OPTIMIZATION OF LINER STRUCTURE TO ENHANCE THE PENETRATION PERFORMANCE OF SHAPED CHARGE WARHEAD

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Abstract

In this study, the Gravitational Search Algorithm (GSA) programmed on MATLAB software is applied to solve the optimal problem of determining the liner structure to enhance the penetration performance of the shaped charge warhead. The obtained results show that when using the optimal set of parameters found by the proposed approach (the internal apex angle α_0 = 16°, the thickness δ = 4.9 mm), the penetration performance of the shaped charged warhead is significantly improved (the penetration depth b = 553 mm, 38% higher than the original plan).

Keywords: *Shaped charge warhead; liner structure; penetration; GSA; optimization.*

1. Introduction

The liner's structural parameters are the very important factors affecting the penetration performance of shaped charge warheads. Studying the structural parameters of the shaped charge warhead's liner to maximize the penetration depth is an essential requirement in the design process of a new model or the improvement of existing ones [1-3].

It is possible to determine the penetration performance of shaped charge warheads through testing, but the number of tests needs to be large, and it is costly in terms of time and money [1-3]. Thanks to the remarkable development of science and technologies, numerical simulation applications have provided very high accuracy in calculation and shortened the time and lower cost. However, with complex problems such as determining the depth of penetration of shaped charge warheads, the simulation still encounters many difficulties and takes much time, especially in the case of a large number of parameters or a wide range of parameters [4-6].

In recent years, various heuristic optimization methods have been developed, many of which are inspired by the swarm behavior in nature, typically Particle Swarm Optimization (PSO), Genetic Algorithm (GA), Ant Colony Optimization algorithm (ACO), Gravitational Search Algorithm (GSA) [4-7]… These methods are suitable for solving complex optimization problems of many space dimensions, intermittent objective function, with many local extremes, fast convergence speed.

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In this paper, the Gravitational optimization algorithm (GSA) is applied to optimize the liner's parameters, namely to determine the thickness and internal apex angle of the liner, in order to achieve the maximum penetration performance of the shaped charge warhead. The calculation is based on a proposed model using MATLAB software.

2. Method for determination of shaped charge's penetration performance

The penetration depth of the shaped charge is determined based on hydrodynamic theory. Consider the warhead model as shown in Fig. 1. The case is cylindrical, the inside is filled with explosives. The liner has a cone shape with an internal apex angle α_0, and the thickness δ from the vertex to the base of the liner is constant. The process of calculating the penetration depth is based on the method of Baltic State University of Technology [1]: Divide the shaped charge into n equal segments by the cross-sections perpendicular to the symmetry axis of the warhead. These cross-sections are evenly spaced and divide the liner into many small elements with corresponding explosive parts. Each element of the liner penetrates the target with a corresponding depth. The shaped charge's penetration performance is determined by the thickness of the target it penetrates, which is the total thickness that all the liner elements penetrate.

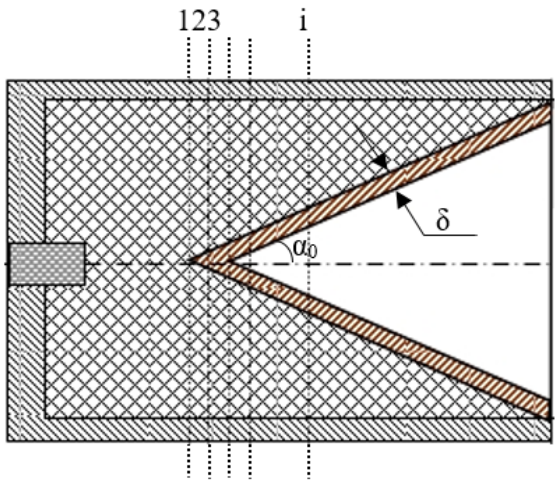

Fig. 1. Shaped charge warhead model

For each segment, in turn, determine the following parameters:

- Active explosive mass m_{ai} of the i^{th} segment:

$$m_{ai} = \frac{m_{TNi}}{2}\left(1 + \frac{m_{ti} - m_{phi}}{m_{ti} + m_{phi} + m_{TNi}}\right) \quad (1)$$

where m_{ai} , m_{Tni} , m_{ti} , m_{phi} are the masses corresponding to active explosive mass, total explosive mass, shell mass, and liner mass of the i^{th} element.

- Ratio of explosive mass to liner mass of the i^{th} element:

$$\beta_i = \frac{m_{ai}}{m_{phi}} \tag{2}$$

- Collapse velocity of the cumulative liner of the i^{th} element:

$$v_{phi} = 0,5D\sqrt{\frac{\beta i}{2+\beta i}} \tag{3}$$

where D is detonation velocity of the explosive charge.

- Liner collapse angle α_i of the i^{th} liner element along the symmetry axis of the warhead:

$$tg\alpha_i = \frac{r_{ni+1} - r_{phi+1}\left(\frac{r_{ni}}{v_{phi}\cos\alpha_0} - \frac{\Delta h}{D}\right)\cos\alpha_0}{h - r_{ni}tg\alpha_0 + v_{phi+1}\left(\frac{r_{ni}}{v_{phi}\cos\alpha_0} - \frac{\Delta h}{D}\right)\sin\alpha_0} \tag{4}$$

where r_{ni}, r_{ni+1} are the external radius of the i^{th} and $(i+1)^{th}$ liner segment, Δh is the height of the liner segment, α_0 is initial apex angle of the liner.

- Jet tip velocity v_{ddi} of i^{th} liner element:

$$v_{ddi} = v_{phi}\frac{1+\cos\alpha_i}{\sin\alpha_i} \tag{5}$$

- Travel distance x_{ddi} of the jet tip of i^{th} liner element:

$$x_{ddi} = F + h + \sum_1^n b_i - \frac{r_{ni}}{tg\alpha_0} - r_{ni}tg\alpha_0 \tag{6}$$

where F is stand-off distance, h is the height of the liner.

- Travel time x_{ddi} of the jet tip of i^{th} liner element:

$$t_{ddi} = \frac{x_{ddi}}{v_{ddi}} \tag{7}$$

- Length of jet formation of i^{th} liner segment:

$$l_{ci} = \begin{cases} l_0 + (v_{ddi} - v_{ddi+2})t_{ddi} & when\ v_{ddi} \geq v_{th} \\ 0 & when\ v_{ddi} < v_{th} \end{cases} \tag{8}$$

where v_{th} is limited velocity of the jet tip, whose value depends on jet material; l_0 is initial length of the jet formation.

- Active jet length l_{hqi} of i^{th} jet element:

$$l_{hqi} = \begin{cases} l_{ci} & when\ l_{ci} \le l_{th} \\ l_{th} & when\ l_{ci} > l_{th} \end{cases} \tag{9}$$

where $l_{th} = k.\Delta l$ is limited length of the jet formation, k is coefficient depending on ammunition structure, Δl is length of the liner pathogenesis.

- The penetration depth of i^{th} jet element:

$$b_i = l_{hqi}\sqrt{\frac{\rho_{ph}}{\rho_{bt}}} \tag{10}$$

where ρ_{ph}, ρ_{bt} are the density of the liner material and the steel target, respectively.

- The total penetration depth of all jet elements:

$$b = \sum_{i=1}^{n} b_i \tag{11}$$

3. Gravitational Search Algorithm

The Gravitational Search Algorithm (GSA) is a heuristic optimization algorithm that is based on the law of gravity and mass interactions. GSA is a nature-motivated algorithm proposed by Rashedi et al. in 2009 and has been gaining interest among many scientists nowadays. The algorithm consists of a collection of separated agents that interact with each other via Newton's gravitational force [7]. The gravitational force motives a global action in which all agents move towards other agents with heavier mass. The slow motion of heavier masses ensures the exploitation step of the algorithm and corresponds to good options.

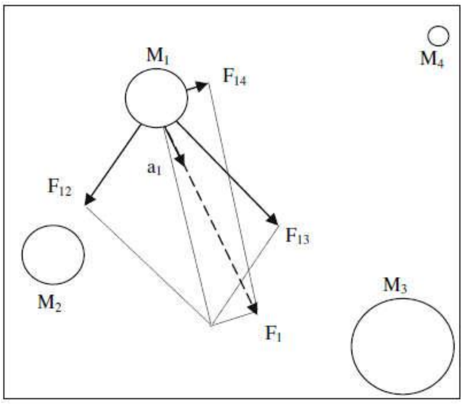

Fig. 2. The acceleration of agent 1 toward the resultant force that acts it from the other agents

In GSA, each agent has four parameters: position, inertial mass, active gravitational mass, and passive gravitational mass. The agent's position corresponds to a solution of the problem, and its gravitational and inertial masses are determined using a fitness function. The algorithm is navigated by adequately adjusting the gravitational and inertia masses, whereas each mass presents a solution. The lighter agents are attracted by the heavier agents. By lapse of time, the lighter agents are attracted by the heaviest agent which presents the optimum solution in the search space.

The steps of Gravitational Search Algorithm are as follows:

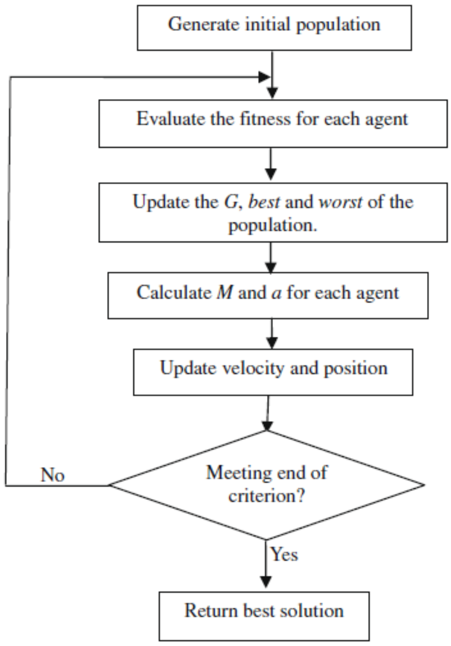

Fig. 3. General principle of GSA

Step 1: Agent's initialization

The positions of N agents (masses) are randomly generated in the search space:

$$X_i = (x_i^1, ..., x_i^d, ..., x_i^n), \quad for \; i = 1, 2, ..., N$$

where x_i^d is the position of i^{th} agent in the dth dimension, n is the number of dimensions of the search space.

At a specific time t, the force acting on i^{th} mass from jth mass is defined as follows:

$$F_{ij}^d(t) = G(t)\frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t) + \varepsilon}\left(x_j^d(t) - x_i^d(t)\right) \tag{12}$$

where M_{aj} is the active gravitational mass related to agent j, M_{pi} is the passive gravitational mass related to agent i, $G(t)$ is the gravitational constant at time t, ε is a small constant, and $R_{ij}(t)$ the Euclidian distance between two agents i and j

Step 2: Optimal problem determination

For a minimization problem:

$$best(t) = \min_{j \in (1,\ldots,N)} fit_j(t) \tag{13}$$

$$worst(t) = \max_{j \in (1,\ldots,N)} fit_j(t) \tag{14}$$

For a maximization problem:

$$best(t) = \max_{j \in (1,\ldots,N)} fit_j(t) \tag{15}$$

$$worst(t) = \min_{j \in (1,\ldots,N)} fit_j(t) \tag{16}$$

where $fit_j(t)$ is the fitness value of the jth agent at time t, $worst(t)$ and $best(t)$ are the best and worst fitness value at time t.

Step 3: Update Gravitational constant (G)

The gravitational constant G is initialized and decreases by the time to control the search accuracy. In other words, G is a function of the initial value G_0 and time t:

$$G(t) = G(G_0, t) \tag{17}$$

Step 4: Masses of the agents' calculation

The gravitational and inertial masses are updated by the following equations:

$M_{ai} = M_{pi} = M_{ii} = M_i$ for $i = 1, 2, \ldots, N$

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \tag{18}$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \tag{19}$$

Step 5: Accelerations of agents' calculation

The acceleration $a_i^d(t)$ of i^{th} agent at time t, and in dth direction, is determined as follow:

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)} \tag{20}$$

$F_i^d(t)$ is the total force acting on i^{th} agent calculated as:

$$F_i^d(t) = \sum_{j \in Kbest;\, j \neq i}^{N} rand_j F_{ij}^d(t) \tag{21}$$

Kbest is the set of first K agents with the best fitness value and biggest mass. With the initial value K_0 at the start, *Kbest* decreases linearly, and at the end, there is only one agent applying force to the others.

Step 6: Update the velocity and position

The next velocity of an agent is considered a fraction of its current velocity added to its acceleration. Hence, its position and its velocity could be given as follows:

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \tag{22}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{23}$$

where $rand_i$ is random number in the interval [0, 1], which is used to give a randomized characteristic to the search.

Step 7: Repeat steps 2 to 6 until the iterations reach their maximum limit. The best fitness value at the final iteration is computed as the global fitness while the position of the corresponding agent at specified dimensions is computed as the global solution of that particular problem.

Some remarks about GSA are noted:

- GSA is a heuristic search algorithm (variables are initialized randomly in the search space).

- A heavy mass has a large effective attraction radius and hence a great intensity of attraction. Therefore, agents with higher performance have a greater gravitational mass. As a result, the agents tend to move toward the best agent.

- The inertial mass is against the motion and causes the mass to move slowly. Therefore, agents with heavy inertia mass move slowly and hence search the space more locally. Hence, it can be considered as an adaptive learning rate.

- The test results show the excellent convergence of GSA [7].

- Here, we assume that the gravitational and the inertial masses are the same. However, for some applications, different values may be used. A larger inertial mass provides a slower movement of agents in the search space and, therefore, a more precise search. Conversely, the greater gravitational mass is, the higher attraction of agents is. This allows a faster convergence.

4. Applying GSA to solve the optimization problem of liner structure to improve the efficiency of shaped charge warhead

The main goal of the optimization of the liner structure is to determine the geometrical parameters of the liner so that the penetration depth in the steel target reaches the maximum value (maximization problem). In the scope of this study, two values of the liner (thickness δ and internal apex angle α_0) are simultaneously surveyed on the model of the shaped charge warhead described in the second part.

Consider a specific structure of the shaped charge warhead with the following parameters: The cylindrical ammunition body is made of steel with density ρ_{th} = 7.8 g/cm^3, outer radius R_n = 50 mm, inner radius R_t = 47 mm. The ammunition body is filled by OKFOL explosive with density ρ_{TN} = 1.82 g/cm^3, detonation velocity D = 9200 m/s. Conical liner is made of copper with density ρ_{ph} = 8.93 g/cm^3, height h = 120 mm, thickness δ = 2.5 mm, internal apex angle α_0 =15°, stand-off distance F = 150 mm. Limited jet formation velocity v_{th} = 2200 m/s. The penetration depth of the shaped charge warhead is calculated according to the method presented in part 2 using MATLAB software, resulting in the penetration depth b = 402 mm.

4.1 Optimization of the liner structure according to one parameter

Conventional analytical methods can only determine the optimal value by evaluating each parameter sequentially while fixing all remaining parameters, i.e., optimizing for each parameter only. The theory [1-3] has shown that, for a given liner thickness, there is an optimal internal apex angle at which the penetration depth is maximum, and for a given internal apex angle, there is an optimal liner thickness at which the penetration depth is maximum.

Firstly, a program using the GSA presented in part 3 is written in MATLAB to solve the optimization problem with a system of N agents, each agent has the position of x_i with i = 1, 2,…, N where x_i represents the internal apex angle α_0 of the i^{th} agent. The number of the search space dimension is 1, the maximum number T of iterations is 50, the number N of agents is 30. The optimization problem of the internal apex angle α_0 is to find the value of angle α_0 within the range [10°; 20°] so that the shaped charge warhead with parameters described above (thickness δ is 2.5 mm) provides maximum penetration depth. The fitness function is the penetration depth b of variable α_0 ($b = f(\alpha_0)$). The results of 5 runs of the program are shown in Tab. 1.

Tab. 1. The penetration depth with N=30; T=50; $\alpha_0 \in$ [10°; 20°]; δ=2.5 *mm*

No.	Internal apex angle α_o(deg)	Thickness δ (mm)	Penetration depth b (mm)
1	18.182	2.5	505.7
2	18.178	2.5	505.6
3	18.178	2.5	505.6
4	18.175	2.5	505.5
5	18.183	2.5	505.7

The optimal internal apex angle α_0 is found about 18.2° then the maximum penetration depth *b* is about 506 mm (26% higher compared to the original plan). This value is compared to other cases with some different values of α_0, the results are shown in Tab. 2. It can be seen that the result is determined by GSA are better than other results in the survey range (Fig. 4). Therefore, for a given liner thickness, there exists an optimal internal apex angle at which the penetration depth is maximum, and the program applying GSA determined this value accurately and quickly.

Tab. 2. The penetration depth with different values of $\alpha_0 \in [10°; 20°]$; δ = 2.5 mm

No.	Internal apex angle α_0 (deg)	Thickness δ (mm)	Penetration depth *b* (mm)
1	10	2.5	207.4
2	11	2.5	232.4
3	12	2.5	265.5
4	13	2.5	310.3
5	14	2.5	359.6
6	15	2.5	402.1
7	16	2.5	439.1
8	17	2.5	471.8
9	18	2.5	500.7
10	19	2.5	486.5
11	20	2.5	475.1

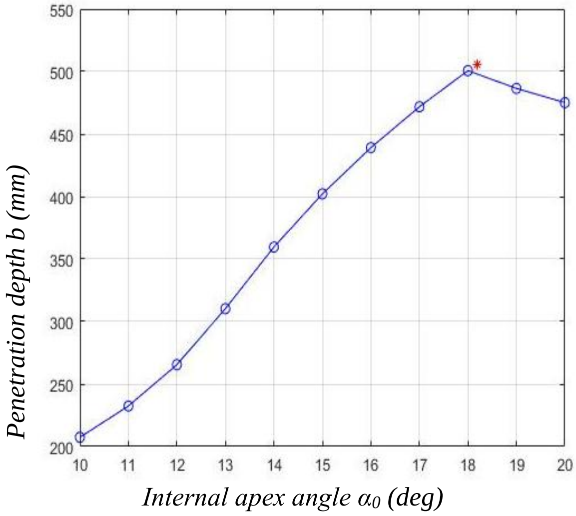

Fig. 4. The penetration depth with different values of $\alpha_0 \in [10°; 20°]$; δ = 2.5 mm
**- optimal value; o- other values*

Tab. 3. The penetration depth with different values of $\delta \in [0.5; 5]$ mm; α_0 = 18.2°

No.	Internal apex angle α_o (deg)	Thickness δ (mm)	Penetration depth *b* (mm)
1	18.2	0.5	297.3
2	18.2	1.0	375.4
3	18.2	1.5	430.0
4	18.2	2.0	472.2
5	18.2	2.5	500.4
6	18.2	3.0	511.2
7	18.2	3.5	511.4
8	18.2	4.0	507.9
9	18.2	4.5	506.9
10	18.2	5.0	497.5

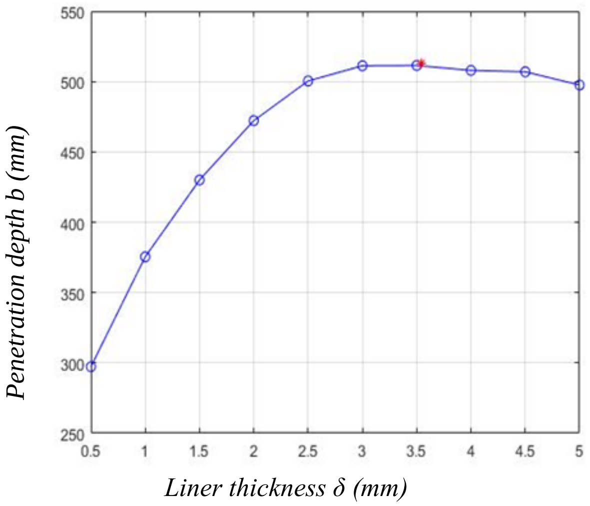

Fig. 5. The penetration depth with different values of $\delta \in [0.5; 5]$ mm; α_0 = 18.2°
**- optimal value; o- other values*

Likewise, with the internal apex angle α_0 of 18.2°, and the liner thickness δ varies from 0.5 mm to 5 mm, the maximum penetration depth b is found approximately 513 mm when the liner thickness δ is 3.54 mm. The comparison results are shown in Tab. 3 and Fig. 5.

Note that the internal apex angle α_0 determined above (18.2°) is optimal only for liner thickness of 2.5 mm, as well as the 3.54 mm thickness value is only optimal for internal apex angle α_0 of 18.2°. The change of these parameters (α_0 and δ) leads to the change of corresponding optimal values accordingly.

4.2 Optimization of the liner structure according to two parameters

The optimization problem now is to find the optimal liner structure with the thickness varying within the range [0.5; 5] mm and internal apex angle varying within the range [10°; 20°] for maximum penetration depth. Applying GSA helps to find the optimal internal apex angle α_0 and the thickness δ simultaneously so that the shaped charge warhead with parameters described above provides maximum penetration depth.

The number of the search space dimensions is 2, the maximum number T of iterations is 50, the number N of agents is 30. The search space limit of the first dimension corresponding to the internal apex angle α_0 of the liner is within the range [10°; 20°]. The search space limit of the second dimension corresponding to the liner thickness δ is within the range [0.5; 5] mm. The fitness function is the penetration depth b of two variables α_0 and δ ($b = f(\alpha_0, \delta)$).

Tab. 4. The penetration depth with N = 30; T = 50; $\alpha_0 \in [10°; 20°]$; $\delta \in [0.5; 5]$ mm

No.	Internal apex angle α_0(deg)	Thickness δ (mm)	Penetration depth b(mm)
1	16.06	4.85	552.6
2	16.03	4.94	554.7
3	16.16	4.93	552.1
4	16.26	4.65	551.3
5	16.23	4.67	551.5

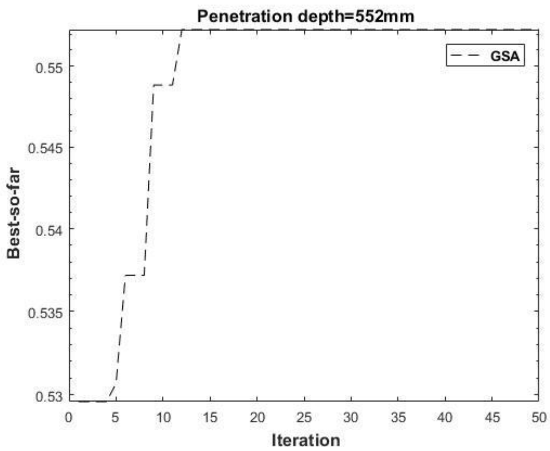

Fig. 6. Example of the program's result

From the results of 5 runs of the program in Tab. 4, it can be seen that, within the given limits of the thickness δ and internal apex angle α_0 of the liner, it is possible to choose the value α_0 = 16°, the thickness δ = 4.9 mm for easy processing and manufacturing of products. With these parameters, the calculation result for the total penetration depth generated by this shaped charge warhead is b = 553 mm. Despite slightly smaller than the value of 554.7 mm with the parameter set of α_0 = 16.02 degrees and δ = 4.94 mm as shown in Tab. 4, this value b = 553 mm is about 38% higher than the original plan b = 402 mm.

This value is compared to other cases with some different values of α_0 and δ to verify the performance of GSA. The results are shown in Tab. 5, Tab. 6, and Fig. 7, Fig. 8.

Tab. 5. The penetration depth with different values of $\alpha_0 \in [10°; 20°]$; δ = 4.9 mm

No.	Internal apex angle α_0 (deg)	Thickness δ (mm)	Penetration depth b (mm)
1	10	4.9	286
2	11	4.9	340
3	12	4.9	399
4	13	4.9	448
5	14	4.9	489
6	15	4.9	523
7	16	4.9	553
8	17	4.9	527
9	18	4.9	508
10	19	4.9	481
11	20	4.9	462

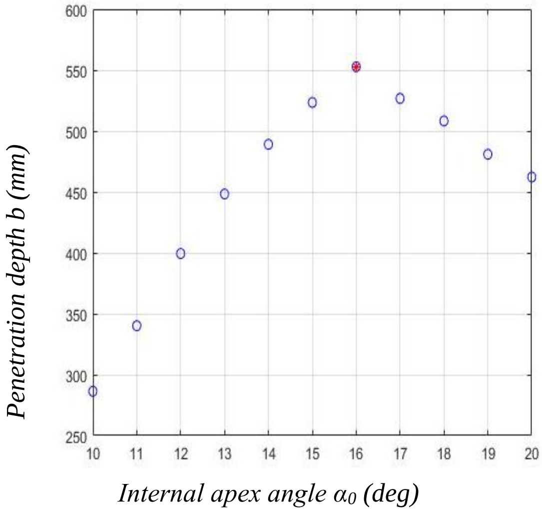

Fig. 7. The penetration depth with different values of $\alpha_0 \in [10°; 20°]$; δ = 4.9 mm
**- optimal value; o- other values*

Tab. 6. The penetration depth with different values of $\delta \in [0.5; 5]$ mm; α_0 = 16°

No.	Internal apex angle α_0 (deg)	Thickness δ (mm)	Penetration depth b (mm)
1	16	0.5	217
2	16	1.0	297
3	16	1.5	356
4	16	2.0	402
5	16	2.5	439
6	16	3.0	471
7	16	3.5	497
8	16	4.0	520
9	16	4.5	539
10	16	5.0	550

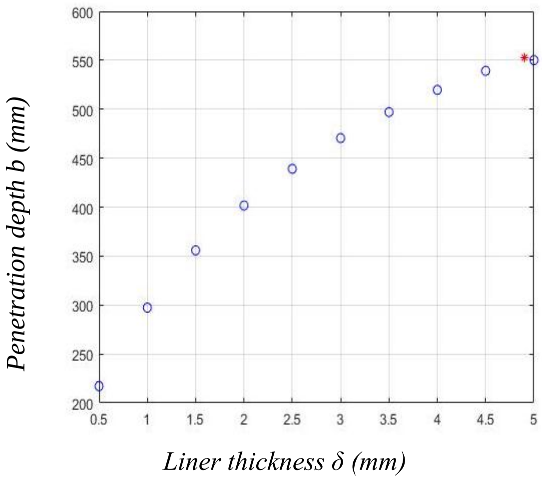

Fig. 8. The penetration depth with different values of $\delta \in [0.5; 5]$ mm; α_0 = 16°
**- optimal value; o- other values*

5. Conclusions

The research to solve the optimization problem of optimizing the liner structure to improve the efficiency of shaped charge warhead by gravitational search algorithm on MATLAB software gives some conclusions as follows:

- Applying GSA helps to quickly find the optimal set of the liner parameters (the thickness δ = 4.9 mm and internal apex angle α_0 = 16°) within the given limits (thickness varying within the range [0.5; 5] mm and internal apex angle varying within the range [10°; 20°]) so that the penetration depth *b* = 553 mm of the shaped charge warhead is close to the maximum value with small error.

- It is possible to reduce the amount of computation without affecting the result significantly by gradually reducing the number of agents through iterations.

- The results of the study allow faster survey and construction of structural options for shaped charge warheads, meeting requirements in design or improvement of ammunition.

The development of GSA-related researches has been promising. Based on the results, GSA has been widely applied to solve highly nonlinear optimization problems of complex engineering systems. The obtained results open up future studies such as using GSA to evaluate more parameters, for example, liner with its thickness changing from vertex to base, liner height, stand-off distance, the material of liner/explosive... and solving the optimization problems of the above parameters simultaneously. Besides, the simulation can be performed to evaluate and compare to the results of applying GSA.

In addition, it is necessary to continue to study different modifications of the GSA algorithm to enhance its performance. In terms of scope of applications, there are still many other potentials in the military field, in particular, and in the areas of life, in general, where GSA algorithms can be applied, such as in finance, economics, medical and engineering areas that need to be further studied.

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TỐI ƯU HÓA KẾT CẤU PHỄU LÓT NÂNG CAO KHẢ NĂNG XUYÊN CỦA ĐẠN XUYÊN LÕM

Tóm tắt: *Trong nghiên cứu này, thuật toán tìm kiếm lực hấp dẫn (Gravitational Search Algorithm) lập trình trên phần mềm MATLAB được áp dụng để giải quyết bài toán tối ưu hóa xác định kết cấu phễu lót nhằm nâng cao khả năng xuyên của đạn xuyên lõm. Kết quả thu được cho thấy khi sử dụng bộ tham số tối ưu do phương pháp đề xuất tìm được (góc mở phễu lót $\alpha_0 = 16°$, bề dày phễu lót $\delta = 4{,}9$ mm), uy lực xuyên của phần chiến đấu đạn xuyên lõm được cải thiện đáng kể (chiều sâu xuyên đạt được là 553 mm, tăng 38% so với phương án gốc ban đầu).*

Từ khóa: Đạn xuyên lõm; kết cấu phễu lót; xuyên lõm; GSA; tối ưu hóa.

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