

# **FINITE TIME TRAJECTORY TRACKING CONTROL OF A QUADROTOR BASED ON ARTIFICIAL NEURAL NETWORK**

**To Xuan Dinh<sup>1</sup>, Kyoung Kwan Ahn<sup>2</sup>, Nguyen Thanh Tien<sup>1\*</sup>**

*<sup>1</sup>Le Quy Don Technical University; <sup>2</sup>University of Ulsan, South Korea*

## **Abstract**

In this paper, a robust hierarchical method for trajectory tracking control of a quadrotor unmanned aerial vehicle (UAV) subjected to parameter uncertainties and external disturbances is presented. A robust control scheme based on a fast nonsingular terminal sliding mode strategy is designed to achieve fast response and excellent tracking accuracy. Moreover, a radial basis function artificial neural network with online adaptive schemes to estimate unknown aerodynamic parameters and external disturbances is developed to improve the control performance and reduce the chattering phenomenon. Numerical simulation and experimental results are used to validate the effectiveness of the proposed control method.

***Keywords:** Quadrotor unmanned aerial vehicle; trajectory tracking control; terminal sliding mode; radial basis function neural network.*

## **1. Introduction**

In the past decades, researches on quadrotor UAVs have gained significant interest due to their wide range of military and civil field applications including surveillance, reconnaissance, rescue mission, power plant inspection, agriculture services, aerial imagery, mapping, drone delivery and law enforcement [1-3]. Trajectories tracking control of quadrotor UAVs is not a straightforward task due to underactuated and nonlinear coupled characteristics in dynamics of quadrotor. Moreover, the quadrotor dynamics is prone to being affected by external disturbances and parameter uncertainties such as wind gusts, drag payload and model uncertainties. To deal with these problems, various control strategies have been proposed for the quadrotor. Linear control methods have been applied to solve the path following and attitude stabilization of quadrotor UAVs such as conventional proportional integral derivative (PID) and linear quadratic regulators (LQRs) [4-6]. However, these control strategies have limited ability to attenuate the nonlinear coupled behavior among variables and they are inadequate to guarantee the robustness of the system in the

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\* Email: ngttienktd@gmail.com

presence of external disturbances and highly nonlinearities. To eliminate the drawbacks of linear control methods, a variety of nonlinear flight control techniques have been developed recently such as model predictive control [7, 8], backstepping control [9, 10], sliding mode control [11, 12]. However, these works provided asymptotically stable control laws, which means the tracking errors reach to zero as time goes to infinity. This motivates us to investigate the finite time control method on trajectory tracking of quadrotor helicopter.

Due to finite time convergence property, terminal sliding mode control (TSMC) methods [13-15], which employ nonlinear sliding surfaces instead of linear surfaces, have been developed recently. To reduce the chattering phenomenon, uncertainty estimation methods have been developed to reduce the switching gain of the controller. With the advantages of simple structure, fast learning algorithm and better approximation ability than conventional neural network, RBFNNs have been employed to estimate the uncertainties in the quadrotor dynamics in several works [13, 16]. However, to the best of the authors' knowledge, there is no study in literature on the incorporation of the Fast Nonsingular Terminal Sliding Mode Control (FNTSMC) and Radial Basis Function Neural Network (RBFNN) method to control quadrotor helicopters.

In light of the significant advantages and drawbacks, this paper investigates a finite time trajectory tracking control scheme for position and attitude subsystem of uncertain quadrotor helicopter based on FNTSMC combined with a RBFNN estimator for uncertainties, modeling errors, and bounded disturbances.

The remainder of this paper is organized as follows. In Section 2, the mathematical model of a quadrotor UAV is described. Section 3 introduces the design of position and attitude controller for the quadrotor UAV system and stability analysis of the closed loop system. Simulation and experimental results are provided to verify the effectiveness and robustness of the proposed control method in Section 4 and Section 5, respectively. Finally, conclusion remarks and future work are given in Section 6.

## **2. Quadrotor model and notation**

### ***2.1. Dynamic equations***

Quadrotor is a typical under-actuated, nonlinear coupled system because it has six degrees of freedom but only four actual inputs. The six degrees of freedom consist of

translational motion in three directions and rotational motion around three axes. The two reference frames are shown in Fig. 1: The Earth inertial frame  $E$  which is represented by  $O_E x_E y_E z_E$  and the body-fixed frame  $B$  which is represented by  $O_B x_B y_B z_B$  with its origin at the center of the mass.

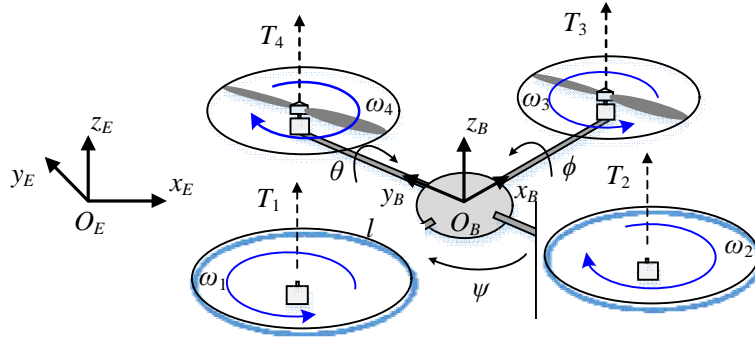


Fig. 1. Quadrotor UAV configuration

Let the vector  $v = (v_x, v_y, v_z)^T$  denotes the linear velocities in the earth frame and vector  $\Omega = (p, q, r)^T$  represents the angular velocity of roll, pitch, and yaw in the body-fixed frame  $B$ . Using Newton-Euler formalism, the dynamic equations of the quadrotor system can be described as follows [1, 3, 9, 20]:

$$\begin{cases} \dot{P} = v \\ \dot{v} = u_p + \eta_1(\Theta, v, u_p, t) + d_t \\ \dot{\Theta} = \Omega \\ \dot{\Omega} = u_\Theta(t) + \eta_2(\Theta, \Omega, u_\Theta, t) + d_a \end{cases} \quad (1)$$

where  $\eta_1(\Theta, v, t) = (A_t + T(R - R_d)z_e)/m$ ,  $\eta_2(\Theta, \Omega, t) = I_f^{-1}(A_r - G) + I_f^{-1}(\Omega \times I_f \Omega)$ ;  $m$  is the mass;  $I_f = \text{diag}(I_{xx}, I_{yy}, I_{zz})$  is a symmetric positive definite constant matrix express in body-fixed frame with  $I_{xx}$ ,  $I_{yy}$ , and  $I_{zz}$  being the rotary inertia with respect to the  $O_b x_b$ ,  $O_b y_b$ , and  $O_b z_b$  axes, respectively;  $g$  is the gravity acceleration;  $z_e = [0, 0, 1]^T$  is the unit vector expressed in the Earth inertial frame;  $T \in R$  and  $\tau = [\tau_1, \tau_2, \tau_3]^T \in R^3$  are the total thrust and the total torque produced by four rotors in free air, respectively;  $G$  denotes the gyroscopic torque vector;  $A_t$  and  $A_r$  denote the drag force and torque coefficients for velocities and angular velocities of the inertial frame;

$u_p(t) \triangleq gz_e - \frac{1}{\bar{m}} TR_d z_e = [u_x, u_y, u_z]^T$ ,  $u_\Theta(t) \triangleq \bar{I}_f^{-1} \tau = [u_\phi, u_\theta, u_\psi]^T$  denote the virtual control signals of the position subsystem and the attitude subsystem, respectively;  $d_t$  and  $d_a$  are total uncertainties and disturbances in the position and attitude subsystem, respectively.

In the view of the under-actuated characteristic of the quadrotor helicopter system, the desired attitude signals  $\Theta_d = (\phi_d, \theta_d, \psi_d)^T$  were computed based on the virtual control signal  $u_p$  and need to be followed by the attitude subsystem. According to the result from [11, 17], with the desired yaw angle selected in advanced, the commanded roll and pitch angle can be calculated as

$$\begin{cases} \phi_d = \arcsin \left( \frac{-u_x \sin \psi_d + u_y \cos \psi_d}{\sqrt{u_x^2 + u_y^2 + (u_z - g)^2}} \right) \\ \theta_d = \arctan \left( \frac{u_x \cos \psi_d + u_y \sin \psi_d}{u_z - g} \right) \end{cases} \quad (2)$$

## 2.2. Notations

The power of a scalar as follows:

$$a^{[c]} = |a|^c \text{sign}(a) \quad (3)$$

where  $c > 0$ . The power of a vector is defined as:

$$\chi^{[c]} = [\chi_1^{[c]}, \chi_2^{[c]}, \dots, \chi_n^{[c]}] \in \mathfrak{R}^n \quad (4)$$

## 3. Control algorithms

From (5), the compact affine nonlinear equation of the aerial robot system is given as the following:

$$\begin{cases} \dot{X}_1 = X_2 \\ \dot{X}_2 = \varsigma(\dot{X}_2) + \eta(X_1, X_2, u, t) + d(t) + u(t) \end{cases} \quad (5)$$

where  $X_1 = [P^T, \Theta^T]^T$ ,  $\eta = [\eta_1^T, \eta_2^T]^T$ ,  $u = [u_p^T, u_\Theta^T]^T$ ,  $d = [d_1^T, d_2^T]^T$  denote the state variables, the control signals, and the uncertainties and the external disturbances of the whole system, respectively;  $\varsigma(\dot{X}_2) = [0_{1 \times 3}, \Gamma(\Omega)^T]^T$  denotes the nominal nonlinear function.

In this section, a fast finite time sliding mode control algorithm for position and attitude tracking of the quadrotor system is presented. The design procedure of the FNTSMC includes of two steps [18, 19]. The appropriate fast nonsingular terminal sliding surface is designed in the first step. The second step is to develop a control law that forces the system states to reach the sliding surface in a finite time.

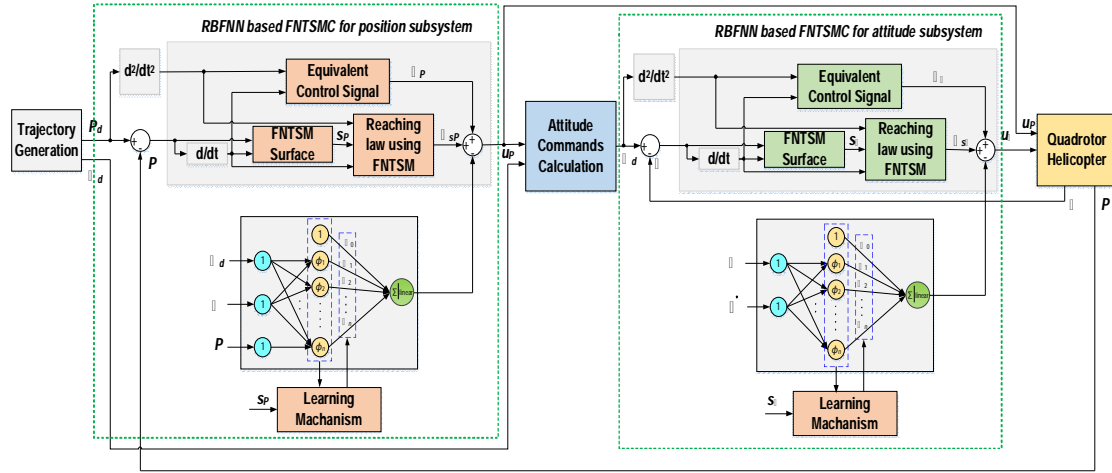


Fig. 2. Cascade control architecture for the quadrotor system

The terminal manifold for the system defined in (5) is selected as follows [19, 20]:

$$s = \tilde{X}_2 + \beta \tilde{X}_1^{\alpha_1} \quad (6)$$

where  $\alpha = \alpha_1/\alpha_2$ , where  $\alpha_1, \alpha_2$  are positive odd integers, and  $\alpha_1 < \alpha_2$ . The derivative of the  $i^{\text{th}}$  element of  $s$  is given as follows:

$$\dot{s}_i = \dot{\tilde{X}}_{2i} + \beta \tilde{X}_{1i}^{\alpha_1 - 1} \tilde{X}_{2i}, \quad i = 1, 2, \dots, 6 \quad (7)$$

To overcome the singular problem in the conventional terminal sliding mode system, motivated by the work [21], we proposed a nonsingular terminal sliding manifold, which provided a smooth switch from the terminal sliding manifold to conventional sliding manifold as the following:

$$s_i = \tilde{X}_{2i} + \beta_i \vartheta(\tilde{X}_{1i}), \quad i = 1, 2, \dots, 6 \quad (8)$$

where the smooth switch function  $\vartheta(\tilde{X}_{1i})$  is defined as follows:

$$\vartheta(\tilde{X}_{1i}) = \begin{cases} \tilde{X}_{1i}^{\alpha} & \text{if } \bar{s}_i = 0 \text{ or } \bar{s}_i \neq 0 \text{ and } |\tilde{X}_{1i}| \geq \sigma_i \\ \kappa_{1i} \tilde{X}_{1i} + \kappa_{2i} \tilde{X}_{1i}^2 & \text{if } \bar{s}_i \neq 0 \text{ and } |\tilde{X}_{1i}| < \sigma_i \end{cases} \quad (9)$$

