Pr\left(X > \theta_{\max}\left(Z\rho + 1\right)\right) = \int\limits_0^\infty \left[1 - F_X\left(Z\rho + 1\right)\right] f_Z\left(z\right) dz \tag{14}$$

Due to $|h_0|$ is an i.i.d. Nakagami-m distributed random variable, $|h_0|^2$ has the Gamma distribution [26] with the Nakagami multipath fading parameter m_0. CDF can be represented as follows:

$$F_X\left(\theta_{\max}\left(|h_{RI}|^2\rho + 1\right)\right) = 1 - \frac{1}{\Gamma\left(m_0\right)} \times \Gamma\left(m_0, \frac{m_0\theta_{\max}\left(|h_{RI}|^2\rho + 1\right)}{\left(1 - \tau_0\right)\Omega_0}\right) \tag{15}$$

where $\Gamma\left(\cdot\right)$ is the Gamma function and $\Gamma\left(\cdot,\cdot\right)$ is incomplete Gamma function determined in [27]. h_{RI} has the Rayleigh distribution random variable, so the PDF of $|h_{RI}|^2$ be written as [26]:

$$f_{|h_{RI}|^2}(z) = \frac{1}{\Omega_{RI}} \exp\left(-\frac{z}{\Omega_{RI}}\right) \tag{16}$$

Substituting (15) and (16) into (14), we have:

$$\Pr\left(X > \theta_{\max}\left(Z\rho + 1\right)\right) = \frac{1}{\Omega_{RI}} \exp\left(-\theta^*\right) \sum_{k=0}^{m_0-1} \left(\theta^*\right)^k \frac{1}{k!} \times \int\limits_0^\infty \left(z\rho + 1\right)^k \exp\left(-\theta^* z - \frac{z}{\Omega_{RI}}\right) dz \tag{17}$$

with $\theta^* = \frac{m_0\theta_{\max}}{(1-\tau_0)\Omega_0}$.

Based on [27, Eq. (1.111)] and after some mathematical transformations, (17) is obtained as:

$$\Pr\left(X > \theta_{\max}\left(Z\rho + 1\right)\right) = \frac{1}{\Omega_{RI}} \exp\left(-\theta^*\right) \sum_{k=0}^{m_0-1} \sum_{t=0}^{k} \frac{\left(\theta^*\right)^k}{k!} \times \binom{k}{t} \rho^t t! \left(\theta^*\rho + \frac{1}{\Omega_{RI}}\right)^{-t-1} \tag{18}$$

Similarly, $\Pr\left(Y_m > \theta_{\max}\right)$ is given by:

$$\begin{aligned}\Pr\left(Y_m > \theta_{\max}\right) &= 1 - \Pr\left(Y_m \le \theta_{\max}\right)\\ &= 1 - \int\limits_0^{\infty} [F_{Y_m}\left(\theta_{\max}\right)] d\left(\theta_{\max}\right)\\ &= \exp\left(-\theta^*\right) \sum_{k=0}^{m_m-1} \frac{\left(\theta^*\right)^k}{k!}\end{aligned} \tag{19}$$

Substituting (18) and (19) into (10), we obtain the OP_{UE_m} of the system as:

$$\begin{aligned}OP_m = 1 - \frac{1}{\Omega_{RI}} \exp\left(-\theta^*\right) \sum_{k=0}^{m_0-1} \sum_{t=0}^{k} \frac{\left(\theta^*\right)^k}{k!} \left(\; kt \; \right) \rho^t t!\\ \times \left(\theta^* \rho + \frac{1}{\Omega_{RI}}\right)^{-t-1} \exp\left(-\theta^*\right) \sum_{k=0}^{m_m-1} \frac{\left(\theta^*\right)^k}{k!}\end{aligned} \tag{20}$$

3.1.2. Outage probability of $\mathrm{UE_M}$*:* The OP_M of $\mathrm{UE_M}$ is expressed as:

$$\begin{aligned}OP_M &= 1 - \Pr\left(\left(X > \theta_M\right) \cap \left(Y_M > \theta_M\right)\right)\\ &= 1 - \Pr\left(X > \theta_M\left(Z\rho + 1\right)\right) \Pr\left(Y_M > \theta_M\right)\end{aligned} \tag{21}$$

where

$$\theta_M = \frac{\gamma_{thM}}{\rho\left(a_M - \sum\limits_{i=1}^{M-1} a_i \gamma_{thM}\right)} \tag{22}$$

Similar to calculating of OP_m , we can give OP_M as:

$$\begin{aligned}OP_M = 1 - \frac{1}{\Omega_{RI}} \exp\left(-2{\theta_M}^*\right) \sum_{k=0}^{m_0-1} \sum_{t=0}^{k} \frac{\left({\theta_M}^*\right)^k}{k!} \begin{pmatrix} k \\ t \end{pmatrix}\\ \times \rho^t t! \left(\theta^* \rho + \frac{1}{\Omega_{RI}}\right)^{-t-1} \sum_{k=0}^{m_M-1} \frac{\left({\theta_M}^*\right)^k}{k!}\end{aligned} \tag{23}$$

3.2. Ergodic capacity

The ergodic capacity of the users is determined by:

$$\begin{aligned}\overline{C}_m &= \min\left(\overline{C}_R^{x_m}, \overline{C}_{UE_m}^{x_m}\right)\\ &= \min\left(E\left[\log_2\left(1 + \gamma_R^{x_m}\right)\right], E\left[\log_2\left(1 + \gamma_{UE_m}^{x_m}\right)\right]\right)\end{aligned} \tag{24}$$

To calculate $\overline{C}_m$ of the users, we start with the first subexpression in (24).

$$\overline{C}_R^{x_m} = \frac{1}{\ln 2}\left(\underbrace{\int_0^\infty \frac{1-F_U(u)}{1+u}du}_{\mathrm{I}_1} - \underbrace{\int_0^\infty \frac{1-F_V(v)}{1+v}dv}_{\mathrm{I}_2} \right) \tag{25}$$

where

$$U = X\rho\left(\sum_{i=1}^{m-1} a_i + \sum_{k=m+1}^{M} a_k\varepsilon + a_m\right) + Z\rho \tag{26}$$

$$V = X\rho\left(\sum_{i=1}^{m-1} a_i + \sum_{k=m+1}^{M} a_k\varepsilon_m\right) + Z\rho \tag{27}$$

$F_U(u)$, $F_V(v)$ are the cumulative distribution function CDF of U and V, respectively. First, $F_U(u)$ is expressed as follows:

$$\begin{aligned} F_U(u) &= \Pr\left(X < \frac{u - z\rho}{\rho\left(\sum\limits_{i=1}^{m-1} a_i + \sum\limits_{k=m+1}^{M} a_k\varepsilon + a_m\right)} \right) \\ &= \int_0^{\frac{u}{\rho}} F_X\left(\frac{u - z\rho}{\rho\left(\sum\limits_{i=1}^{m-1} a_i + \sum\limits_{k=m+1}^{M} a_k\varepsilon + a_m\right)} \right) f_Z(z)\,dz \end{aligned} \tag{28}$$

with the condition $z \le \frac{u}{\rho}$. After transformation, the function $F_U(u)$ is defined as follows:

$$\begin{aligned} F_U(u) = 1 - \exp\left(\frac{-u}{\rho\Omega_{RI}}\right) - \frac{1}{\Omega_{RI}}\exp(-b_{11})\sum_{k=0}^{m_0-1}\sum_{t=0}^{k}\frac{b_{11}^k}{k!} \\ \times \binom{k}{t}(-1)^t u^{k-t}\rho^t \int_0^{x/\rho} z^t \exp(-c_{11}z)\,dz \end{aligned} \tag{29}$$

where

$$b_{11} = \frac{m_0}{(1-\tau)\,\Omega_0\rho\left(\sum\limits_{i=1}^{m-1} a_i + \sum\limits_{k=m+1}^{M} a_k\varepsilon + a_m\right)} \tag{30}$$

$$c_{11} = -\frac{m_0}{(1-\tau)\,\Omega_0} + \frac{1}{\Omega_{RI}} \tag{31}$$

- In the case of $c_{11} \neq 0$, applying [27, Eq. (3.383.10)], I1 expression is expressed as:

$$\begin{aligned} I_1 = {} & \frac{1}{\Omega_{\mathrm{RI}}} \sum_{k=0}^{m_0-1} \sum_{t=0}^{k} \frac{b_{11}^k}{k!} \left(\begin{array}{c} k \\ t \end{array} \right) \frac{(-1)^t \rho^t t!}{c_{11}^{t+1}} \\ & \times \left(\mathrm{e}^{b_{11}}\ \Gamma(k-t+1)\,\Gamma(t-k,b_{11}) - \mathrm{I}_{11} \right) e^{\frac{1}{\rho\Omega_{\mathrm{RI}}}} \mathrm{Ei}\left(-\frac{1}{\rho\Omega_{\mathrm{RI}}}\right) \end{aligned} \tag{32}$$

where

$$\mathrm{I}_{11} = \sum_{m=0}^{t} \left(\frac{c_{11}}{\rho}\right)^m \frac{\mathrm{e}^{d_{11}}\,(k+m-t)!\,\Gamma(t-k-m,d_{11})}{m!} \tag{33}$$

$$d_{11} = b_{11} + \frac{c_{11}}{\rho} \tag{34}$$

I_2 is similarly transformed. Therefore, the ergodic capacity of the mth user at R is determined as follows:

$$\begin{aligned} \overline{C}_R^{x_m} = {} & \frac{1}{(\ln 2)\,\Omega_{RI}} \sum_{k=0}^{m-1} \sum_{t=0}^{k} \left(\begin{array}{c} k \\ t \end{array} \right) \frac{(-1)^t \rho^t t!}{k!} \\ & \times \left(\frac{b_{11}^k}{c_{11}^{t+1}} \mathrm{e}^{b_{11}} \Gamma(k-t+1)\,\Gamma(t-k,b_{11}) - \frac{b_{12}^k}{c_{12}^{t+1}} \mathrm{e}^{b_{12}} \right. \\ & \left. \times\ \Gamma(k-t+1)\,\Gamma(t-k,b_{12}) - \frac{b_{11}^k}{c_{11}^{t+1}} \mathrm{I}_{11} + \frac{b_{12}^k}{c_{12}^{t+1}} \mathrm{I}_{12} \right) \end{aligned} \tag{35}$$

- In the case of $c_{11} = 0$, we have:

$$\mathrm{I}_1 = \int_0^\infty \frac{e^{-\frac{u}{\rho\Omega_{RI}}}}{1+u} dx + \frac{1}{\Omega_{\mathrm{RI}}} \sum_{k=0}^{m_0-1} \sum_{t=0}^{k} \frac{b_{11}^k}{k!} \left(\begin{array}{c} k \\ t \end{array} \right) \frac{(-1)^t}{(t+1)\,\rho} \int_0^\infty \frac{\mathrm{e}^{-b_{11}u} u^{k+1}}{1+u} du \tag{36}$$

$$\begin{aligned} = {} & \frac{1}{\Omega_{\mathrm{RI}}} \sum_{k=0}^{m_1-1} \sum_{t=0}^{k} \frac{b_{1i}^k}{k!} \left(\begin{array}{c} k \\ t \end{array} \right) \frac{(-1)^t}{(t+1)\,\rho} \left(\mathrm{e}^{b_{1i}} \Gamma(k+2)\,\Gamma(-1-k,b_{1i}) \right) \\ & - e^{\frac{1}{\rho\Omega_{\mathrm{RI}}}} \mathrm{Ei}\left(-\frac{1}{\rho\Omega_{\mathrm{RI}}}\right) \end{aligned} \tag{37}$$

I_2 is defined similarly.

As a result, the ergodic capacity of the mth user at R is calculated as follows:

$$\begin{aligned} \overline{C}_R^{x_m} = {} & \frac{1}{(\ln 2)\,\Omega_{RI}} \sum_{k=0}^{m-1} \sum_{t=0}^{k} \left(\begin{array}{c} k \\ t \end{array} \right) \frac{(-1)^t \rho^t t!}{k!} \left(\frac{b_{11}^k}{c_{11}^{t+1}} \mathrm{e}^{b_{11}} \Gamma(k+2)\,\Gamma(-1-k,b_{11}) \right. \\ & \left. - \frac{b_{12}^k}{c_{12}^{t+1}} \mathrm{e}^{b_{12}} \Gamma(k+2)\,\Gamma(-1-k,b_{12}) \right) \end{aligned} \tag{38}$$

We continue with the second subexpression in (24):

$$\overline{C}_{UE_m}^{x_m} = \frac{1}{\ln 2}\left(\int\limits_0^\infty \frac{1-F_{\mathrm{W}}(\mathrm{w})}{1+\mathrm{w}}d\mathrm{w} - \int\limits_0^\infty \frac{1-F_S(s)}{1+s}ds\right) \tag{39}$$

where

$$\mathrm{W} = Y_m\rho\left(\sum_{i=1}^{m-1} a_i + \sum_{k=m+1}^{M} a_k\varepsilon + a_m\right) \tag{40}$$

$$S = Y_m\rho\left(\sum_{i=1}^{m-1} a_i + \sum_{k=m+1}^{M} a_k\varepsilon_m\right) \tag{41}$$

and $F_{\mathrm{W}}(\mathrm{w})$, $F_{\mathrm{S}}(\mathrm{s})$ are CDP of W and S, respectively. After some mathematical transformations of $F_{\mathrm{W}}(\mathrm{w})$, we get:

$$F_{\mathrm{W}}(\mathrm{w}) = 1 - \exp(-b_{m_1}w)\sum_{k=0}^{m_m-1}(b_{m_1}w)^k\frac{1}{k!} \tag{42}$$

where

$$b_{m_1} = \frac{m_m}{(1-\tau)\Omega_m\left(\sum\limits_{i=1}^{m-1} a_i + \sum\limits_{k=m+1}^{M} a_k\varepsilon + a_m\right)} \tag{43}$$

Using a similar representation, we can compute $F_S(s)$.

$$F_S(s) = 1 - \exp(-b_{m_2}s)\sum_{k=0}^{m_m-1}(b_{m_2}s)^k\frac{1}{k!} \tag{44}$$

where

$$b_{m_2} = \frac{m_m}{(1-\tau)\Omega_m\left(\sum\limits_{i=1}^{m-1} a_i + \sum\limits_{k=m+1}^{M} a_k\varepsilon\right)} \tag{45}$$

Substituting (42) and (44) into (39), we have the ergodic capacity of $\mathrm{UE_m}$ as:

$$\overline{C}_{UE_m}^{x_m} = \frac{1}{\ln 2}\left(\sum_{k=0}^{m_m-1} b_{m_1}e^{b_{m1}}\Gamma(-k,b_{m_1}) - \sum_{k=0}^{m_m-1} b_{m_2}e^{b_{m1}}\Gamma(-k,b_{m_2})\right) \tag{46}$$

4. Simulation result and discussion

In this section, we provide analytical results as well as Monte-Carlo simulation results for comparison to validate the mathematical equations derived in section 3. Furthermore, the performance of the FDR-NOMA system is explored to comprehend the effect of RSI, imperfect SIC and CSI. The system simulation model includes BS, R, and 3 users $\mathrm{UE}_1, \mathrm{UE}_2, \mathrm{UE}_3$. The system simulation parameters are set as shown in Table 1.

Table 1. Simulation parameters

Parameters	Value
Power allocation coefficients	$a_1 = 0.1,\ a_2 = 0.3,\ a_3 = 0.6$
Target rates	1 bpcu
Channel gains	$\Omega_0 = 2,\ \Omega_1 = 3,\ \Omega_2 = 2,\ \Omega_3 = 1$
Imperfect CSI	0.01
Imperfect SIC	0.02
Residual self-interference RSI	-30 dB
Nakagami multipath fading parameter	$m_0 = m_1 = m_2 = m_3 = 2$

Figure 2 illustrates the impact of RSI on the OP of FDR-NOMA network for different values of Ω_{RI}, i.e. $\Omega_{RI} = -10, -30, -50$ dB. As shown in the figure, the analytical curves perfectly match the simulation one. It is obvious that RSI has a significant effect on OP. When Ω_{RI} is low, such as $\Omega_{RI} = -50$ dB, the influence of RSI is negligible. However, when the RSI is high, such as $\Omega_{RI} = -10$ dB, the system performance drops sharply and reaches the outage floor at $\mathrm{SNR} = 25$ dB. At the value of $\Omega_{RI} = -30$ dB, the system performance is less affected by the RSI at the low $\mathrm{SNR} < 20$ dB, but at the high SRN region, the influence of the RSI increases with the transmit power and tends to reach saturation at SNR = 40 dB. Therefore, for a full-duplex system, we should choose suitable transmit power to improve system quality and avoid wasting energy. It can be seen that the RSI is a crucial factor in the implementation of the FD technique.

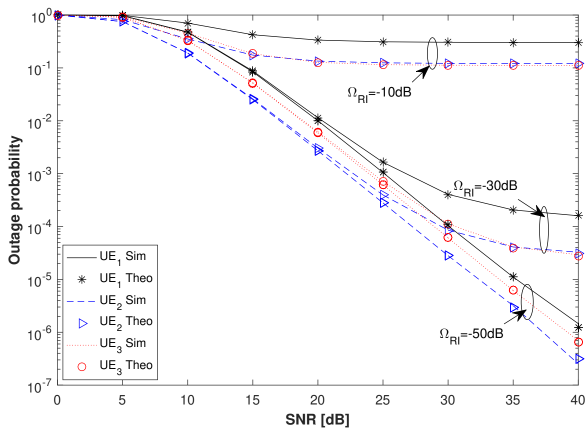

Fig. 2. The impact of RSI on the OP of the FDR-NOMA system.

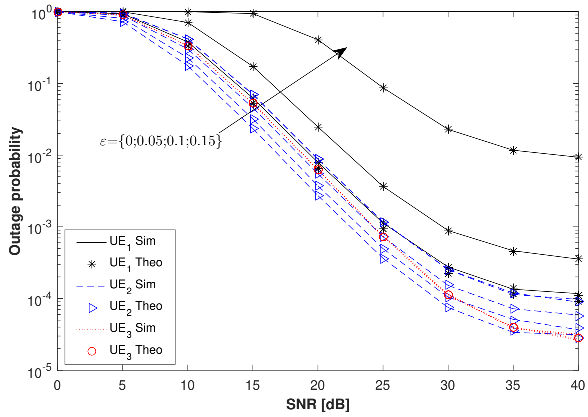

Fig. 3. The impact of SIC on the OP of the FDR-NOMA system.

Figure 3 shows the impact of SIC on the system outage performance. In the case of a small coefficient ε, i.e. $\varepsilon = 0.05$, it has a negligible effect on the OP system; however when ε is large, i.e. in this article at $\varepsilon > 0.1$, the system cannot successfully decode the signal x_1. Through the mathematical analysis in the section 3, we see that to successfully decode the signal, it is necessary to satisfy the condition of the expression (13). This is a prerequisite for the implementation of the SIC technique in NOMA. The results in Figure 3 clearly show that user UE_1 has the greatest influence on the SIC coefficient due to the final decoding, while the system quality of the user UE_3 is not affected by the SIC factor because of the first split. Specifically, at $OP = 3 * 10^{-2}$, we find that the imperfect SIC is lower than the perfect SIC by about 2 dB in terms of gain at $\varepsilon = 0.05$, about 10 dB at $\varepsilon = 0.1$ for UE_1; about 1 dB for UE_2 in both cases $\varepsilon = 0.05$ and $\varepsilon = 0.1$.

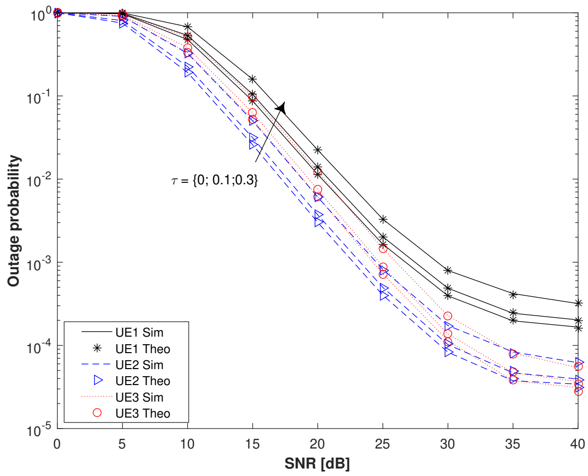

Fig. 4. The OP of the FDR-NOMA system for different values of channel estimation error parameter.

Figure 4 shows the probability of unsuccessfully decoding the user's signal in cases of the channel estimation error are $\tau = 0; 0.1; 0.3$. In this figure, the gain of the system decreases proportionally with the increase of τ and the gain of the system decreases by 2.5 dB when $\tau = 0.3$ at the value of $OP = 10^3$. Thus, it is clear that when the channel estimation error increases, it greatly affects the signal recovery of the users.

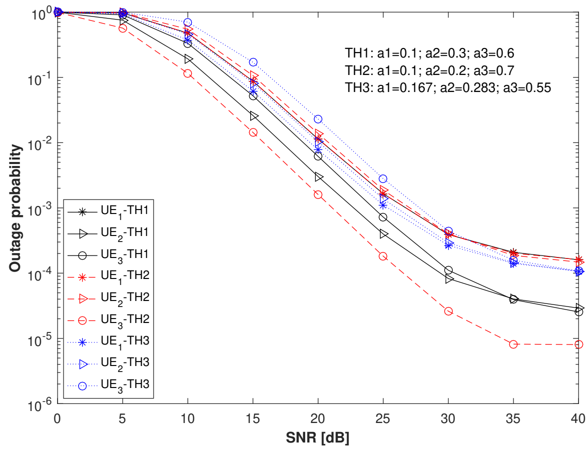

Fig. 5. The impact of power allocation on the OP of the FDR-NOMA system.

Figure 5 plots the OP performance of the system with various values of power factors. They need to satisfy the expression (13) to ensure that the signals can be decoded. When power allocation is reasonable, the quality of the user is significantly improved. However, for multi-user FD-NOMA systems, power optimization is a complex problem. Due to the complexity of the multivariable optimization problem with the constraints, there is currently no research work on this issue.

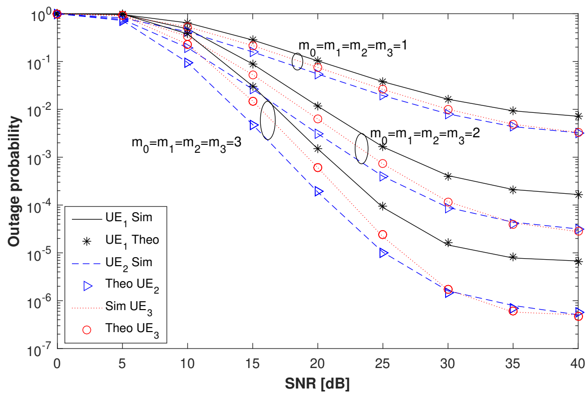

Fig. 6. The OP of the FDR-NOMA system for different values of the channel fading factors.

In Figure 6, the system performance is significantly impacted by the fading channel parameters. As can be seen in the figure, an increase in the value of m causes the OP to decrease, which means that the system quality is improved. By varying the values of m_i, we can survey different systems in practice; for example, when $m_i = 1$, the channel becomes a Rayleigh fading channel.

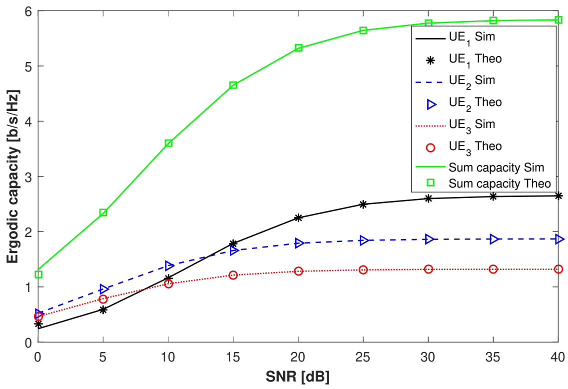

Fig. 7. The ergodic capacity of the FDR-NOMA system.

Figure 7 shows the user capacity and total capacity of the FDR-NOMA system. It is obvious that the analytical curves perfectly match the simulation one, which validates the expressions of ergodic capacity. Although the channel gain of the UE_3 is the smallest, the ergodic capacity of UE_3 is larger than that of the UE_1 due to the largest power allocation factor.

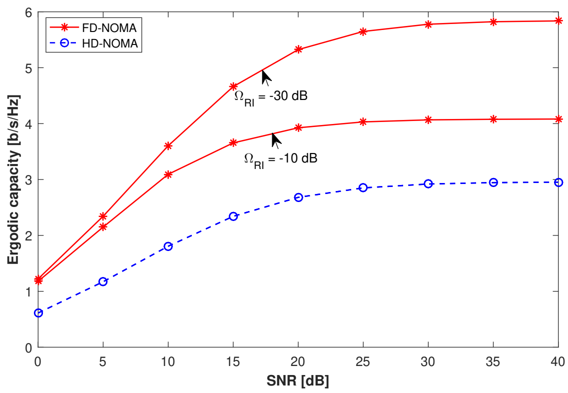

Fig. 8. The ergodic capacity of FDR-NOMA and HDR-NOMA systems.

Figure 8 compares the capacity of the NOMA system in the case of using full-duplex and half-duplex at the R. The simulation results show that the system capacity is significantly improved when deploying the full-duplex communication system compared to the half-duplex system. In particular, when the residual noise at the R is well suppressed, such as the simulation in the article at $\Omega_{RI} = -30$ dB, the FD system capacity can be twice the HD system capacity. However, when the RSI noise is large, i.e., $\Omega_{RI} = -10$ dB, the system capacity is reduced by more than 1 b/s/Hz at $\mathrm{SNR} = 20$ dB. Figure 9 shows the capacity of the users in the system when the SIC technique is

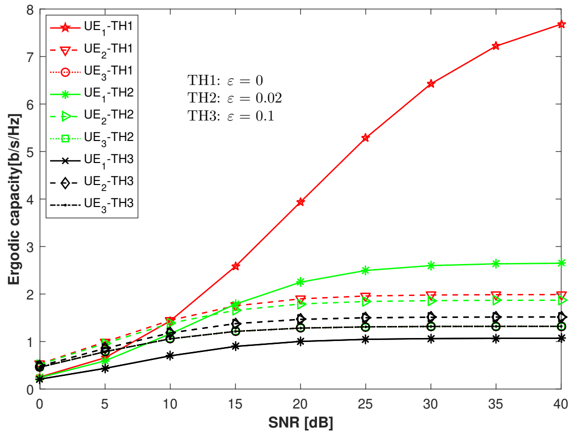

Fig. 9. The ergodic capacity of FDR-NOMA and HDR-NOMA systems for different values of SIC.

perfect, corresponding to the case $\varepsilon = 0$ and the imperfect SIC at $\varepsilon = 0.02$ and $\varepsilon = 0.1$. It is obvious that the capacity of UE_1 is most affected by SIC. This is understandable because $\mathrm{UE_m}$'s data is decrypted last, so the decoding speed depends a lot on previous SIC times. In contrast, the capacity of the last user UE_3 allocated the largest capacity is split first, so it is not affected by the SIC factor.

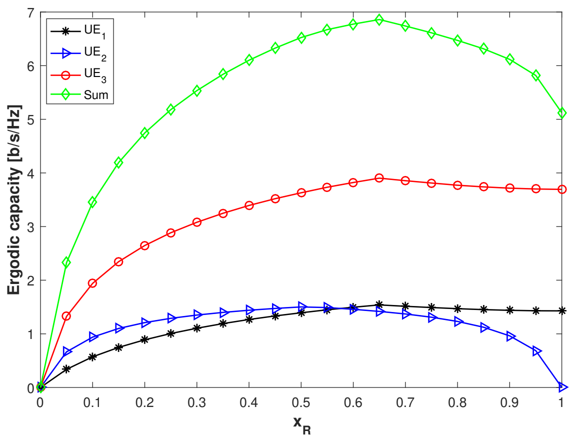

Fig. 10. The capacity of the users as a function of location R at SNR = 15dB.

Figure 10 surveys the impact of R location on system capacity. It is noticed that in the figure, there exists a position R where the maximum system capacity at $\mathrm{SNR} = 15$ dB. In the implementation of the relay system, it is very important to determine the optimal position of the relay node R, so this result can make the survey of the R location more convenient. When implementing a relay system, it is very significant to determine the optimal position of the relay node R, so this result can make the survey of the R location more convenient. In Figure 10, it can be seen that the best position to place R is at $x_R = 0.65$. This is also the position at which the capacities of UE_1 and UE_3 reach their maximum values, while for UE_2 it is at $x_R = 0.5$. Thus, the position of R to ensure maximum system capacity is not necessarily achieved at each user. Outside the optimal position of R, the system capacity tends to decrease significantly when R is located far from BS and near the users.

5. Conclusion

The NOMA systems combined with the FD technique at the relay node are a promising solution for wireless networks in both increasing capacity and expanding coverage. The article analyzed and investigated the impacts of residual noise RSI, imperfect SIC, channel estimation error, and channel parameter on the general channel Nakagami-m on the system performance. Through the mathematical framework, the article derived exact closed-form expressions for OP and the ergodic capacity of the users in the system and compared them with that of the HDR-NOMA system. Moreover, we determined the conditional expression to ensure the successful decoding of the signals. The results of the numerical analysis and simulation showed that the quality and capacity of the system depend heavily on RSI and SIC. The system had saturation points, so it is essential to choose the values of the RSI and SIC parameters to ensure system performance. In addition, the article also investigated the best position of R in the specific system. To further improve the research content for this model, we propose to optimize the transmit power to maximize the capacity of the multi-user FDR-NOMA system. Moreover, we would like to consider the FDR-NOMA system that combines multiple inputs and multiple outputs (MIMO) when deploying multi-antenna for BS station and multi-user FDR-NOMA user terminal and relay stations.

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HỆ THỐNG NOMA CHUYỂN TIẾP SONG CÔNG ĐA NGƯỜI DÙNG DƯỚI ẢNH HƯỞNG CỦA CÁC ĐIỀU KIỆN THỰC TẾ

Nguyễn Thị Thu Hằng, *Trần Xuân Nam*, *Hoàng Thị Hồng Hà*

Tóm tắt

Bài báo đưa ra biểu thức tường minh xác suất dừng và dung lượng ergodic của hệ thống đa truy nhập không trực giao NOMA (non-orthogonal multiple access) kết hợp truyền thông song công hoàn toàn FD (full-duplex) tại nút chuyển tiếp (relay), gọi tắt là hệ thống FDR-NOMA trong trường hợp tổng quát đa người dùng trên kênh Nakagami-m với các tham số không hoàn hảo trong hệ thống, bao gồm nhiễu dư RSI (residual self-interference) khi thực hiện kỹ thuật song công, loại bỏ nhiễu nối tiếp SIC (successive interference cancellation) trong NOMA và có sai số ước lượng kênh truyền. Các kết quả phân tích được kiểm chứng bằng chương trình mô phỏng Monte-Carlo trên phần mềm Matlab. Bên cạnh đó, bài báo cũng đánh giá ảnh hưởng của các tham số không hoàn hảo đối với chất lượng của hệ thống và khảo sát vị trí tối ưu của nút chuyển tiếp để đạt dung lượng hệ thống tốt nhất trong điều kiện của bài toán.

Từ khóa

Đa truy nhập không trực giao (NOMA), loại bỏ nhiễu nối tiếp (SIC), truyền thông song công (FD), nhiễu tự giao thoa còn dư (RSI), giải mã và chuyển tiếp (DF), truyền thông chuyển tiếp, thông tin trạng thái kênh truyền (CSI).