COMPARATIVE SOLUTIONS FOR OPTIMIZING THE CUTTING PARAMETERS WHEN TURNING TITANIUM ALLOY Ti-6Al-4V

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Abstract

This paper introduces some traditional and modern cutting parameters optimization (CPO) methods, specifically applied for the Ti-6Al-4V alloy, the most common material in the Ti-alloys, that also belong to the group of typically difficult-to-machine materials. The single and multi-objective optimization models have been mathematically rigorously built, using the reliable experimental data set according to the full factorial model. Five optimization methods were used for comparison. They give quite similar and realistic solutions. The average value of the calculated results only deviates (0.2 ÷ 2.3)% from the confirmation test. In the case study, the economic benefit from optimization is significant, the machining cost is only 56% of the average value of the experimental options; the cost for cutting tools accounting for about (17 ÷ 45)%, which cannot be ignored as in the case of conventional structural steel. Similar results can be predicted for the materials in the same group, such as stainless steel, Ni, or Co-based alloys.

Keywords: *Cutting parameters optimization; genetic algorithm; linear programming; titanium alloy.*

1. Introduction

Ti-alloys have many excellent mechanical, physical, and chemical properties. They are still difficult to replace in many industries, such as aerospace, automobile, food, chemical, medicine, etc. [1]. However, Ti-alloys are well known as difficult-to-machined materials because of their high strength, high toughness, strong chemical activity, strong adhesion, poor thermal conductivity, causing high cutting force, high energy consumption, high temperature in the cutting zone, poor surface roughness, and extremely fast tool wear [2]. For machining such difficult-to-machine materials, where it is difficult to determine what is a reasonable technology regime, CPO is increasingly urgent.

Unfortunately, few optimization models satisfy both mathematical rigor and practical application. Reviewing published works on CPO we can see two opposing trends. Traditional optimization models are more mathematically complete, mostly multi-variable models (usually cutting speed, feed rate, and depth of cut), using economic or productivity objective function (OF) with constraints [3]. In contrast, all the "new" models use a simple OF, without constraints [4].

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Looking beyond the wider scope of the literature, we can see a similar picture. In many studies, models without constraints are also encountered, with an OF of material removal rate [5], surface roughness [6-8], or cutting force [9]. About OF, the model in [10] takes into account multiple objectives (maximum tool life, minimum cutting force, and minimum roughness) but they were separated into single-objective problems. A few authors choose more complete models, using an OF of total machining cost or total machining time with constraints on surface roughness, cutting force, and cutting power, but described them rather faintly [11, 12]. The model in [13], has up to 3 OF: highest productivity, lowest machining cost, minimum surface roughness with the constraints of the spindle power, and cutting force but it was converted back to the single-objective optimization problem using the weighted sum as the only OF.

The choice of the CPO model depends on the research purpose and actual conditions of the authors, it is difficult to judge it is right or wrong, good or bad. However, there are a few comments as follows:

- Manufacturers often have to trade-off between three conflicting requirements of the machining process: productivity, quality, and cost. They usually try to achieve the highest productivity, lowest cost but just satisfactory quality. Accordingly, the optimization objective may be the lowest machining cost, and/or the highest machining productivity, but should not be a quality criterion. In other words, the quality criteria such as surface roughness, dimension accuracy, etc. should be constraints rather than OFs.

- Constraints ensure that the optimization results are suitable for practice. Surface roughness, allowable deformation of parts, cutting force, tool life,... are factors that ensure product quality or the normal operation of the system. But if many constraints are considered, the problem will be quite complicated. It may be the reason why many authors prefer CPO models without constraint.

- The choice of problem solving method depends first of all on the model. For simple models with no constraints, simple methods, such as Taguchi, RSM can be used [7-9]. Complicated models with constraints require more "professional" tools. Traditional methods including the class of mathematical programming usually give stable, accurate results but are algorithmically complicated. They are gradually being combined with or replaced by heuristic methods that are based on evolutionary computing and artificial intelligence. The most commonly used methods of this type are GA [7, 9], particle swarm optimization (PSO) [5, 6, 10], etc. They are quite simple and easy to implement even with complicated, nonlinear problems.

To fully reflect the economic and technical aspects of the CPO problem, in this paper we introduce the models for optimization of the cutting parameters with the OF of total machining cost and total machining time, taking into account the technological factors as the basic constraints: cutting force, spindle power, surface roughness, tool life, and boundary conditions. The model will be established for the turning process of the Ti-6Al-4V alloy. Such a complicated CPO model is verified by different solution methods, both the traditional and the modern ones. The article will present in turn, from setting up the problem, applying algorithms and solving methods, and finally, analyzing the results.

2. Setting up the problem

As the general optimization problem, the CPO one is stated as follows:

Minimizing the OF (a) subject to (b) in (1):

$$\mathbf{y} = \left[y_1(\mathbf{x})...y_z(\mathbf{x}) \right]^T \rightarrow \min \qquad \text{(a)}$$
$$\mathbf{f}(\mathbf{x}) = \begin{cases} \mathbf{g} = \left[g_1(\mathbf{x})...g_{m1}(\mathbf{x}) \right]^T \leq 0 \\ \mathbf{h} = \left[h_1(\mathbf{x})...\mathrm{h}_{m2}(\mathbf{x}) \right]^T = 0 \\ \mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{x}_{\max} \end{cases} \qquad \text{(b)} \tag{1}$$

where $\mathbf{x} = [x_1 \ \ldots \ x_i \ \ldots \ x_n]^T$ is the vector of input variables, $\mathbf{y} = [y_1 \ldots \ y_j \ldots y_z]^T$ is the objective vector representing optimization criteria; in (b) are the m_1 vectors representing the constraints of the inequality form and m_2 vectors for the equality, and the boundary conditions respectively.

2.1. Objective functions

In the CPO models, the OFs should be the machining productivity (highest) and/or the machining cost (lowest).

2.1.1. Objective function of highest machining productivity

The highest machining productivity is usually represented by minimum operation time. That is the time required to machine one workpiece at the given operation.

$$T_{nc} = T_0 + T_{ph} + T_{md} + \frac{T_{ck}}{n} \rightarrow \min \tag{2}$$

where T_0, T_{ph}, T_{md}, T_{ck} are basic time, auxiliary time, tool sharpening time, time for preparation and end of machining series respectively, n is the number of the machined workpiece in a series. Since T_{ph} and T_{ck} do not depend on the cutting parameters, they can be ignored in this case. Then (2) becomes

$$T_{nc} = T_0 + T_{md} = T_0 + \frac{T_0}{T} t_{md} = \frac{V}{Z}(1 + \frac{t_{md}}{T}) \rightarrow \min \tag{3}$$

where t_{md} (min) is the time for one tool sharpening, T (min) is tool life, V (cm^3) is the volume of material removed from the workpiece, $Z = vsa$ (cm^3/min) is the material removal rate (MRR).

Instead of T_{nc}, it is common to use the machining time per unit of volume of material removed from the workpiece

$$\frac{T_{nc}}{V} = \frac{1}{Z} + \frac{t_{md}}{ZT} \rightarrow \min \quad (4)$$

Tool life is calculated using Taylor's formula

$$T = C_T v^{mt} s^{nt} a^{kt} \quad (5)$$

where C_T is coefficient; v (m/min), s (mm/rev) and a (mm) are cutting speed, feed rate, and depth of cut respectively; mt, nt, kt are their corresponding exponents.

This formula is valid only when the exponents are negative, and usually $|mt| >> |nt| > |kt|$, thus the effect of s and a can be ignored.

Substituting (5) into (4) and notice that $Z = vsa$, we get the OF

$$\frac{T_{nc}}{V} = \frac{1}{vsa} + \frac{t_{md}}{C_T v^{mt+1} s^{nt+1} a^{kt+1}} \rightarrow \min \quad (6)$$

2.1.2. Objective function of lowest machining cost

Machining cost is the one required to perform the operation, expressed in monetary units, in this case, is thousand VND (written as “10^3 VND”).

$$K = K_m + K_d + K_{md} \ (10^3\text{VND}) \quad (7)$$

where K_m is the cost of the machine operation, including the machine tool related to cost and the salary of the machinist

$$K_m = AT_0 = \frac{AV}{Z},$$

where A is the cost for one minute of the machine operation (10^{3}VND/min).

The cost of tool buying for the whole operation, K_d is calculated as follows:

$$K_d = \frac{B_z}{z}.\frac{T_0}{T} = B\frac{T_0}{T} = \frac{BV}{ZT},$$

where z is the number of sharpening during the entire cycle life of the tool, B_z (10^{3}VND) is the cost of buying tool, $B=B_z/z$ is the average tool buying cost per sharpening;

The cost of tool sharpening for the whole operation, K_{md} is calculated as follows:

$$K_{md} = \frac{AT_0}{T} t_{md} = \frac{AV}{ZT} t_{md} = \frac{CV}{ZT},$$

where t_{md} (min) and $C = A.t_{md}$ are the time and the cost for one sharpening respectively.

Substitute the components K_m, K_d, K_{md} into (7), we get the OF

$$K = \frac{AV}{Z} + \frac{BV}{ZT} + \frac{CV}{ZT} \to \min \quad (8)$$

Since B and C are both constants and have the same dimension, they can be lumped together into the constant $C_d = B + C = B + A.t_{md}$, collectively called the tool cost, to obtain the OF in a simpler form:

$$K = \frac{AV}{Z} + \frac{C_d V}{ZT} \to \min \quad (9)$$

It is also common to use the standard of the machining cost of 1cm^3 materials

$$\frac{K}{V} = \frac{A}{Z} + \frac{C_d}{ZT} \to \min \quad (10)$$

Substituting the tool life from (5) into (10) we get the final OF

$$\frac{K}{V} = A\left(\frac{1}{vsa} + \frac{C_d / A}{C_T v^{mt+1} s^{nt+1} a^{kt+1}}\right) \to \min \quad (11)$$

From (6) and (11) we can see that the OFs have 2 components. The first is the basic one which is related only to Z. The second is related to the T. If only the first component is taken into account, the model is very simple but incomplete, leading to erroneous results, especially when cutting difficult-to-machined materials, the tool is expensive, and when the tools wear quickly. It is also obvious that the first components of the OFs (6) and (11) differ only in the constant A, so if we ignore tool wear, the productivity problem and the economic problem are the same. If we ignore the constraints again as mentioned in many works in section 1, we can easy to see that the optimal value of v, s, a will coincide with their upper bound (v_{max}, s_{max}, a_{max}). In other words, if the tool wear and the constraints are not taken into account, the scientific and practical significance of the CPO problem will be very low. Conversely, if the above factors are fully calculated, the problem will be difficult to solve, so many authors erroneously avoid them.

The OFs (6) and (11) can be used directly in nonlinear models, but they are difficult to solve with traditional methods. With the linear model, their sum form is an obstacle to logarithmic linearization. The way to overcome this one is converting them into a product.

According to (4) and (10) both OFs depend on the tool life, T. If T is constant, then the optimal criteria $T_{nc}/V \to min$ and $K/V \to min$ have the same form as the criterion $Z \to max$ or $1/Z \to min$. The value of T giving the highest productivity is called the productive tool life T_{ns}. The one giving the lowest cost is called the economic tool life T_e. In the following, we will see that T_{ns} and T_e are both constant, independent of cutting parameters, and can be calculated independently before solving the CPO problem.

To calculate the productive tool life T_{ns}, we take the derivative of both sides of (6) concerning v (ignoring the weak influence of s and a) and solve the equation with the right side equal to zero, getting the formula

$$T_{ns} = -t_{md}(mt+1) \tag{12}$$

Thus T_{ns} is proportional to the sharpening time and depends on the exponent of v in Taylor's formula.

Same with (11) we get the formula for economic tool life

$$T_e = -\frac{C_d}{A}(mt+1) \tag{13}$$

Note that the formulas for T_{ns} and T_e are valid only when $mt < -1$.

Substituting T_{ns} from (12) into (4) and T_e from (13) into (10), we get the OFs:

- Minimum total machining time (maximum productivity)

$$\frac{T_{nc}}{V} = \frac{1}{Z}\left(1+\frac{t_{md}}{T_{ns}}\right) = \frac{t_\Sigma}{Z} \to \min \tag{14}$$

- Minimum total machining cost

$$\frac{K}{V} = \frac{1}{Z}\left(A+\frac{C_d}{T_e}\right) = \frac{A_\Sigma}{Z} \to \min \tag{15}$$

The OFs (14) and (15) are equivalent to (4) and (10) respectively, but (14), (15) have product form, which can be taken logarithmic to form a linear function.

2.2. Constraints and boundary conditions

Constraints ensure that the machining process meets technical requirements, ensuring the model is realistic. In metal cutting theory, relationships between output quantities and the cutting parameters are usually expressed in exponential form. The following are common constraints. By meaning, they are classified into 3 groups.

2.2.1. The group describing the technical conditions

- The constraint that the surface roughness does not exceed the allowable limit:

$$R_a = R_1 v^{mr} s^{nr} a^{kr} \le R_{max} \tag{16}$$

- The constraints on size and shape errors, if any, have similar forms.

2.2.2. The group ensuring the normal working conditions of the system

- The constraint that the cutting force does not exceed the allowable threshold:

$$F = F_1 v^{mf} s^{nf} a^{kf} \leq F_{max} \tag{17}$$

This condition ensures that there is no excessive deformation of the workpiece, the safety of the system, etc. In the same type, there are constraints on workpiece deformation, vibration, cutting zone temperature, and so on.

- Constraints that the tool life is equal to the productive tool life (T_{ns}) or the economic one (T_e) used in the OF (14) or (15) as the linear models:

$$T = C_T v^{mt} s^{nt} a^{kt} = \begin{cases} T_{ns} & in(14) \\ T_e & in(15) \end{cases} \tag{18}$$

2.2.3. The group depending on the features of the technological system

- The condition cutting power does not exceed the spindle motor power *(P_m)*:

$$P = \frac{F_1}{60.1000} v^{(mf+1)} s^{nf} a^{kf} \leq P_m \eta \tag{19}$$

- The range of cutting speed, feed rate, depth of cut, called boundary conditions:

$$v_{\min} \leq v \leq v_{\max}\, ;\ s_{\min} \leq s \leq s_{\max}\, ;\ a_{\min} \leq a \leq a_{\max} \tag{20}$$

3. Problem solution methods

The CPO problems in section 2 are traditional ones. Since the forms of the productivity problem and the cost problem are similar, only the cost problem is solved as the case study, using the OF (11).

3.1. Experimental model

The experimental study is based on the Design of Experiment (DoE) model with 3 factors (v, s, a) and 3 levels, whose values are selected in accordance with the pair of the workpiece and tool materials, that is Ti-6Al-4V alloy – BK6 in Tab. 1.

Tab. 1. Factors and their levels

Factor (k = 3)	Levels (L = 3)		
	Level 1	Level 2	Level 3
v (m/min)	20	35	50
s (mm/rev)	0.10	0.20	0.30
a (mm)	0.50	1.00	1.50

The experiment conditions are as follows: CNC lathe EMCO-E25 with spindle power 5.5 kW, maximum spindle speed 6300 rev/min, maximum spindle torque 35 Nm. Workpiece material is Ti-6Al-4V alloy, size D×L = 50×300 (mm). The tool nose material is BK6. The measured parameters include the cutting force *F* by the Kistler's (Switzerland) 3-component force sensor 9257BA, the surface roughness R_a by the Mitutoyo's (Japan) roughness tester SJ-201. The tool life *T* is defined as the continuous cutting time from the moment the tool is newly sharpened until the height of the flank wear reaches h = 0.3 mm. Carrying out 27 experiments according to the full factorial design L_{27}, we get the experimental data in Tab. 2.

Tab. 2. Experimental data

Experiments	Coded variables			Cutting Conditions			Measurement data			*K/V*
	v	*s*	*T*	*v*	*s*	*t*	*F*	R_a	*T*	
1	-1	-1	-1	20	0.10	0.50	183.11	0.69	1110.91	2.532
2	-1	-1	0	20	0.10	1.00	246.69	0.75	438.83	1.290
3	-1	-1	1	20	0.10	1.50	293.68	0.79	254.88	0.879
4	-1	0	-1	20	0.20	0.50	287.33	1.88	308.16	1.307
5	-1	0	0	20	0.20	1.00	387.10	2.05	121.73	0.697
6	-1	0	1	20	0.20	1.50	460.84	2.16	70.70	0.499
7	-1	1	-1	20	0.30	0.50	373.98	3.38	145.55	0.913
8	-1	1	0	20	0.30	1.00	503.83	3.70	57.49	0.518
9	-1	1	1	20	0.30	1.50	599.80	3.90	33.39	0.394
10	0	-1	-1	35	0.10	0.50	311.60	0.88	281.99	1.499
11	0	-1	0	35	0.10	1.00	419.80	0.97	111.39	0.804
12	0	-1	1	35	0.10	1.50	499.76	1.02	64.70	0.579
13	0	0	-1	35	0.20	0.50	488.96	2.41	78.22	0.842
14	0	0	0	35	0.20	1.00	658.74	2.64	30.90	0.519
15	0	0	1	35	0.20	1.50	784.21	2.78	17.95	0.424
16	0	1	-1	35	0.30	0.50	636.40	4.34	36.95	0.657
17	0	1	0	35	0.30	1.00	857.38	4.75	14.59	0.466
18	0	1	1	35	0.30	1.50	1020.69	5.01	8.48	0.421
19	1	-1	-1	50	0.10	0.50	437.28	1.04	117.69	1.119
20	1	-1	0	50	0.10	1.00	589.12	1.13	46.49	0.651
21	1	-1	1	50	0.10	1.50	701.33	1.20	27.00	0.506
22	1	0	-1	50	0.20	0.50	686.16	2.83	32.65	0.714
23	1	0	0	50	0.20	1.00	924.42	3.10	12.90	0.521
24	1	0	1	50	0.20	1.50	1100.50	3.27	7.49	0.478
25	1	1	-1	50	0.30	0.50	893.07	5.10	15.42	0.636
26	1	1	0	50	0.30	1.00	1203.18	5.58	6.09	0.550
27	1	1	1	50	0.30	1.50	1432.35	5.88	3.54	0.551
									Average K/V	**0.777**

The limit values of the parameters are as follows: cutting force F_{max}= 800 N, spindle power P_{max} = 5.5 kW, surface roughness R_{max} = 2.5 μm, the ranges of cutting parameters are: v = [20 ÷ 50] m/min, s = [0.10 ÷ 0.30] mm/rev, a = [0.5 ÷ 1.50] mm. From reality, these constants have been determined: C_d = 35 (10^3VND), A = 2.5 (10^3VND /min).

Using the linear regression method, the empirical relationships are determined:

$$F = 64v^{0.95}s^{0.65}a^{0.43} \tag{21}$$

$$R_a = 5.5v^{0.45}s^{1.45}a^{0.13} \tag{22}$$

$$T = 9546v^{-2.45}s^{-1.85}a^{-1.34} \tag{23}$$

From (19) and (21), we get

$$P = 0.0013v^{1.95}s^{0.65}a^{0.43} \tag{24}$$

Substituting all relations (21), (22), (24) and limiting values (20) into (1), we get the nonlinear single objective CPO (NS-CPO) model as follows:

$$\begin{aligned} &y = \frac{2.5}{v.s.a} + \frac{35}{9546v^{(-2.45+1)}s^{(-1.85+1)}a^{(-1.34+1)}} \to \min && \text{(a)} \\ &f(x) = \begin{cases} 64v^{0.95}s^{0.65}a^{0.43} \le 800 \\ 0.0013v^{1.95}s^{0.65}a^{0.43} \le 5.5 \\ 5.5v^{0.45}s^{1.45}a^{0.13} \le 2.5 \\ 20 \le v \le 50 \\ 0.1 \ge s \le 0.3 \\ 0.5 \le a \le 1.5 \end{cases} && \text{(b)} \end{aligned} \tag{25}$$

As known, the OF (a) in (25) consists of two components: the direct machining cost and the tool cost. The influence of the cutting parameters on each one is the opposite. In rough machining (productivity $Z = v.s.a$ is high), the first component decreases while the second one increases (due to rapid tool wear). In finishing, the opposite trend happens. The investigation of these trends allows the manufacturer to choose a suitable machining plan. In this case, each cost component is examined as an independent OF, thus obtaining a nonlinear multi-objective CPO (NM-CPO) model with the same constraint system (b) in (25), only the OF (a) is decoupled into y_1 and y_2:

$$\begin{cases} y_1 = \dfrac{2.5}{v.s.a} \\ y_2 = \dfrac{35}{9546v^{(-2.45+1)}s^{(-1.85+1)}a^{(-1.34+1)}} \end{cases} \tag{26}$$

For the linear model, from (13) and (15) we get T_e = 20.3 min, A_Σ= 4.22 (10^3VND), and OF (27) respectively:

$$y = \frac{4.22}{v.s.a} \to min \tag{27}$$

Substituting T_e into (18), we get the constraint of economic tool life as follows:

$$9546v^{-2.45}s^{-1.85}a^{-1.34} = 20.3 \tag{28}$$

Finally, substituting (27) into (a) and adding (28) into (b) of (25), then taking the logarithm of the obtained equations and setting x_1 = ln(v), x_2 = ln(s), x_3 = ln(t), we get the linear single objective CPO (LS-CPO) model as follows:

$$y = 1.44 \quad -x_1 \quad -x_2 \quad -x_3 \quad \to \quad min \qquad \text{(a)}$$

$$f(x) = \begin{cases} 4.16 + 0.95x_1 + 0.65x_2 + 0.43x_3 \leq 6.68 \\ -6.62 + 1.95x_1 + 0.65x_2 + 0.43x_3 \leq 1.70 \\ 1.70 + 0.45x_1 + 1.45x_2 + 0.13x_3 \leq 0.92 \\ 9.16 - 2.45x_1 - 1.45x_2 - 1.34x_3 = 3.01 \\ 3.0 \leq x_1 \leq 3.91 \\ -2.3 \leq x_2 \leq -1.20 \\ -0.69 \leq x_3 \leq 0.41 \end{cases} \qquad \text{(b)} \tag{29}$$

Similarly, we get the linear multi-objective (LM-CPO) model with the same constraints as in (29), and the OFs are obtained by taking the logarithm (26) as follows:

$$\begin{cases} y_1 = 0.92 - x_1 - x_2 - x_3 \\ y_2 = -5.61 + 1.45x_1 + 0.85x_2 + 0.34x_3 \end{cases} \tag{30}$$

3.2. Solving the CPO problem

In this section, the CPO problem will be solved by using two different methods: by Excel's Solver, and GA in Matlab. There are 8 model-method combinations, but based on the capacity of the software, only five of them are implemented, as shown in Tab. 3.

Tab. 3. CPO problem-solving methods

Models / Methods	Linear (L)		Non-linear (N)	
	Single obj. (S)	Multi-obj. (M)	Single obj. (S)	Multi-obj. (M)
Solver - Excel	LS-Solver	-	-	-
GA - Matlab	LS-GA	LM-GA	NS- GA	NM-GA

The following summarizes the steps to solve the problem according to the above methods. The results will be compared to evaluate the performance of each method and the effectiveness of the CPO. Detailed information on methods can be found in [14].

3.2.1. Solving LS-CPO problem using Simplex LP method

Linear programming (LP) is a classic optimization model based on linear algebra. It is usually solved by the so-called "Simplex LP" method, which is supported by most of the computing platforms, from Microsoft's Excel to specialized technical software, such as Matlab. To increase the visibility for readers, this paper introduces Excel's Solver.

For the LS-CPO problem, the model (29) is imported into Excel's spreadsheet as in Tab. 4: the OF is in array L50:O50; initially, x_1, x_2, x_3 (M50:O50) are assigned arbitrarily, for example with [1 1 1], which will be updated during solution. The constraints and boundary conditions are in L40:O49 and their limit values in Q40:Q49.

Tab. 4. LP problem solving with Excel's Solver

	J	K	L	M	N	O	P	Q	R	S
39	Constraints		y_0	x_1	x_2	x_3	y	Limit value		Opt. value
40	$F <= F_{max}$	N	4.16	0.95	0.65	0.43	6.62	6.68	800.00	751.48
41	$P <= P_m$	kW	-6.62	1.95	0.65	0.43	-0.59	1.70	5.50	0.55
42	$R_a <= R_{max}$	µm	1.70	0.45	1.45	0.13	**0.92**	**0.92**	**2.50**	**2.50**
43	$v <= v_{max}$	m/min		1			3.56	3.91	50.00	35.26
44	$s <= s_{max}$	mm/rev			1		-1.69	-1.20	0.30	0.19
45	$a <= a_{max}$	mm				1	**0.41**	**0.41**	**1.50**	**1.50**
46	$v >= v_{min}$	m/min		1			3.56	3.00	20.00	35.26
47	$s >= s_{min}$	mm/rev			1		-1.69	-2.30	0.10	0.19
48	$a >= a_{min}$	mm				1	0.41	-0.69	0.50	1.50
49	$T = T_e$	min	9.16	-2.45	-1.85	-1.34	**3.01**	**3.01**	**20.30**	**20.30**
50	Objective function		**1.44**	**3.56**	**-1.69**	**0.41**	**-0.84**	**K/V =**	**0.4310**	
51	Confirmation test with			**v**	**s**	**a**	**Z**	**A/Z**	**C_d/ZT**	**K/V_{min}**
52	the optimal parameters			**35.26**	**0.19**	**1.50**	**9.800**	**0.255**	**0.176**	**0.431**
53	Cutting cost A (10^3VND)			2.5	Tool cost Cd (10^3VND)			35.0	Te (Tns)	20.30

Follow the instructions on the software interface, then press the *"Solve"* command, the array M50:O50 will be updated with the optimal values: x_1 = 3.56, x_2 = -1.69, x_3 = 0.41, corresponding to v = 35.26 m/min, s = 0.19 mm/rev, a = 1.5 mm. The OF value is -0.84, corresponding to the lowest machining cost K/V = exp(-0.84) = 0.431 (10^3VND/cm^3), which equals only 56% in comparison to the average value of all the experimental ones.

3.2.2. Solving LS-CPO problem using GA

Genetic algorithm is a branch of evolution computing. The theory of GA was first published in 1975 by Holland, inspired by the laws of nature *"Survival of the Fittest"*. The laws of evolution in nature are inherited and simulated by GA as follows:

1- The living environment is always changing. In order to survive and thrive, beings must constantly adapt;

2- The well-adapted individuals will survive, while the less-adapted ones will be eliminated or not selected to continue the lineage;

3- Characteristics of the best individuals will be passed on to the offspring. Selection and inheritance for the next generation is continued in loops, making the next generations have better characteristics than their parent;

4- Sometimes mutations can occur, making the evolution faster.

The essence of the above selection and inheritance process is optimization, called the Genetic Algorithm (GA), which is depicted in Fig. 1.

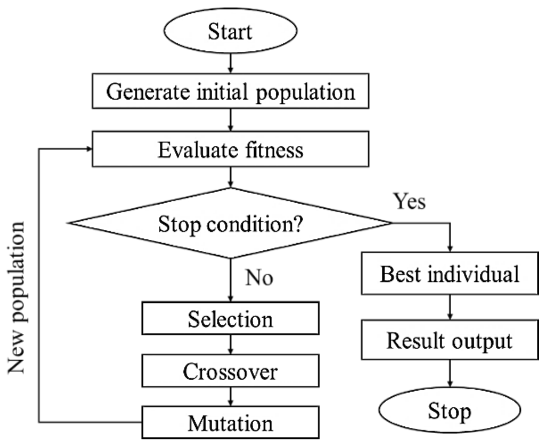
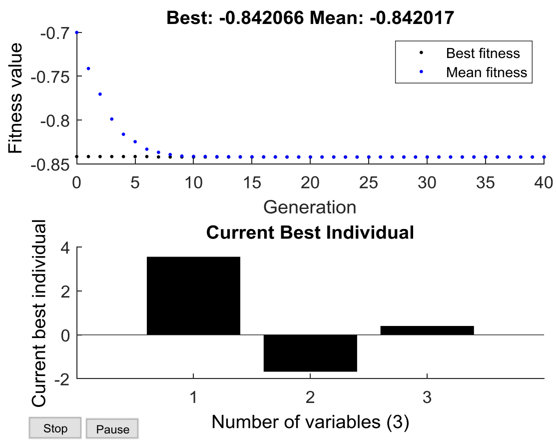


Fig. 1. Optimization with GA.

In this section, GA is used to solve the same LS-CPO problem with the model (29) A program is written in Matlab with the number of populations (PopulationSize: 40), and the number of generations being 40 (Generations: 40).



= OPTIMAL CUTTING PARAMETERS BY LINEAR GA =
Cutting speed : v = 35.07 (m/min)
Feed rate : s = 0.19 (mm/rev)
Depth of cut : a = 1.50 (mm)
Cutting productivity : Z = 9.99 (cm^3/min)
Surface roughness : R_a = 2.50 (μm)
Cutting force : F = 751.21 (N)
Spindle motor power : P = 0.55 (kW)
Tool life : T = 20.28 (min)
Economic tool life : Te = 20.30 (min)
Direct cutting costs : K1 = 0.255 (10^3VND/cm^3)
Tool costs : K2 = 0.176 (10^3VND/cm^3)
Total machining costs : K/V= 0.431 (10^3VND/cm^3)
Calculation time : t = 3.107 (s)
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=

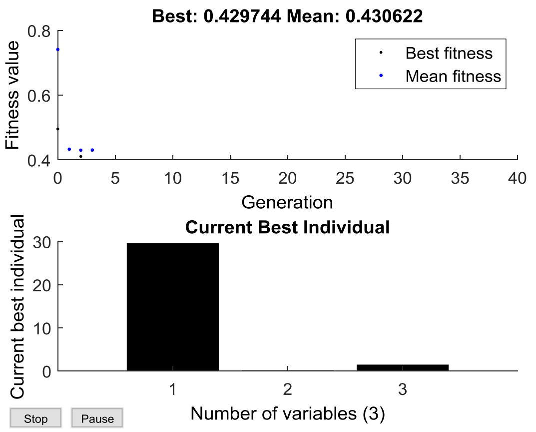
Fig. 2. Results of the CPO with linear GA.

The progression of the CPO is shown in the upper-left of Fig. 2 whereby after about 10 generations the OF has reached the optimal value of -0.84, which approximates the smallest value of ln(*K/Z*), ie. K/Z_{min} = 0.431. The lower-left graph shows the optimal parameters: x_1 = 3.56, x_2 = -1.69, x_3 = 0.41, corresponding to v = 35.07 m/min, s = 0.19 mm/rev,

a = 1.5 mm. This result is also printed on the computer screen like the right part of the figure.

3.2.3. Solving NS-CPO problem using GA

For the NS-CPO problem, the model (25) is used. The solving process is shown in the upper-left part of Fig. 3. Similarly, the lower-left part plot and onscreen information show the optimal cutting parameters: v = 35.26 m/min, s = 0.19 mm/rev, a = 1.5 mm corresponding to the minimum cost K/V = 0.431 (10^3 VND/cm^3). We can see that the non-linear GA shows a similar result as linear, but the computing time is longer.



OPTIMAL CUTTING PARAMETERS BY N-LINEAR GA

Cutting speed	: v = 35.26 (m/min)
Feed rate	: s = 0.19 (mm/rev)
Depth of cut	: a = 1.50 (mm)
Cutting productivity	: Z = 9.800 (cm^3/min)
Surface roughness	: Ra = 2.50 (μm)
Cutting force	: F = 751.48 (N)
Spindle motor power	: P = 0.55 (kW)
Tool life	: T = 20.30 (min)
Economic tool life	: Te = 20.30 (min)
Direct cutting costs	: K1 = 0.255 (10^3VND/cm^3)
Tool costs	: K2 = 0.176 (10^3VND/cm^3)
Total machining costs	: K/V= 0.431 (10^3VND/cm^3)
Calculation time	: t = 4.684 (s)

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Fig. 3. Results of the CPO with non-linear GA.

3.2.4. Solving LM-CPO problem using GA

Multi-objective optimization presents a set of options, allowing the user to choose the most suitable one in his particular production context. For the LM-CPO problem, the model (29), in which (30) is used as the OFs. Using GA, a Pareto solution set is represented in Fig. 4. There are 21 options are shown, each corresponding to a point on the graph. The coordinate axes represent the two objective functions respectively.

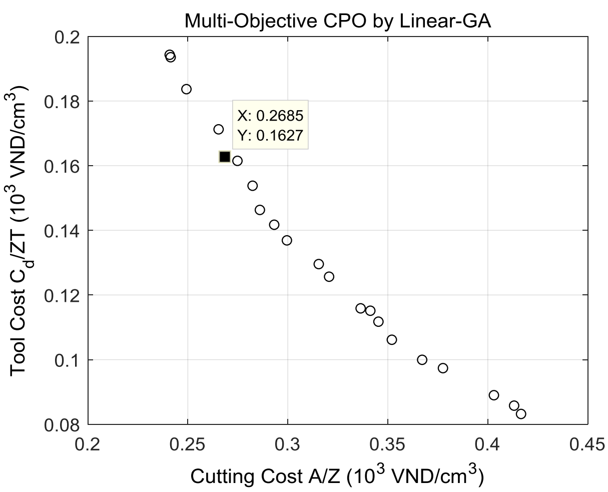

Fig. 4. Pareto set in multi-objective CPO.

Their values are directly exported to an Excel spreadsheet as in Tab. 5 with some rows hidden for compactness.

Tab. 5. Linear Multi-objective CPO results with GA

N	v(m/min)	s(mm/rev)	a(mm)	A/Z	C_d/AZ	K/V	Z(mm^3)	T(min)	R_a(μm)	F(N)	P(kW)	K_2/K(%)
1	35.52	0.18	1.49	0.265	0.171	0.437	9.420	21.613	2.345	735.542	0.544	39.22
2	22.91	0.20	1.49	0.367	0.100	0.467	6.809	51.232	2.274	522.337	0.249	21.40
3	38.28	0.18	1.50	0.241	0.194	0.435	10.376	17.293	2.498	800.765	0.639	44.64
4	27.71	0.19	1.49	0.321	0.126	0.446	7.797	35.586	2.286	603.821	0.349	28.16
16	30.37	0.19	1.49	0.286	0.146	0.432	8.740	27.263	2.463	668.518	0.423	33.84
17	**33.18**	**0.19**	**1.49**	**0.269**	**0.163**	**0.431**	**9.310**	**23.011**	**2.468**	**715.154**	**0.494**	**37.74**
18	32.44	0.18	1.49	0.282	0.154	0.436	8.853	25.607	2.345	687.401	0.465	35.26
19	21.59	0.19	1.49	0.403	0.089	0.492	6.203	63.156	2.103	482.702	0.217	18.09
20	28.49	0.19	1.49	0.315	0.130	0.445	7.925	33.957	2.278	615.367	0.365	29.11
21	20.19	0.20	1.49	0.417	0.083	0.500	6.001	69.837	2.149	463.201	0.195	16.65

The upper-left point on the graph and row 3 in the table represent the minimum cutting cost (*A/Z*), corresponding to maximum tool cost (C_d/AZ), highest productivity (*Z*), shortest tool life (*T*), maximum cutting force (*F*) and maximum power consumption (*P*). In contrast, the lower-right point on the graph and row 21 in the table show the opposite situation. The overall optimal solution is the one with the lowest total cost (*K/V*) in row 17 of the table and the black square on the graph. This trade-off the cutting and tool costs, and uses resources (reflected in constraints and boundary conditions) harmoniously. In addition to the three special options mentioned above, the user can choose any option according to his/her preference, such as cutting productivity, surface roughness, or tool life.

3.2.5. Solving NM-CPO problem using GA

Similarly, for the NM-CPO problem, a model (25) with the OF replaced by (26) is used. The Pareto set of solutions is summarized in Tab. 6. As shown in the table, the overall optimal solution is the one with the lowest total cost (*K/V*) in row 17.

Tab. 6. Non-linear Multi-objective CPO results with GA

N	v(m/min)	s(mm/rev)	a(mm)	A/Z	C_d/AZ	K/V	Z(mm^3)	T(min)	R_a(μm)	F(N)	P(kW)
1	23.55	0.18	1.40	0.415	0.094	0.510	6.020	61.595	2.015	492.134	0.241
2	38.72	0.18	1.47	0.244	0.195	0.440	10.234	17.506	2.499	798.992	0.644
3	23.57	0.19	1.41	0.402	0.097	0.499	6.214	58.265	2.088	501.744	0.246
13	34.59	0.18	1.47	0.271	0.167	0.438	9.212	22.780	2.395	721.046	0.520
14	38.03	0.18	1.44	0.252	0.190	0.442	9.926	18.533	2.499	782.550	0.620
15	25.98	0.19	1.49	0.339	0.116	0.455	7.376	41.046	2.276	571.250	0.309
16	26.04	0.19	1.43	0.354	0.114	0.468	7.060	43.551	2.246	560.506	0.304
17	**29.85**	**0.19**	**1.49**	**0.292**	**0.143**	**0.435**	**8.565**	**28.644**	**2.460**	**656.283**	**0.408**
18	36.82	0.18	1.47	0.259	0.180	0.440	9.637	20.158	2.410	756.974	0.581

4. Summary and discussion on the results

Tab. 7 synthesizes the results of the 5 different methods. It can be seen that the first three, i.e. the single-objective models give the closest results, fully satisfying the requirements on $R_a = R_{amax} = 2.5$ µm, the tool life is approximately the economic value $T \approx T_e$ = 20.30 minutes. The minimum machining cost is K/V = 0.431 (10^3VND). The results of the two multi-objective models are slightly different from the others. The deviations arise from two main sources. Firstly, GA is an approximation method that only guarantees a near-optimal solution. Secondly, the Pareto solution sets are discrete, the truly optimal solutions may not coincide with the output points. In return, users can flexibly choose a certain solution from the Pareto set, although this solution may not be optimal, but suitable for their requirements.

Tab. 7. Summary of the results with different methods

Method	v(m/min)	s(mm/rev)	a(mm)	Z	T	Ra	A/Z	Cd/ZT	K/V
LS-Solver	35.26	0.19	1.50	9.80	20.30	2.50	0.255	0.176	0.431
LS-GA	35.07	0.19	1.50	9.99	20.28	2.50	0.255	0.176	0.431
NS-GA	35.26	0.19	1.50	9.80	20.30	2.50	0.255	0.176	0.431
LM-GA	33.18	0.19	1.49	9.31	23.01	2.47	0.269	0.163	0.431
NM-GA	29.85	0.19	1.49	8.57	28.64	2.46	0.292	0.143	0.435
Average (A)				9.86	20.29	2.50	0.255	0.176	0.431
Confirm. (B)	35.20	0.19	1.50	10.03	20.25	2.55	0.249	0.172	0.422
|A-B|/A (%)				1.7%	0.2%	2.0%	2.3%	2.1%	2.2%

The calculation results have been verified as shown in the last 3 rows of Tab. 7. Firstly, assuming that the first 3 methods are the most accurate, the average of the calculated criteria from only 3 top rows is taken. Next, do 3 confirmation tests with the same optimal cutting parameters, measure the quantities T and R_a, calculate the remaining quantities following (21) to (24), and then also take the average. Finally, calculate the error between the actual values and the calculated ones. We can see, the maximum error is 2.3%, which is acceptable.

From the multi-objective CPO data in Tab. 5, in addition to the value of the total machining cost, it is possible to evaluate the portion of the components. Column K_2/K records the percentage of the tool cost (C_d/ZT) in the total cost (K/V), which is the smallest (16.65%) in the lightest cutting option (maximum tool life), and the largest (44.64%) in the heaviest cutting option (minimum tool life). With the optimal parameters, this percentage is 37.74%. Obviously, with this distribution of tool cost, it cannot be ignored in any situation.

5. Conclusions

- By taking into account tool wear and tool life, and considering practical constraints, the CPO problems are acceptable in both mathematics and practical applications despite the complexity of models and solutions.

- This paper presents 3 models and 5 solving methods to compare their capabilities and results. The LS model is simple, gives accurate and reliable results but requires many manual processing steps, while the NS, LM and NM models give less accurate results but are more flexible in practical applications.

The above CPO models and their solutions have been applied to Ti-6Al-4V Titanium alloy, a typical difficult-to-machine material. The results show that:

- The economic benefits of CPO are significant: the machining cost with optimal cutting parameters equals only 56% in comparison to the average of the values under non-optimal cutting conditions.

- The tool cost contributes significantly to the total machining cost, which is about (17÷45)%, cannot be ignored in the CPO models as in the case of conventional materials.

Based on the static CPO problem presented above, our further research will focus on dynamic problems to meet the needs of online monitoring and adaptive control of the machining process.

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CÁC GIẢI PHÁP SO SÁNH CHO TỐI ƯU HÓA CHẾ ĐỘ CẮT KHI TIỆN HỢP KIM TITAN Ti-6Al-4V

Đào Văn Hiệp

Tóm tắt: *Bài báo giới thiệu một số phương pháp truyền thống và hiện đại trong tối ưu hóa chế độ cắt, ứng dụng cụ thể cho hợp kim Ti-6Al-4V, một trong những vật liệu thông dụng nhất trong số các hợp kim Ti, cũng thuộc nhóm các vật liệu điển hình khó gia công. Các mô hình tối ưu hóa một mục tiêu và đa mục tiêu đã được xây dựng một cách chặt chẽ về toán học, sử dụng bộ số liệu thực nghiệm tin cậy theo mô hình quy hoạch thực nghiệm đa yếu tố toàn phần. Năm phương pháp tối ưu hóa đã được sử dụng để so sánh. Chúng cho lời giải khá tương đồng và sát thực tế. Giá trị trung bình của các kết quả tính toán chỉ sai lệch (0,2÷2,3)% so với kết quả thí nghiệm kiểm chứng. Với trường hợp nghiên cứu, lợi ích kinh tế từ tối ưu hóa là đáng kể, chi phí gia công chỉ bằng 56% giá trị trung bình của các phương án thí nghiệm; chi phí cho dụng cụ cắt chiếm khoảng (17÷45)% là không thể bỏ qua như với thép kết cấu thông thường. Kết quả tương tự có thể dự đoán ở các vật liệu cùng nhóm, như thép không gỉ, các hợp kim nền Ni hoặc Co.*

Từ khóa: Tối ưu hóa chế độ cắt; giải thuật di truyền; quy hoạch tuyến tính; hợp kim titan.

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