

STATIC INVESTIGATION OF A FUNCTIONALLY GRADED CARBON NANOTUBES REINFORCED COMPOSITE CYLINDRICAL SHELL, DOUBLE-ENDED CLAMPED SUBJECTED TO EXTERNAL PRESSURE LOADS

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Abstract

In this article, static analysis of clamped-clamped (C-C) functionally graded carbon nanotubes reinforced composite (FG-CNTRC) cylindrical shell subjected to external pressure was conducted. The governing equations were established by using higher-order shear deformation theory (HSDT) taking transverse normal stress effect into account. In this theory, the transverse displacement w is not a constant but rather is the second order polynomial of the coordinate along the thickness direction. Distribution of carbon nanotubes (CNT) across the shell thickness is assumed to be uniform (UD) or functionally graded in four types: FG- Λ , FG-V, FG-O, and FG-X. Effective material properties of FG-CNTRC cylindrical shells were estimated by the rule of mixture. An analytical solution using the simple trigonometric series and the Laplace transformation to solve governing equations of shell with clamped boundary condition at the both ends is presented. The validation of the applied approach was examined by comparing the results based on 3D exact model. The effects of the CNT distribution, the CNT volume fraction, and the geometrical parameters on the static behaviour of cylindrical shells subjected to external pressure were investigated. The result is remarkable that the stress components near the outer or inner surface vary most strongly, and in the case of a short or/and thick shell, the geometric parameter greatly affects the stress of shell.

Keywords: Static analysis; the higher-order shear deformation theory; functionally graded carbon nanotubes reinforced composite; cylindrical shell; FG-CNTRC.

1. Introduction

The high strength and stiffness, and thermal properties of CNTs make it to be as an excellent reinforcement material for high-performance structural composites with great potential application. However, material properties in traditional composites do not vary spatially at the macroscopic level because the distribution of reinforcements is either uniform or random in the matrix phase. Functionally graded materials (FGMs) are

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a new generation of composite materials with specific properties in the preferred direction/orientation.

Combining the unique properties of FGM and the excellent properties of CNT, FG-CNTRCs become an advanced material that can be applied widely in engineering. Attracted by the potential provided by FG-CNTRCs, many researchers have studied the mechanical behaviour of FG-CNTRC structures in the recent decade. The first model of FG-CNTRC material was proposed by Shen in 2009 [1] with CNT distribution within an isotropic matrix design specifically to grade them with certain rules along the desired directions. Based on this model, many scientists have studied FG-CNTRC structures in the past few years and their results were recorded in the recent reviews by Liew et al. [2, 3], Zhang et al. [4]. The static analysis of FG-CNTRC plates and shells has been conducted based on the different plates theory by many researchers.

Based on classical shell theory (CST), Nguyen Dinh Duc et al. [5] studied thermal and mechanical stability of a FG-CNTRC truncated conical shells surrounded by the elastic foundations. Using the same theory, Do Quang Chan et al. [6] carried out the nonlinear buckling and post-buckling behavior of FG-CNTRC truncated conical shells. They used the Airy stress function and Galerkin method to solve the governing equation. Duong Tuan Manh et al. [7] presented the nonlinear post-buckling analysis of carbon nanotubes (CNTs) reinforced sandwich composite annular spherical (AS) shells supported by elastic foundations in the thermal environment. In this work, the governing equations are established by using CST and von Kármán's geometrical nonlinearity. Dinh Gia Ninh and Dao Huy Bich [8] used the CST and Galerkin method to investigate of electro-thermo-mechanical vibration of simply supported FG-CNTRC cylindrical shells surrounded by an elastic medium.

The first-order shear deformation plate theory (FSDT) was employed to study mechanical behavior of FG-CNTRC structures in various article. Liew et al. [9] presented the postbuckling of FG-CNTRC cylindrical panels under axial compression by using FSDT. The Ritz method is employed to obtain the discretized governing equation. Based on the FSDT, Ansari and Torabi [10] studied the vibration and buckling of FG-CNTRCs conical shells subjected to compressive axial loads. The DQM and periodic differential operators were used to solve the governing equations. Vu Thanh Long and Hoang Van Tung [11] investigated thermomechanical postbuckling behavior of CNTRC sandwich plate models resting on elastic foundations. Governing equations are established within the framework of FSDT taking into account von Kármán's nonlinearity. Galerkin method is used to solve these equations for plate with simply supported boundary conditions.

In order to acquire more accurate results for studying plate/shell structures, higher-order shear deformation theories (HSDTs) were created as improvements of the FSDT. Mehrabadi and Aragh [12] investigated the bending behaviour of FG-CNTRC open cylindrical shells subjected to mechanical load. Both the three-dimensional theory of elasticity and the third-order shear deformation theory (TSDT) were employed. The generalized differential quadrature method (GDQM) was used to solve the governing equations. Zghal et al. [13] studied static behaviour for various types of FG-CNTRC structures (plate, skew plate, and cylindrical shell) using discrete double director shell elements. Moradi-Dastjerdi et al. [14] carried out static analysis of FG-CNTRC cylinders under internal and external pressure by a mesh-free method. In this method, moving least squares shape functions were used for approximation of displacement field in the weak form of equilibrium equation. Based on the HSDT which was proposed by Reddy, Nguyen Dinh Dat et al. [15] investigated nonlinear magneto-electro-elastic vibration of smart sandwich plate with carbon nanotube reinforced nanocomposite core in hygrothermal environment. The Galerkin method is used to obtain natural frequency the smart sandwich plate with simply supported at four edges. Huu Quoc Tran et al. [16] developed a new four-variable refined plate theory for static analysis of laminated FG-CNTRC plates integrated with a piezoelectric composite (PFRC) actuator under electro-mechanical loadings. Duong Thanh Huan et al. [17] also used this theory to study free vibration analysis of FG-CNTRC plates submerged in a fluid medium.

The three-dimensional (3D) theory of elasticity was used by several authors to examine FG-CNTRCs structures. Based on 3D theory and Navier's method, Alibeigloo et al. [18] conducted static analysis of the FG-CNTRC plates embedded in piezoelectric layer subjected to mechanical load with simply-supported edges. Using the same theory and method, Alibeigloo et al. [19] studied the bending of the simply supported FG-CNTRC cylindrical panels. The static and free vibration of FG-CNTRC cylindrical shells based on the three-dimensional theory of elasticity was carried out by Alibeigloo and Jafarian [20]. In this study, the authors employed the differential quadrature method to solve the governing equation with various boundary conditions.

As can be seen from the above studies, the model based on the 3D theory is the most exact. Nevertheless, 3D solution is difficult to compute for structures with non-simply supported boundary conditions. In addition, the CST can be applied for thin plate/shell structure, and the FSDT requires the shear correction coefficients because transverse shear deformations are assumed to be a constant variation through the structure thickness. Using HSDTs is more convenient in most boundary conditions thanks to the low computational effort compared to 3D models. Consequently, many

HSDTs are continuing to be introduced to study the mechanical behavior of structures. However, the transverse normal stress effect was ignored in most of HSDTs with constant transverse displacement. This assumption about the displacement field is inaccurate, especially for thick plates/shells. To solve this issue, the HSDT type quasi-3D which accounts for the transverse normal stress effect was presented. Doan et al. used this theory to conduct static analysis of laminated cylindrical shell [21], and FGM cylindrical shell [22]. Results showed that the stresses in clamped boundary condition in these studies were much different compared to those based on other HSDTs and very closed with those based on 3D theory.

Motivated by the efficiency and accuracy of the HSDT type quasi-3D which accounts for the transverse normal stress effect, it was applied to the static analysis of the clamped-clamped FG-CNTRC cylindrical shells in this article. The principle of virtual work is used to establish governing equations and associated boundary conditions. An analytical solution for a cylindrical shell with clamped boundary conditions at both ends is presented. In this solution, the simple trigonometric series and the Laplace transform were employed. Effective material properties of the FG-CNTRC cylindrical shell were estimated by the rule of mixture. Present approach is validated by comparing the results based on 3D exact model in the open literature. Numerical examples are carried out to investigate the effects of CNT distribution and CNT volume fraction on the bending behaviour of cylindrical shells subjected to external pressure.

2. Material properties

The CNT reinforcements can be uniformly (UD) or functionally graded in four types: FG- Λ , FG-V, FG-O, and FG-X along the radial direction of the cylindrical shell as shown in Fig 1. The effective mechanical properties of FG-CNTRC are determined based on the rule of mixture as follows [1, 20]:

$$\begin{aligned} E_{11} &= \eta_1 V_{CNT} E_{11}^{CNT} + V_m E_m, \\ \frac{\eta_2}{E_{22}} &= \frac{V_{CNT}}{E_{22}^{CNT}} + \frac{V_m}{E_m}, \quad \frac{\eta_3}{G_{12}} = \frac{V_{CNT}}{G_{12}^{CNT}} + \frac{V_m}{G_m} \end{aligned} \quad (1)$$

where $\eta_i (i=1,2,3)$ are CNT efficiency parameters, $E_{11}^{CNT}, E_{22}^{CNT}, G_{12}^{CNT}, V_{CNT}, E_m, G_m, V_m$ are Young's modulus, shear modulus, the volume fraction of carbon nanotube, and matrix, respectively.

The relation between the CNT and matrix volume fractions is stated as

$$V_{CNT} + V_m = 1 \quad (2)$$

Relations of CNT volume fraction distribution for five cases are as follows [20]:

$$\text{For UD: } V_{CNT} = V_{CNT}^* \quad (3.1)$$

$$\text{For FG-}\Lambda\text{: } V_{CNT} = 2((r_o - r) / h) V_{CNT}^* \quad (3.2)$$

$$\text{For FG-V: } V_{CNT} = 2((r - r_i) / h) V_{CNT}^* \quad (3.3)$$

$$\text{For FG-O: } V_{CNT} = 2(1 - 2|r - r_m| / h) V_{CNT}^* \quad (3.4)$$

$$\text{For FG-X: } V_{CNT} = 4(|r - r_m| / h) V_{CNT}^* \quad (3.5)$$

where r_m and h are the mid radius and thickness of the cylindrical shell, respectively.

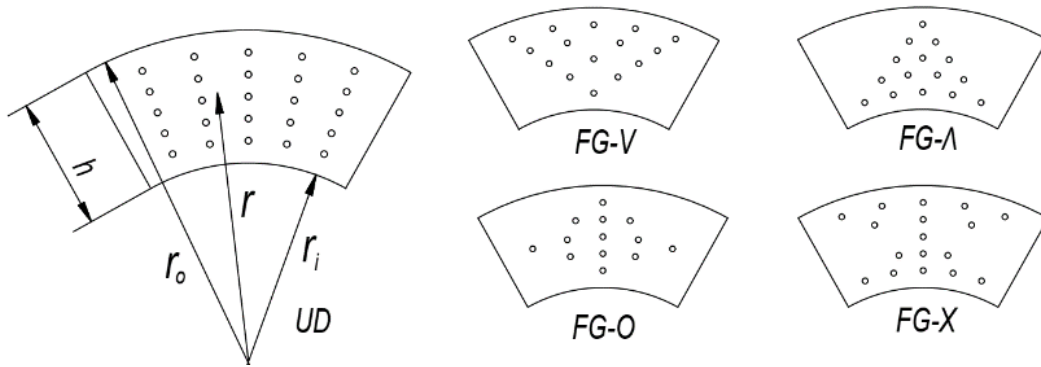


Fig. 1. Volume fraction distribution for five cases.

Effective Poisson's ratio, ν_{12} , and effective density, ρ are defined by [1, 20]:

$$\nu_{12} = V_{CNT}^* \nu_{12}^{CNT} + V_m \nu_{12}, \quad \rho = V_{CNT}^* \rho_{CNT} + V_m \rho_m \quad (4)$$

where ν_{12}^{CNT} , ν_m are Poisson's ratio of CNT and polymer matrix.

The other effective elastic constants for FG-CNTRC are [20]:

$$E_{22} = E_{33}, G_{12} = G_{13} = G_{23}, \nu_{12} = \nu_{13}, \nu_{31} = \nu_{21}, \nu_{32} = \nu_{23} = \nu_{21}, \nu_{21} = \nu_{12} E_{22} / E_{11} \quad (5)$$

3. Governing equations

Considering the FG-CNTRC cylindrical shell, and an orthogonal curvilinear coordinate system $O\xi\theta z$ as shown in Fig. 2. The displacements of an arbitrary point along the ξ , θ and z directions are denoted by u , v and w , respectively. The shell is subjected to transverse normal loads $q^+(\xi, \theta)$ and $q^-(\xi, \theta)$ on the outer and inner surfaces, respectively. The displacement field in $O\xi\theta z$ was expressed by the Taylor series [21]:

$$\begin{aligned}
 u(\xi, \theta, z) &= u_0(\xi, \theta) + u_1(\xi, \theta)z + u_2(\xi, \theta)\frac{z^2}{2!} + u_3(\xi, \theta)\frac{z^3}{3!}, \\
 v(\xi, \theta, z) &= v_0(\xi, \theta) + v_1(\xi, \theta)z + v_2(\xi, \theta)\frac{z^2}{2!} + v_3(\xi, \theta)\frac{z^3}{3!}, \\
 w(\xi, \theta, z) &= w_0(\xi, \theta) + w_1(\xi, \theta)z + w_2(\xi, \theta)\frac{z^2}{2!},
 \end{aligned} \tag{6}$$

This model has 11 variables according to 11 displacement components. The transverse displacement w is not a constant but rather is the second order polynomial of the coordinate along the thickness direction. Therefore the transverse stress in present model is not equal to zero.

The linear strain-displacement relation is defined as follows:

$$\begin{aligned}
 \varepsilon_\xi &= \frac{1}{R} \frac{\partial u}{\partial \xi}, \quad \varepsilon_\theta = \frac{1}{R+z} \left(\frac{\partial v}{\partial \theta} + w \right), \quad \gamma_{\xi\theta} = \frac{1}{R} \frac{\partial v}{\partial \xi} + \frac{1}{R+z} \frac{\partial u}{\partial \theta}, \\
 \gamma_{\theta z} &= \frac{1}{R+z} \frac{\partial w}{\partial \theta} + \frac{\partial v}{\partial z} - \frac{v}{R+z}, \quad \gamma_{\xi z} = \frac{1}{R} \frac{\partial w}{\partial \xi} + \frac{\partial u}{\partial z}, \quad \varepsilon_z = \frac{1}{R} \frac{\partial w}{\partial z}.
 \end{aligned} \tag{7}$$

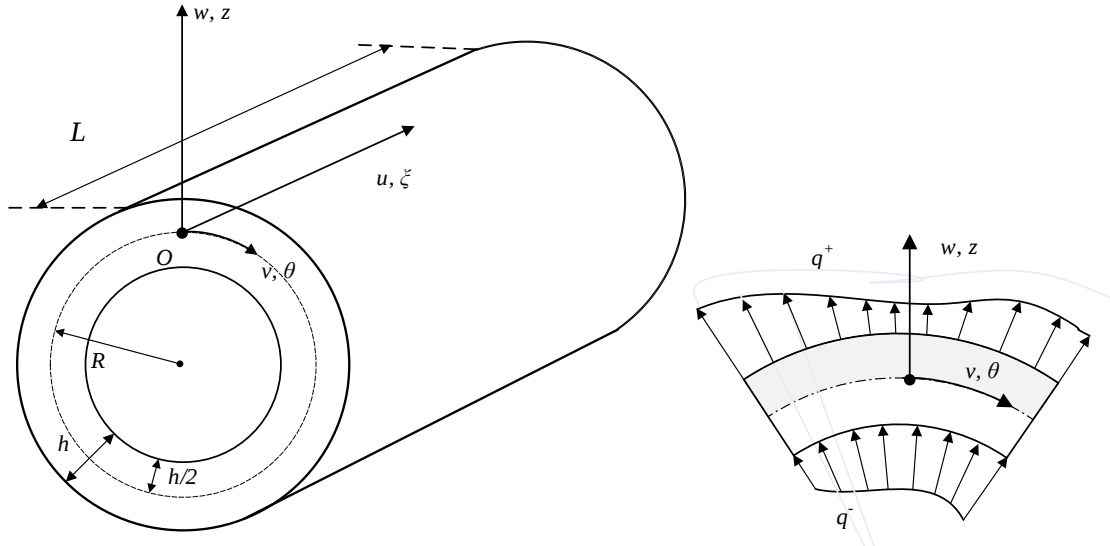


Fig. 2. The geometry of the FG-CNTRC cylindrical shell.

The stress-strain relation is given by Hooke's law as:

$$\begin{Bmatrix} \sigma_\xi \\ \sigma_\theta \\ \sigma_z \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_\xi \\ \varepsilon_\theta \\ \varepsilon_z \end{Bmatrix}, \quad \begin{Bmatrix} \tau_{z\theta} \\ \tau_{\xi z} \\ \tau_{\xi\theta} \end{Bmatrix} = \begin{bmatrix} C_{44} & 0 & 0 \\ 0 & C_{55} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \gamma_{z\theta} \\ \gamma_{\xi z} \\ \gamma_{\xi\theta} \end{Bmatrix}. \tag{8}$$

where the stiffness elements, C_{ij} can be obtained as follows [20]:

$$\begin{aligned} C_{11} &= E_{11}(1-\nu_{23}\nu_{32})/\Delta, C_{22} = E_{22}(1-\nu_{31}\nu_{13})/\Delta, C_{33} = E_{33}(1-\nu_{21}\nu_{12})/\Delta, \\ C_{55} &= G_{13}, C_{66} = G_{12}, C_{12} = E_{11}(\nu_{21} + \nu_{23}\nu_{31})/\Delta, C_{13} = E_{11}(\nu_{31} + \nu_{21}\nu_{32})/\Delta, \\ C_{44} &= G_{23}, C_{23} = E_{22}(\nu_{32} + \nu_{12}\nu_{31})/\Delta, \Delta = 1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} - 2\nu_{12}\nu_{23}\nu_{32} \end{aligned} \quad (9)$$

Using the principle of minimum potential energy to establish the equilibrium equations. This principle is defined in the following formula:

$$\delta\Pi = \delta(U - A) = 0 \quad (10)$$

The elastic potential energy U , and the work done by external forces A can be obtained as follows [21, 22]:

$$\begin{aligned} \delta U &= \iiint [\sigma_{\xi}\delta\varepsilon_{\xi} + \sigma_{\theta}\delta\varepsilon_{\theta} + \sigma_z\delta\varepsilon_z + \tau_{\xi\theta}\delta\gamma_{\xi\theta} + \tau_{\xi z}\delta\gamma_{\xi z} + \tau_{\theta z}\delta\gamma_{\theta z}] (1+z/R) R^2 d\xi d\theta dz \\ \delta A &= \int \int_{\xi \theta} [q^+ \delta w^+ (1+h/2R) + q^- \delta w^- (1-h/2R)] R^2 d\xi d\theta \end{aligned} \quad (11)$$

in which $w^+ = w_0 + w_1 h/2 + w_2 h^2/8$, $w^- = w_0 - w_1 h/2 + w_2 h^2/8$.

Integrating separately the expression (10) according to displacement components, then taking independently the possible displacement equals zero. The equilibrium equations are derived as follows:

$$\begin{aligned} \delta u_0 : \frac{\partial N_{\xi}}{\partial \xi} + \frac{\partial N_{\theta\xi}}{\partial \theta} &= 0, & \delta v_0 : \frac{\partial N_{\theta}}{\partial \theta} + \frac{\partial N_{\xi\theta}}{\partial \xi} + Q_{\theta} &= 0, \\ \delta w_0 : \frac{\partial Q_{\xi}}{\partial \xi} + \frac{\partial Q_{\theta}}{\partial \theta} - N_{\theta} - R p_0 &= 0, & \delta u_1 : \frac{\partial M_{\xi}}{\partial \xi} + \frac{\partial M_{\theta\xi}}{\partial \theta} - R Q_{\xi} &= 0, \\ \delta v_1 : \frac{\partial M_{\xi\theta}}{\partial \xi} + \frac{\partial M_{\theta}}{\partial \theta} - R Q_{\theta} &= 0, & \delta w_1 : \frac{\partial S_{\xi}}{\partial \xi} + \frac{\partial S_{\theta}}{\partial \theta} - M_{\theta} - R Q_z - R p_1 &= 0, \\ \delta u_2 : \frac{\partial N_{\xi}^*}{\partial \xi} + \frac{\partial N_{\theta\xi}^*}{\partial \theta} - R S_{\xi} &= 0, & \delta v_2 : \frac{\partial N_{\xi\theta}^*}{\partial \xi} + \frac{\partial N_{\theta}^*}{\partial \theta} - R S_{\theta} - Q_{\theta}^* &= 0, \\ \delta v_3 : \frac{\partial M_{\xi\theta}^*}{\partial \xi} + \frac{\partial M_{\theta}^*}{\partial \theta} - R Q_{\theta}^* - 2 S_{\theta}^* &= 0, & \delta u_3 : \frac{\partial M_{\xi}^*}{\partial \xi} + \frac{\partial M_{\theta\xi}^*}{\partial \theta} - R Q_{\xi}^* &= 0, \\ \delta w_2 : \frac{\partial Q_{\xi}^*}{\partial \xi} + \frac{\partial Q_{\theta}^*}{\partial \theta} - N_{\theta}^* - R S_z - R p_2 &= 0 \end{aligned} \quad (12)$$

In Equation (12), the symbols are used for extending internal forces as follows:

$$\begin{aligned}
 (N_\xi, M_\xi, N_\xi^*, M_\xi^*) &= \int_{-h/2}^{+h/2} \sigma_\xi \left(1 + \frac{z}{R}\right) \left(1, z, \frac{z^2}{2}, \frac{z^3}{6}\right) dz, \\
 (N_\theta, M_\theta, N_\theta^*, M_\theta^*) &= \int_{-h/2}^{+h/2} \sigma_\theta \left(1, z, \frac{z^2}{2}, \frac{z^3}{6}\right) dz, \\
 (N_{\theta\xi}, M_{\theta\xi}, N_{\theta\xi}^*, M_{\theta\xi}^*) &= \int_{-h/2}^{+h/2} \tau_{\xi\theta} \left(1, z, \frac{z^2}{2}, \frac{z^3}{6}\right) dz, \\
 (N_{\xi\theta}, M_{\xi\theta}, N_{\xi\theta}^*, M_{\xi\theta}^*) &= \int_{-h/2}^{+h/2} \tau_{\xi\theta} \left(1 + \frac{z}{R}\right) \left(1, z, \frac{z^2}{2}, \frac{z^3}{6}\right) dz, \\
 (Q_z, S_z) &= \int_{-h}^{+h} \sigma_z (1, z) \left(1 + \frac{z}{R}\right) dz, \quad (Q_\xi, S_\xi, Q_\xi^*) = \int_{-h/2}^{+h/2} \tau_{\xi z} \left(1, z, \frac{z^2}{2}\right) \left(1 + \frac{z}{R}\right) dz, \\
 (Q_\theta, S_\theta, Q_\theta^*, S_\theta^*) &= \int_{-h/2}^{+h/2} \tau_{\theta z} \left(1, z, \frac{z^2}{2}, \frac{z^3}{6}\right) dz, \\
 p_i &= q^+ \left(1 + \frac{h}{2R}\right) \frac{(h/2)^i}{i!} + q^- \left(1 - \frac{h}{2R}\right) \frac{(-h/2)^i}{i!}, \quad i = 0, 1, 2.
 \end{aligned} \tag{13}$$

For closed cylindrical shells, the boundary conditions for simply supported (S), clamped (C), and free (F) at $\xi = 0, L/R$ edges are assumed as follows [21, 22]:

$$C : u_0 = u_1 = u_2 = u_3 = 0, \quad v_0 = v_1 = v_2 = v_3 = 0, \quad w_0 = w_1 = w_2 = 0. \tag{14}$$

4. Solution procedure

To solve these differential equations, we developed an analytical solution, which can be applied to a cylindrical shell with different boundary conditions, including: simply supported, clamped, and free. For this solution, we employed the simple trigonometric series and the Laplace transform. The process of solving differential equation (12) has been detailed in the article [21, 22].

Substitution of equations (6) ÷ (9) and (11) into equilibrium equations (12), leads to the system of 11 differential equations corresponding to 11 displacement components $u_i, v_i, w_j, i = 0 \dots 3, j = 0 \dots 2$. The general solution with the internal constants is derived by solving these equations. Then, apply the boundary conditions in equation (14) to obtain these constants. The partial differential equations are transformed into a system of ordinary differential equations by using trigonometric series and the Laplace transform.

The displacement field and the load are represented as follows [21, 22]:

$$\begin{aligned}
 u_i(\xi, \theta) &= U_{i0}(\xi) + \sum_{m=1}^{\infty} \left[U_{im}^{(1)}(\xi) \cos m\theta + U_{im}^{(2)}(\xi) \sin m\theta \right], \\
 v_i(\xi, \theta) &= V_{i0}(\xi) + \sum_{m=1}^{\infty} \left[V_{im}^{(1)}(\xi) \sin m\theta - V_{im}^{(2)}(\xi) \cos m\theta \right], \\
 w_j(\xi, \theta) &= W_{j0}(\xi) + \sum_{m=1}^{\infty} \left[W_{jm}^{(1)}(\xi) \cos m\theta + W_{jm}^{(2)}(\xi) \sin m\theta \right], \\
 q^{\pm}(\xi, \theta) &= Q_0^{\pm}(\xi) + \sum_{m=1}^{\infty} \left[Q_m^{(1)}(\xi) \cos m\theta + Q_m^{(2)}(\xi) \sin m\theta \right].
 \end{aligned} \tag{15}$$

Substituting the root expression of the displacement into equations (6) ÷ (8), stresses σ_{ξ} , σ_{θ} and $\tau_{\xi\theta}$ are obtained. The shear stress components are determined by integrals form as follows [21, 22]:

$$\begin{aligned}
 \tau_{\xi z} &= -\frac{1}{R+z} \int_{-h/2}^z \left[\left(1 + \frac{z}{R} \right) \frac{\partial \sigma_{\xi}}{\partial \xi} + \frac{\partial \tau_{\xi\theta}}{\partial \theta} \right] dz, \\
 \tau_{\theta z} &= -\frac{R}{(R+z)^2} \int_{-h/2}^z \left[\left(1 + \frac{z}{R} \right) \frac{\partial \sigma_{\theta}}{\partial \theta} + \left(1 + \frac{z}{R} \right)^2 \frac{\partial \tau_{\xi\theta}}{\partial \xi} \right] dz, \\
 \sigma_z &= -\frac{1}{R+z} \int_{-h/2}^z \left[\left(1 + \frac{z}{R} \right) \frac{\partial \tau_{\xi z}}{\partial \xi} + \frac{\partial \tau_{\theta z}}{\partial \theta} - \sigma_{\theta} \right] dz + \frac{R-h/2}{R+h/2} q^{-}.
 \end{aligned} \tag{16}$$

The equation (16) is derived from the equation of equilibrium based on the 3D theory of elasticity. According to this equation, the stress field can satisfy simultaneously the internal equilibrium state of the shell element and the boundary conditions on the inner and outer surfaces.

5. Numerical results and discussion

5.1. Validation

To validate the present approach, numerical results obtained for the one-layer orthotropic cylindrical shell are compared with those reported by Varadan and Bhaskar [23] based on 3D elasticity solutions. The material properties are assumed to be, $E_1 = 25E_2$, $G_{23} = 0.2E_2$, $G_{13} = G_{12} = 0.5E_2$, Poisson's ratio $\nu_{12} = 0.25$, and non-dimensional parameters are $\bar{w} = (10wE_1)/(Q_0hS^4)$, $\bar{\sigma}_z = 10\sigma_z/(hS^2Q_0)$, $(\bar{\sigma}_{\xi}, \bar{\sigma}_{\theta}) = 10(\sigma_{\xi}, \sigma_{\theta})/(hS^2Q_0)$. Shell has geometrical parameters as: the length to radius ratio $L/R = 4$ and some different radius to thickness ratios

$R/h = 4, 10, 50, 100, 500$. It is observed from Table 1 that the obtained results are in good agreement with those in [23]. Also, when the radius-to-thickness increases, the cylindrical shell behaves as a thin shell, the results of the present model converge well with those of the 3D model.

Table 1. Comparison of displacement and stresses for one layer cylindrical shell

R/h	Method	$\bar{w}$	$\bar{\sigma}_\theta$	$\bar{\sigma}_\theta$	$\bar{\sigma}_\xi$	$\bar{\sigma}_\xi$	$\bar{\sigma}_z$
		$z = 0$	$z = -h/2$	$z = h/2$	$z = -h/2$	$z = h/2$	$z = 0$
4	Varadan [23]	2.873	-6.969	4.859	-0.2295	0.0981	-0.69
	Present	2.7723	-7.0758	4.5767	-0.28403	0.08925	-0.658
10	Varadan [23]	0.9189	-4.509	4.051	-0.0656	0.0663	-1.37
	Present	0.91724	-4.5286	4.0160	-0.08667	0.05528	-1.373
50	Varadan [23]	0.5385	-3.979	3.902	-0.0086	0.0845	-5.38
	Present	0.53838	-3.9825	3.8972	-0.01220	0.08128	-5.383
100	Varadan [23]	0.5170	-3.876	3.843	0.0288	0.1190	-10.13
	Present	0.51693	-3.8784	3.8410	0.02704	0.11734	-10.128
500	Varadan [23]	0.3060	-2.293	2.306	0.1924	0.2459	-29.24
	Present	0.30596	-2.2935	2.3053	0.19218	0.24573	-29.240

5.2. The effect of CNT distribution

In this section, numerical illustration for clamped-clamped FG-CNTRC cylindrical shell subjected to external uniform pressure $Q_0 = 1$ MPa with five cases of CNTs distribution UD, FG- Λ , FG-V, FG-O, and FG-X is carried out. The shell has the following geometrical parameters: $h = 0.1$ m, $R/h = 10$, $L/R = 4$.

The material properties of the (10,10) SWCNTs used here from [13] are $E_{11}^{CNT} = 5.6466$ TPa, $E_{22}^{CNT} = 7.0800$ TPa, $G_{12}^{CNT} = 1.9445$ TPa; and $\nu_{12}^{CNT} = 0.175$. The material properties of matrix are $E_m = 2.1$ GPa, $\nu_m = 0.34$.

Efficiency parameters of CNT depend on CNT volume fraction as follows:

$$V_{CNT}^* = 0.11; \eta_1 = 0.149; \eta_2 = 0.934; \eta_3 = \eta_2$$

$$V_{CNT}^* = 0.14; \eta_1 = 0.150; \eta_2 = 0.941; \eta_3 = \eta_2$$

$$V_{CNT}^* = 0.17; \eta_1 = 0.149; \eta_2 = 1.381; \eta_3 = \eta_2$$

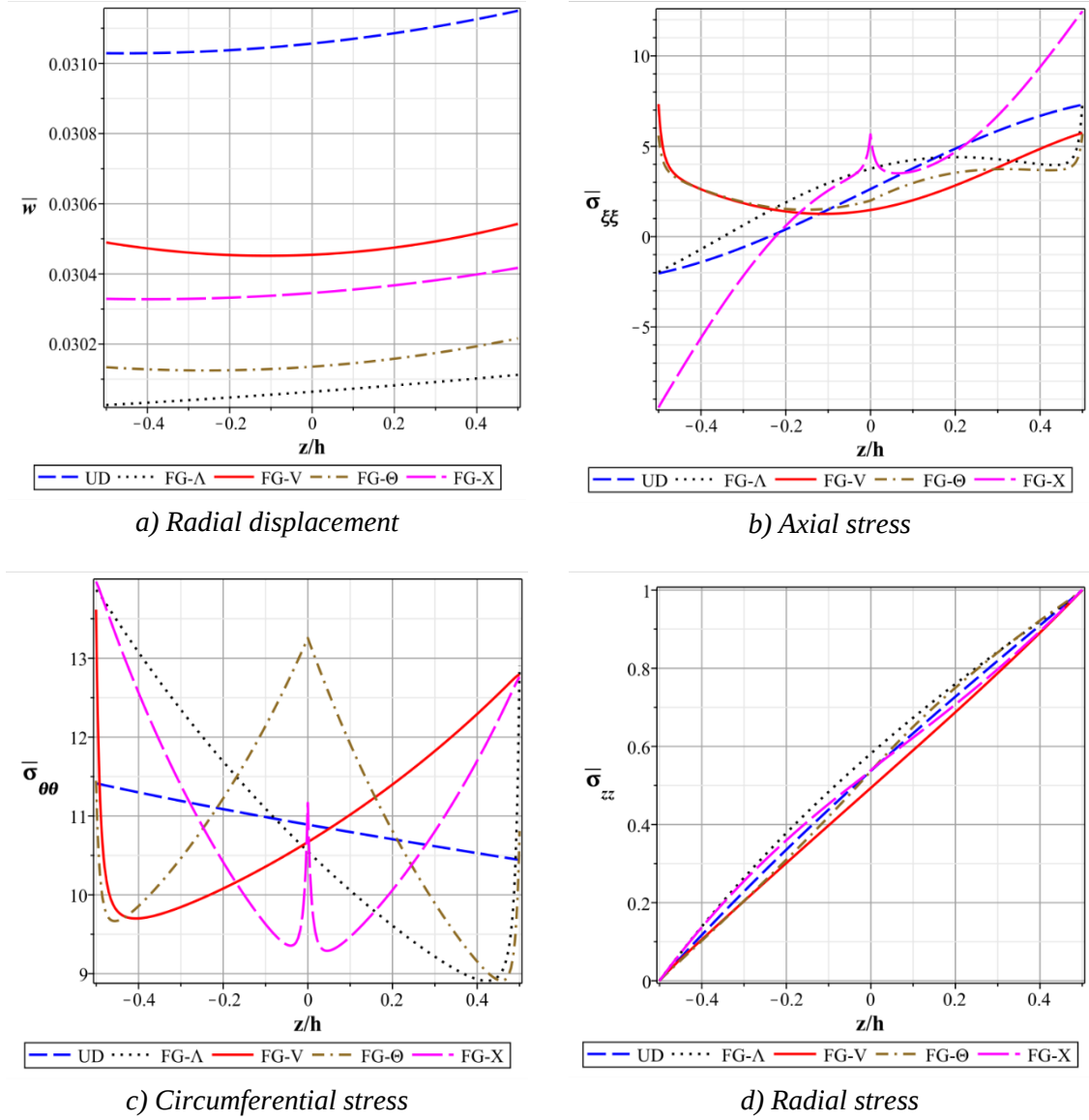


Fig. 3. Effect of CNT distribution on non-dimensional stresses and displacements at the middle position $\xi = L/2R$ with $V_{CNT}^* = 0.17$.

The displacement and stress components are non-dimensionalized as follows:

$$\bar{w} = w/h; (\bar{\sigma}_{\xi\xi}, \bar{\sigma}_{\theta\theta}, \bar{\sigma}_{zz}, \bar{\tau}_{\xi\theta}, \bar{\tau}_{\xi z}, \bar{\tau}_{z\theta}) = (\sigma_{\xi}, \sigma_{\theta}, \sigma_z, \tau_{\xi\theta}, \tau_{\xi z}, \tau_{z\theta})/Q_0 \quad (17)$$

Fig. 3a-d represents the effect of five cases of CNT distribution on the stress and displacement fields at $\xi = L/2R$ along the radial direction of the FG-CNTRC cylindrical shell with $V_{CNT}^* = 0.17$. As can be seen from the figures that:

- The through-thickness distribution of stresses and displacement strongly depends on the case of CNT distribution.

- The displacement of the UD case is the largest, and the displacement of the FG- Λ case is the smallest. The stress components at a given point along the radial direction for the case of UD are situated between the distributions for the case of FG-V and FG- Λ , respectively. The variation of stress components for the case of uniformly distributed UD is smaller than that for other cases of CNT distribution.

- The variation of non-dimensional axial and circumferential stress near the outer/inner surface ($z = \pm h/2$) and middle surface ($z = 0$) is strongly. This variation can be explained by the CNT volume fraction on the outer/inner surface ($z = \pm h/2$) and middle surface ($z = 0$) reaching maximum or minimum values.

- The maximum of stress components of the FG-V shell are the smallest when compared to those of other CNT distribution types for the shell subjected to external pressure. In the case of FG-V, CNTs were concentrated majority near the outer surface.

5.3. Effect of CNT volume fraction

In this section, to investigate the effect of CNT volume fraction on stresses and displacement fields, a clamped-clamped FG-V cylindrical shell with the above-mentioned geometrical dimensions and material properties is analyzed. The numerical results in Fig. 4 show that:

- Increasing of CNT volume fraction leads to a decrease in the value of non-dimensional radial displacement. Moreover, increasing of CNT volume fraction has a significant influence on the non-dimensional axial and circumferential stress near the outer surface and it has a small influence on those near the inner surface. The difference between affect levels, in the FG-V case, can be explained by the CNT volume fraction on the inner surface ($z = -h/2$) for different values of V_{CNT}^* are all zero, and the deviation of the CNT volume fraction on the outer surface ($z = h/2$) is the largest.

- The non-dimensional radial stress increases from zero at the inner surface to the maximum value at the outer surface.

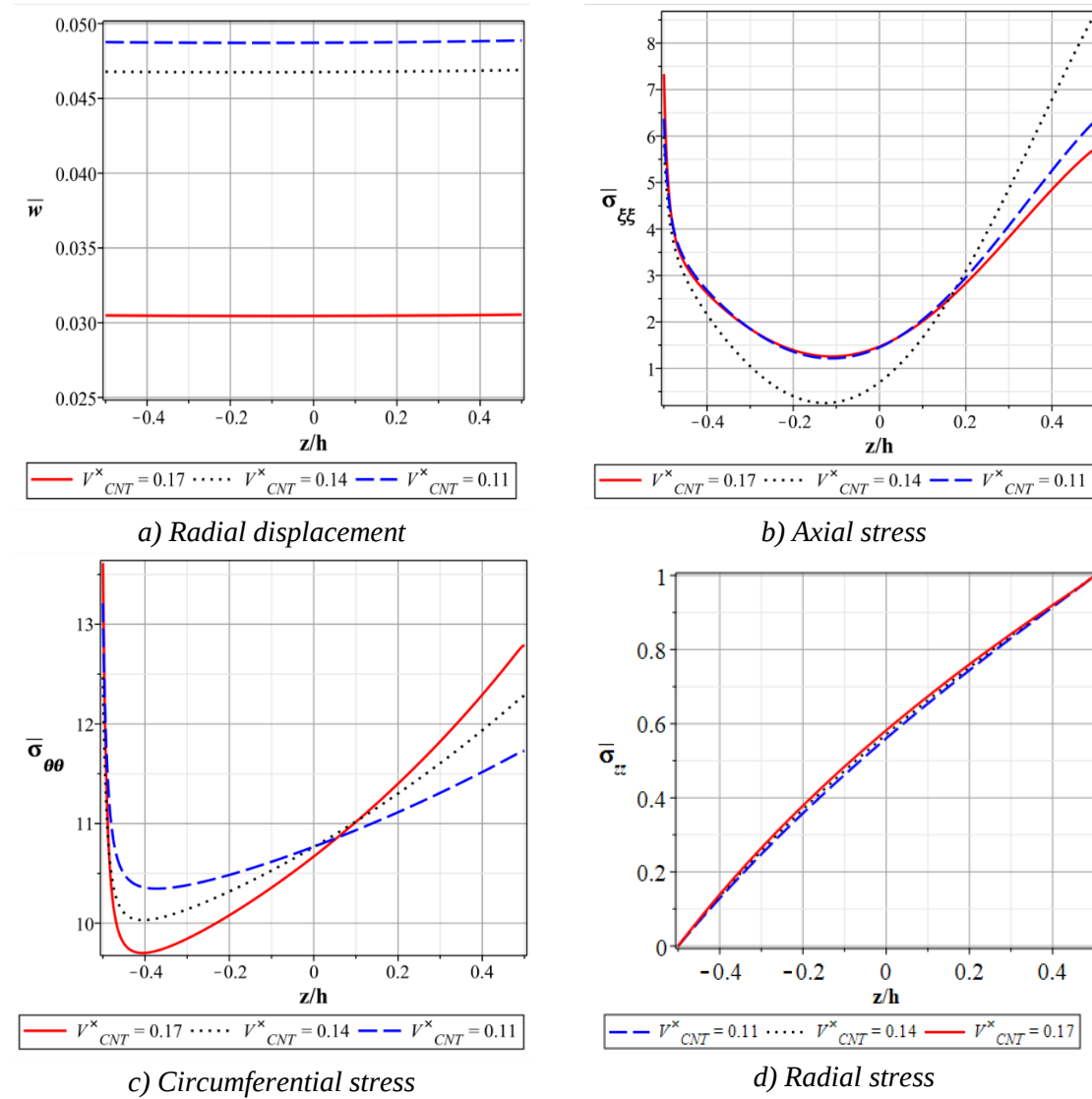


Fig. 4. Effect of CNT volume fraction on non-dimensional stresses and displacements at the middle position $\xi = L/2R$ for FG-V case.

5.4. Effect of geometrical parameters

To investigate effect of the radius to thickness ratio, the clamped-clamped FG-V cylindrical shells with $V_{CNT}^* = 0.17$, $L/R = 4$ and different values $R/h = 5, 10, 20, 50$ are considered. The effects of the radius to thickness ratio on the non-dimensional stresses and displacements at the middle position along the radial direction are shown in Fig. 5. It can be seen from this figure that increasing the value R/h causes the increase of non-dimensional radial displacement, axial stress, and circumferential stress. It is

noteworthy that increasing the value R/h makes non-dimensional radial stress decrease in the case of FG-V shell subjected to external pressure.

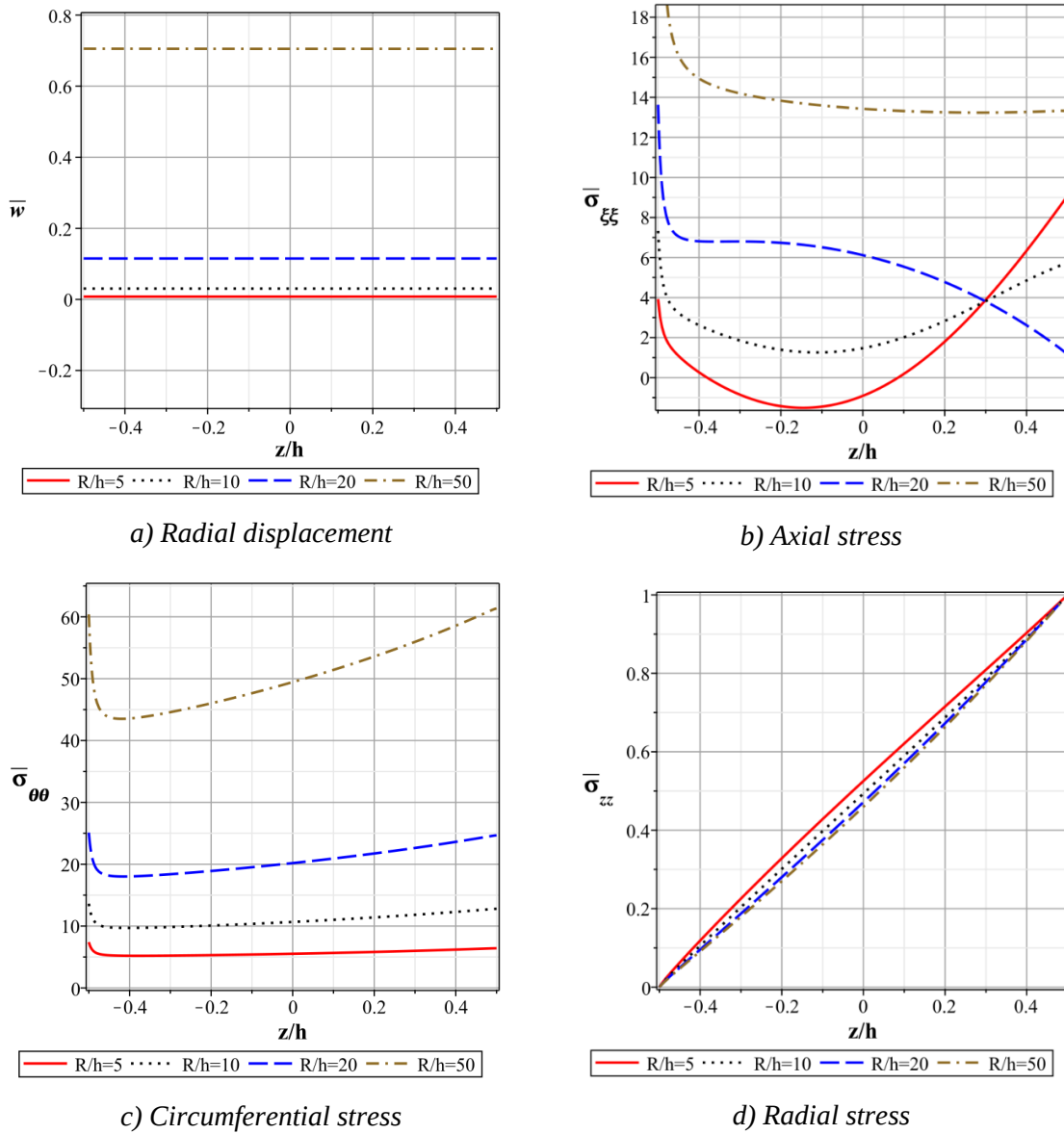


Fig. 5. Effect of the radius to thickness ratio on non-dimensional stresses and displacements at the middle position $\xi = L/2R$ for FG-V case.

To investigate effect of the length to radius ratio, numerical illustration for the clamped-clamped FG-V cylindrical shells with $V_{CNT}^* = 0.17$, $R/h = 10$ and different values $L/R = 0.5, 4, 6, 10$ is carried out. Fig. 6 shows the influence of the length to radius

ratio on the non-dimensional stresses and displacements at the middle position along the radial direction. It can be seen from this figure that:

- For the shell of medium or large relative length ($L/R \geq 4$), the values L/R does not have any significant influence on the displacements and the stresses at the middle position.
- For short shells ($L/R = 0.5$), the values L/R have a large effect on the displacements and the stresses at the middle position. The maximum axial stress of the short shell ($L/R = 0.5$) is equivalent to 300% of the medium and long shell ($L/R \geq 4$).

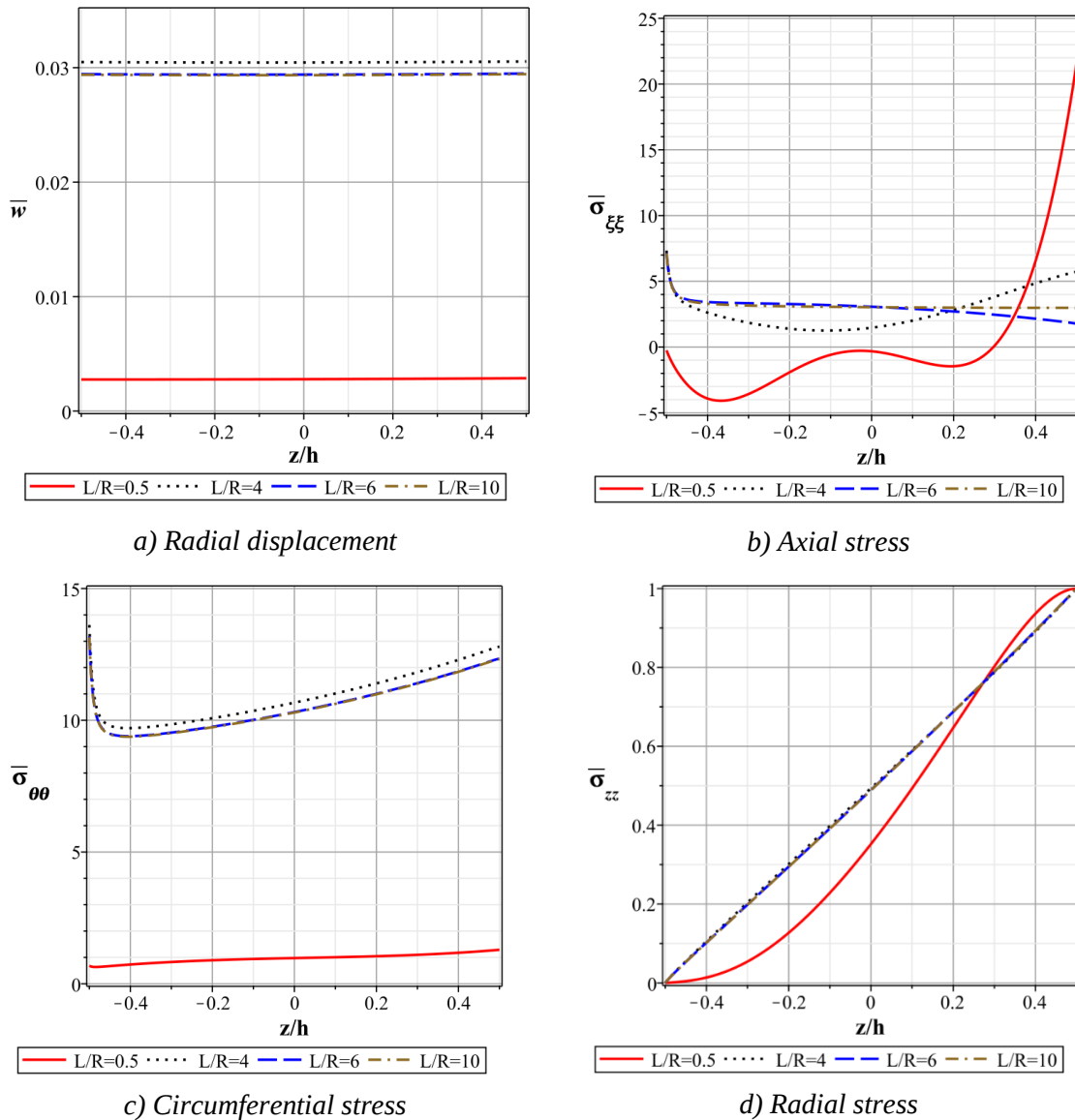


Fig. 6. Effect of the length to radius ratio on non-dimensional stresses and displacements at the middle position $\xi = L/2R$ for FG-V case.

6. Conclusions

In this article, static analysis of clamped-clamped FG-CNTRC cylindrical shells was presented using the higher-order shear deformation theory taking the transverse normal stress effect into account. The transverse displacement was expressed by second order polynomial of the coordinate along the thickness direction. The material properties of the FG-CNTRC were determined by the extended rule of mixture. The governing equations and the boundary conditions were derived based on the virtual work principle. An analytical solution is presented to solve the governing equations of a cylindrical shell with clamped boundary conditions at both ends. The simple trigonometric series and the Laplace transform were employed in this solution. The obtained results of the present approach were validated by comparing its numerical results with the results based on 3D theory of elasticity of published works. Effects of the types of distribution, the volume fraction of CNT on the displacement, and stresses of clamped-clamped FG-CNTRC cylindrical shell subjected to external pressure were investigated. Some notable results were obtained from this investigation:

- The HSDT type quasi-3D which accounts for the transverse normal stress effect is effective for static analysis of thick FG-CNTRC cylindrical shell.
- The present analytical solution can be applied to calculate the cylindrical shell with non-simply supported boundary conditions.
- The type of CNT distribution and the CNT volume fraction have a significant effect on the static behavior of cylindrical shells. The variation of components stress near the outer/inner surface and middle surface is strongly. For the cylindrical shells subjected to external pressure, the maximum value of stress components is smaller when CNTs are concentrated majority near the outer surface.
- The geometrical parameters have a significant effect on the static behavior in the case of short or/and thick cylindrical shells.

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KHẢO SÁT TÍNH VỎ TRỤ COMPOSITE GIA CƯỜNG BẰNG ỚNG NANO CACBON ĐƯỢC PHÂN BỐ BIẾN THIÊN, HAI ĐẦU NGÀM CHỊU TẢI ÁP SUẤT TRÊN BỀ MẶT NGOÀI

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Tóm tắt: Trong bài báo này, phân tích tĩnh của vỏ trụ bằng vật liệu composite gia cường bằng ống nano cacbon được phân bố biến thiên (FG-CNTRC) có hai đầu ngàm (C-C) chịu tải áp suất trên bề mặt ngoài đã được thực hiện. Hệ phương trình cân bằng được thiết lập bằng cách sử dụng lý thuyết biến dạng cắt bậc cao có kể đến ứng suất pháp tuyến. Trong lý thuyết này, chuyển vị cắt theo phương pháp tuyến không phải là hằng số mà là đa thức bậc hai của tọa độ theo chiều dày vỏ. Các kiểu phân bố CNT theo độ dày vỏ được giả thiết là không đổi (UD) hoặc biến đổi theo 4 kiểu: FG-A, FG-V, FG-O và FG-X. Đặc tính hiệu dụng của vỏ trụ bằng vật liệu FG-CNTRC được xác định bằng quy luật trộn lẫn. Phương pháp giải tích sử dụng chuỗi lượng giác và phép biến đổi Laplace để giải hệ phương trình cân bằng của vỏ có điều kiện biên ngàm hai đầu. Độ tin cậy của mô hình sử dụng được kiểm chứng bằng cách so sánh với kết quả dựa trên mô hình chính xác 3D đã được công bố. Sau đó thực hiện khảo sát ảnh hưởng của quy luật phân bố cốt CNT, tỉ lệ thể tích CNT và các thông số hình học đến đáp ứng tĩnh của vỏ trụ dưới tác dụng tải trọng áp suất trên bề mặt ngoài. Kết quả đáng lưu ý rằng ứng suất của vỏ biến thiên mạnh nhất ở mặt trong hoặc mặt ngoài và trong trường hợp vỏ ngắn và/hoặc dày, thông số hình học ảnh hưởng lớn đến giá trị ứng suất của vỏ.

Từ khóa: Phân tích tĩnh; lý thuyết biến dạng cắt bậc cao; composite có cơ tính biến thiên; vỏ trụ; FG-CNTRC.

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