NONLINEAR CONTROL OF A TRI-ROTOR BASED ON THE DECOMPOSITION THE DYNAMIC MODEL AND FEEDBACK LINEARIZATION

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Abstract

This paper presents the tri-rotor UAV dynamic modeling and divides it to the control loops under the condition that the response of the inner loop is faster than the response of the outer loop. From diagram of the control loops, attitude, velocity and position controllers have been synthesized based on feedback linearization and module optimization methods. The stability of the attitude loop is proved by Lyapunov theory. Finally, the simulation results on MATLAB/Simulink confirm that the synthesized controllers are realizable in all flying modes with control parameters such as the settling time is about 5-8s and overshot is approximately equal to zero.

Symbol

Parameters	Unit	Mean
m	kg	Mass of quadrotor
I_{xx}	kg.m^2	The body moment of inertia around the x-axis
I_{yy}	kg.m^2	The body moment of inertia around the y-axis
I_{zz}	kg.m^2	The body moment of inertia around the z-axis
g	m.s^{-2}	Gravitational acceleration
l	m	The distance between the center of the tri-rotor and the center of a propeller
ω_{mi}	rad/s	Speed of propeller rotors
α_i	rad	Tilt angles of the rotor
K_t		Drag moment constant
K_f		Thrust constant

***Keywords**: Tri-rotor UAV; feedback linearization; dynamic model; nonlinear control; module optimization.*

1. Introduction

The tri-rotor UAV is a vertical take-off and landing aircraft with 3 rotors in which rotors's angles can change to allow the flights are more flexible compared to

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other UAVs such as quadrotor, six-rotor, etc. However, dynamics of tri-rotor UAVs are highly coupled and nonlinear, which makes the control design of these UAVs be the key for successful flight and operations [5]. Compared to quadrotor systems, the yaw control of the tri-rotor systems is a further challenge due to the asymmetric configuration of the tri-rotor. For example, the reactive yaw moment in the quadrotor system is decoupled from roll and pitch moment, so which simplifies the yaw control design. In contrast, the yaw, roll and pitch moments are highly coupled in the tri-rotor system. Moreover, the attitude control of these tri-rotors is more complicated compared to quadrotor system due to the gyroscopic and Coriolis terms. The design of the control system is more complicated with coupling between attitude and position control loops.

The design of tri-rotor control systems is published in many works. The authors in [6] propose a tri-rotor configuration in which all rotors of the system tilt simultaneously to the same angle to attain yaw control. The control design focuses only on the attitude stabilization and neglects the position control problem. In [5], the authors propose a tri-rotor system of which the control design is implemented by four loops for attitude control and guidance. This control design is complicated with coupling between attitude and position control loops and high computation load. The control algorithm in [7] is based on nested saturation for decoupled channels from which the attitude control and position control of the UAV are designed independently. The authors in [10, 13] are concerned with the control design of nonlinear systems using feedback linearisation. The paper highlights the destabilisation effect of unmodelled actuator dynamics when applying feedback linearisation. To overcome this difficulty, a two stage feedback linearisation technique is proposed to compensate for actuator dynamics and subsequently linearise nonlinear systems.

From the overview, the problem of tri-rotor control system design is a challenging problem because the dynamics has highly coupled and nonlinear. This paper presents a tri-rotor control system design approach based on the dynamic model decomposition, feedback linearization. To simplify the implementation of feedback linearisation, several assumptions relating to the model of the nonlinear system and its operating point are considered. One of these assumptions, which is widely accepted in literature, is to neglect actuators dynamics [9, 11, 12].

2. Tri-rotor dynamics

Remind the dynamic equation system of tri-rotor in [1, 2] with $\alpha_2 = 0, \alpha_3 = 0$, the translational acceleration equation system (1) and the angle acceleration equation system (2):

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \frac{1}{m} \begin{bmatrix} \begin{array}{c} -K_f \omega_{m1}^2 \sin(\alpha_1)\cos(\theta)\sin(\psi) + \\ + K_f \sin(\theta)(\omega_{m1}^2 \cos(\alpha_1) + \omega_{m2}^2 + \omega_{m3}^2) \\ K_f \omega_{m1}^2 \sin(\alpha_1)(\cos(\psi)\cos(\phi) - \sin(\psi)\sin(\phi)\sin(\theta)) - \\ - K_f \cos(\theta)\sin(\phi)(\omega_{m1}^2 \cos(\alpha_1) + \omega_{m2}^2 + \omega_{m3}^2) \\ - mg + K_f \omega_{m1}^2 \sin(\alpha_1)(\cos(\psi)\sin(\phi) + \sin(\psi)\cos(\phi)\sin(\theta)) + \\ + K_f \cos(\phi)\cos(\theta)(\omega_{m1}^2 \cos(\alpha_1) + \omega_{m2}^2 + \omega_{m3}^2) \end{array} \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{I_{yy} - I_{zz}}{I_{xx}} qr + \frac{\sqrt{3} K_f l(\omega_{m2}^2 - \omega_{m3}^2)}{2I_{xx}} \\ \frac{I_{zz} - I_{xx}}{I_{yy}} pr + \frac{K_t \omega_{m1}^2 \sin(\alpha_1)}{I_{yy}} - \frac{K_f l(2\omega_{m1}^2 \cos(\alpha_1) - \omega_{m2}^2 - \omega_{m3}^2)}{2I_{yy}} \\ \frac{I_{xx} - I_{yy}}{I_{zz}} pq + \frac{K_t(\omega_{m1}^2 \cos(\alpha_1) + \omega_{m2}^2 + \omega_{m3}^2) + K_f l \omega_{m1}^2 \sin(\alpha_1)}{I_{zz}} \end{bmatrix} \tag{2}$$

$$\begin{aligned} & I_1 = (I_{yy} - I_{zz})/I_{xx}; \quad I_2 = (I_{zz} - I_{xx})/I_{yy}; \quad I_3 = (I_{xx} - I_{yy})/I_{zz}; \\ & u_1 = K_f(\omega_{m1}^2 \cos(\alpha_1) + \omega_{m2}^2 + \omega_{m3}^2); \qquad u_2 = K_f \omega_{m1}^2 \sin(\alpha_1); \\ & u_3 = \sqrt{3} l K_f(\omega_{m2}^2 - \omega_{m3}^2)/2; \\ & u_4 = 2K_t \omega_{m1}^2 \sin(\alpha_1) - K_f l(2\omega_{m1}^2 \cos(\alpha_1) - \omega_{m2}^2 - \omega_{m3}^2)/2; \\ & u_5 = K_t\left(\omega_{m1}^2 \cos(\alpha_1) + \omega_{m2}^2 + \omega_{m3}^2\right) + K_f l \omega_{m1}^2 \sin\left(\alpha_1\right) = \frac{K_t}{K_f} u_1 + l u_2 \end{aligned} \tag{3}$$

Rewrite equations (1) and (2) with (3), we receive (4) and (5):

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} -\cos(\theta)\sin(\psi)\frac{u_2}{m} + \sin(\theta)\frac{u_1}{m} \\ \left(\cos(\psi)\cos(\phi) - \sin(\psi)\sin(\phi)\sin(\theta)\right)\frac{u_2}{m} - \cos(\theta)\sin(\phi)\frac{u_1}{m} \\ \left(\cos(\psi)\sin(\phi) + \sin(\psi)\cos(\phi)\sin(\theta)\right)\frac{u_2}{m} + \cos(\phi)\cos(\theta)\frac{u_1}{m} - g \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} I_1 qr + u_3/I_{xx} \\ I_2 pr + u_4/I_{yy} \\ I_3 pq + u_5/I_{zz} \end{bmatrix} \tag{5}$$

$$\dot{\mathbf{X}} = \begin{bmatrix} -\cos(X_5)\sin(X_6)u_2/m + \sin(X_5)/m \\ (\cos(X_6)\cos(X_4)\text{-}\sin(X_6)\sin(X_4)\sin(X_5))u_2/m\text{-}\cos(X_5)\sin(X_4)u_1/m \\ (\cos(X_6)\sin(X_4)\text{+}\sin(X_6)\cos(X_4)\sin(X_5))u_2/m\text{+}\cos(X_4)\cos(X_5)u_1/m\text{-}g \\ X_7 \\ X_8 \\ X_9 \\ I_1X_8X_9 + u_3/I_{xx} \\ I_2X_7X_9 + u_4/I_{yy} \\ I_3X_7X_8 + u_5/I_{zz} \end{bmatrix} \quad (6)$$

In this paper, control system is synthesized with the following conditions: $-\pi/2 < \alpha_1 < \pi/2$. Equation systems (4) and (5) can be written in a state space form $\dot{\mathbf{X}} = f(\mathbf{X},\mathbf{u})$ where $\mathbf{X} \in \mathbb{R}^9 = (\dot{x}, \dot{y}, \dot{z}, \phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi})^T$ is the state vector with state variables which are set following: $X_1 = \dot{x}$; $X_3 = \dot{z}$; $X_2 = \dot{y}$; $X_4 = \theta$; $X_5 = \theta$; $X_6 = \psi$; $X_7 = \dot{\theta}$; $X_8 = \dot{\theta}$; $X_9 = \dot{\psi}$. System of equations (3), (4) in the state space form in (6).

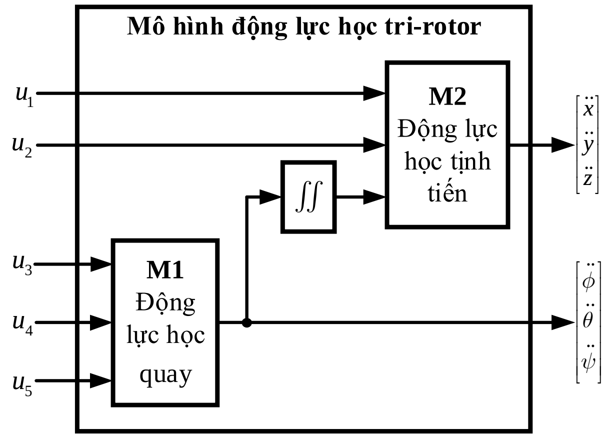


Fig. 1. Diagram shows the links between subsystems M1 and M2.

The decomposition technique is used to transform the state space equations (6), into two subsystems, in which the subsystem M1 consists of equations describing the state of the Euler angles with the inputs are variables u_3, u_4, u_5 (7):

$$\begin{bmatrix} \dot{X}_7 \\ \dot{X}_8 \\ \dot{X}_9 \end{bmatrix} = \begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \mathrm{I}_1\mathrm{X}_8\mathrm{X}_9 + u_3/\mathrm{I}_{xx} \\ \mathrm{I}_2X_7X_9 + u_4/\mathrm{I}_{yy} \\ \mathrm{I}_3X_7X_8 + u_5/\mathrm{I}_{zz} \end{bmatrix} \quad (7)$$

and the second subsystem M2 consists of the translational motion equations of tri-rotor with the inputs are the outputs from the subsystem M1 and inputs u_1, u_2 (8):

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\end{bmatrix}=\begin{bmatrix}-\cos(X_5)\sin(X_6)\frac{u_2}{m}+\sin(X_5)\frac{u_1}{m}\\ \left(\cos(X_6)\cos(X_4)-\sin(X_6)\sin(X_4)\sin(X_5)\right)\frac{u_2}{m}-\cos(X_5)\sin(X_4)\frac{u_1}{m}\\ \left(\cos(X_6)\sin(X_4)+\sin(X_6)\cos(X_4)\sin(X_5)\right)\frac{u_2}{m}+\cos(X_4)\cos(X_5)\frac{u_1}{m}-g\end{bmatrix} \quad (8)$$

The equation systems (7), (8) can be described by a diagram which shows the links between subsystems M1 and M2, also between state variables of M1 and M2 (Fig. 1). The diagram in Fig. 1 will be the basis for synthesizing tri-rotor control loops.

3. Design of tri-rotor control system

This section presents the synthesis of three controllers for the attitude control loop, translational velocity control loop and the position loop. The steps of the controller synthesis present below.

From model shown in Fig. 1, the authors proposed a nested control structure for tri-rotor UAV control. The block diagram of the nested control loops is shown in Fig. 2. In which the inner loop C1-M1 is the control loop for controlling and stabilizing the Euler angles, the middle loop C2-M2 is the translational velocity control loop and the outer loop C3 is the position control loop. With this structure, it is assumed that the inner control loop responses must be much faster than the outer loop responses.

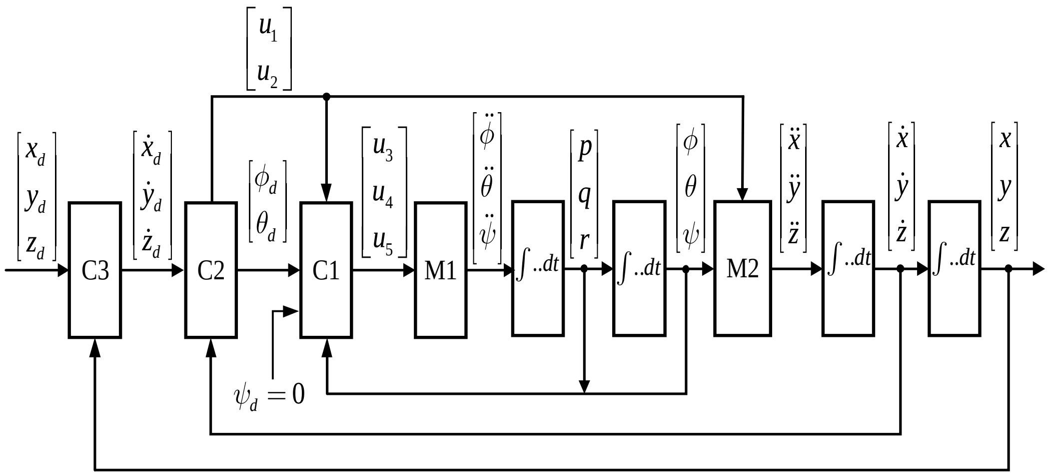


Fig. 2. Nested control structure of the tri-rotor UAV

The following shows the synthesis of controllers for the above control loops based on feedback linearization and module optimization. The synthesis is performed in the order C1, C2 and C3.

3.1. Synthesis of attitude control system

The dynamic equation system of M1 subsystem is shown in (7). The attitude loop stabilizes the Euler angles following a desired vector $(\phi_d, \theta_d, \psi_d)$. To synthesize the controller C1 for this loop, use the feedback linearization method.

From the expression (7), applying the feedback linearization [3], [4] to obtain a linear system (9) with new input variables u_3^*, u_4^*, u_5^*:

$$\begin{aligned} u_3 &= f_3(X_7, X_8, X_9) + u_3^* \\ u_4 &= f_4(X_7, X_8, X_9) + u_4^* \\ u_5 &= f_5(X_7, X_8, X_9) + u_5^* \end{aligned} \tag{9}$$

Substituting (9) into the Eq. (7), we received the Eq. (10):

$$\begin{bmatrix} \dot{X}_7 \\ \dot{X}_8 \\ \dot{X}_9 \end{bmatrix} = \begin{bmatrix} \mathrm{I}_1 X_8 X_9 + \left(f_3(X_7, X_8, X_9) + u_3^* \right) / I_{xx} \\ \mathrm{I}_2 X_7 X_9 + \left(f_4(X_7, X_8, X_9) + u_4^* \right) / \mathrm{I}_{yy} \\ \mathrm{I}_3 X_7 X_8 + \left(f_5(X_7, X_8, X_9) + u_5^* \right) / \mathrm{I}_{zz} \end{bmatrix} \tag{10}$$

In order to obtain a linear system, the new control variables u_3^*, u_4^*, u_5^* are selected in the right side of the equation system (10), which becomes a linear system. For this the following conditions must be fulfilled:

$$\begin{aligned} I_1 X_8 X_9 + f_3(X_7, X_8, X_9) / I_{xx} &= K_3 X_7 \\ I_2 X_7 X_9 + f_4(X_7, X_8, X_9) / I_{yy} &= K_4 X_8 \\ I_3 X_7 X_8 + f_5(X_7, X_8, X_9) / I_{zz} &= K_5 X_9 \end{aligned} \tag{11}$$

with the unknown constant parameters K_3, K_4, K_5. Evaluation of (12) yields the nonlinear feedback for linearization:

$$\begin{aligned} f_3(X_7, X_8, X_9) &= I_{xx}(K_3 X_7 - I_1 X_8 X_9) \\ f_4(X_7, X_8, X_9) &= I_{yy}(K_4 X_8 - I_2 X_7 X_9) \\ f_5(X_7, X_8, X_9) &= I_{zz}(K_5 X_9 - I_3 X_7 X_8) \end{aligned} \tag{12}$$

Substituting (9), (12) into (7) turns into the linear and decoupled system (13):

$$\begin{bmatrix} \dot{X}_7 \\ \dot{X}_8 \\ \dot{X}_9 \end{bmatrix} = \begin{bmatrix} K_3 X_7 + u_3^* / I_{xx} \\ K_4 X_8 + u_4^* / I_{yy} \\ K_5 X_9 + u_5^* / I_{zz} \end{bmatrix} \tag{13}$$

It can be shown that the linearized closed-loop system is stable even for non-modeled components. For that purpose, consider that inputs $u_3^* = u_4^* = u_5^* = 0$ and the operating point $X_7 = X_8 = X_9 = 0$. A Lyapunov function $V\left(X_7, X_8, X_9\right)$ is defined which is C_1 and positive defined around the operating point:

$$V(X_7, X_8, X_9) = (X_7^2 + X_8^2 + X_9^2)/2 \tag{14}$$

Combining (10) with (11), (13), the derivative of the last Lyapunov function has the following form:

$$\dot{V} = X_7\dot{X}_7 + X_8\dot{X}_8 + X_9\dot{X}_9 = K_3X_7^2 + K_4X_8^2 + K_5X_9^2 \tag{15}$$

The derivative is negative defined if K_3, K_4, K_5 < 0, and this guarantees that the operating point of the linearized closed-loop system is asymptotically stable.

Substituting variables $\dot{X}_4 = X_7$, $\dot{X}_5 = X_8$, $\dot{X}_6 = X_9$ into (13), we have:

$$\begin{aligned} \ddot{X}_4 &= K_3\dot{X}_4 + u_3^*/I_{xx} \\ \ddot{X}_5 &= K_4\dot{X}_5 + u_4^*/I_{yy} \\ \ddot{X}_6 &= K_5\dot{X}_6 + u_5^*/I_{zz} \end{aligned} \tag{16}$$

If $X_{4d}; X_{5d}; X_{6d}$ are the desired angles, select the linear controllers: $u_3^* = \varpi_3\left(X_{4d} - X_4\right)$; $u_4^* = \varpi_4\left(X_{5d} - X_5\right)$; $u_5^* = \varpi_5\left(X_{6d} - X_6\right)$ for (13) with constants $\varpi_3; \varpi_5; \varpi_6$. Using Laplace transform with (16), we received the transfer functions for roll, pitch and yaw channels, respectively:

$$\begin{aligned} W_4\left(s\right) &= \frac{X_4(s)}{X_{4d}(s)} = \frac{\varpi_3}{I_{xx}s^2 - I_{xx}K_3s + \varpi_3} \\ W_5\left(s\right) &= \frac{X_5(s)}{X_{5d}(s)} = \frac{\varpi_4}{I_{yy}s^2 - I_{yy}K_4s + \varpi_4} \\ W_6\left(s\right) &= \frac{X_6(s)}{X_{6d}(s)} = \frac{\varpi_5}{I_{zz}s^2 - I_{zz}K_5s + \varpi_5} \end{aligned} \tag{17}$$

The dynamics of these closed-loop systems can now be easily defined by adjustment of the parameter pairs $(K_3,\ \varpi_3), (K_4,\ \varpi_4), (K_5,\ \varpi_5)$, respectively, with the only limitation that the parameters K_3, K_4, K_5 must be negative.

3.2. Synthesis of the translational velocity control system C2

If the attitude control loop is sufficiently fast, i.e. the desired values of the roll, pitch and yaw angles $X_{4d}; X_{5d}; X_{6d}$ are achieved very fast compared to the outer translational velocity control loop. Therefore, the closed inner attitude control loop can be approximately considered as a static block that just transfers the desired values of roll, pitch and yaw angles to the next subsystem M2. The M2 model can be rewritten in simple form:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} -\cos(X_{5d})\sin(X_{6d})u_2/m + \\ +\sin(X_{5d})u_1/m \\ (\cos(X_{6d})\cos(X_{4d}) - \sin(X_{6d})\sin(X_{4d})\sin(X_{5d}))u_2/m - \\ -\cos(X_{5d})\sin(X_{4d})u_1/m \\ (\cos(X_{6d})\sin(X_{4d}) + \sin(X_{6d})\cos(X_{4d})\sin(X_{5d}))u_2/m + \\ +\cos(X_{4d})\cos(X_{5d})u_1/m - \mathrm{g} \end{bmatrix} \tag{18}$$

where X_{4d}, X_{5d}, X_{6d} and u_1, u_2 are input variables. Eq. (18) can be expressed by the following set of nonlinear differential equations

$$\begin{aligned} \dot{X}_1 &= \tilde{u}_1 = f_1(X_{4d}, X_{5d}, X_{6d}, u_1, u_2) \\ \dot{X}_2 &= \tilde{u}_2 = f_2(X_{4d}, X_{5d}, X_{6d}, u_1, u_2) \\ \dot{X}_3 &= \tilde{u}_3 = f_3(X_{4d}, X_{5d}, X_{6d}, u_1, u_2) \end{aligned} \tag{19}$$

with the new input variables $\tilde{u}_1, \tilde{u}_2, \tilde{u}_3$, that depend on the five input variables in a nonlinear form. However, regarding these new input variables, the control task is very simple because it comprises the control of three independent systems of first order which might be solved by pure proportional controllers, respectively:

$$\tilde{u}_1 = b_1(X_{1d} - X_1); \qquad \tilde{u}_2 = b_2(X_{2d} - X_2); \qquad \tilde{u}_3 = b_3(X_{3d} - X_3) \tag{20}$$

Here, the parameters of the controllers b_1, b_2, b_3 can be selected in a way such that allows the outer loop are fast enough but not too fast compared to the inner attitude control loop. From the above conditions and equations, the main task of designing these controllers are to determine the relationship between $X_{4d}, X_{5d}, X_{6d}, u_1, u_2$ and $\tilde{u}_1, \tilde{u}_2, \tilde{u}_3$.

We could know that any desired velocity vector can be achieved without any yaw rotation and therefore we can set $X_{6d} = \psi_d = 0$, so (19) can be rewritten bellow:

$$\begin{aligned} \tilde{u}_1 &= \sin(X_{5d})u_1/m \\ \tilde{u}_2 &= \cos(X_{4d})u_2/m - \cos(X_{5d})\sin(X_{4d})u_1/m \\ \tilde{u}_3 &= \sin(X_{4d})u_2/m + \cos(X_{4d})\cos(X_{5d})u_1/m - g \\ u_5 &= (K_t/K_f)u_1 + lu_2 \end{aligned} \tag{21}$$

From (21) we receive:

$$\sin(X_{5d}) = m\tilde{u}_1 / u_1 \tag{22}$$

$$m\tilde{u}_2 = \cos(X_{4d})u_2 - \cos(X_{5d})\sin(X_{4d})u_1 \tag{23}$$

$$m(\tilde{u}_3 + g) = \sin(X_{4d})u_2 + \cos(X_{4d})\cos(X_{5d})u_1 \tag{24}$$

Take the square of the two equations (23), (24) and add them together, we receive:

$$m^2\tilde{u}_2{}^2 + m^2\left(\tilde{u}_3 + g\right)^2 = u_2^2 + \cos^2\left(X_{5d}\right)u_1^2 \tag{25}$$

Take the square of Eq. (22) and then add it with Eq. (25), we have:

$$m^2\left[\tilde{u}_1{}^2 + \tilde{u}_2{}^2 + \left(\tilde{u}_3 + \mathrm{g}\right)^2\right] = u_1^2 + u_2^2 \tag{26}$$

From the last Eq. (22), we can determine u_2:

$$u_2 = \frac{1}{l}\left(u_5 - \frac{K_t}{K_f}u_1\right) \tag{27}$$

Take the square of Eq. (27) and substitute it into (27), we receive:

$$\left(l^2 + \frac{K_t{}^2}{K_f{}^2}\right)u_1{}^2 - 2u_5\frac{K_t}{K_f}u_1 + u_5^2 - m^2l^2\left(\tilde{u}_1{}^2 + \left(\tilde{u}_3 + g\right)^2 + \tilde{u}_2{}^2\right) = 0 \tag{28}$$

Because $\frac{K_t}{K_f}$ is very small, we simplify the Eq. (28), and can find u_1:

$$u_1 = \pm\sqrt{\frac{-u_5^2 + m^2l^2\left(\tilde{u}_1{}^2 + \left(\tilde{u}_3 + \mathrm{g}\right)^2 + \tilde{u}_2{}^2\right)}{l^2}} \tag{29}$$

u_1 has the same direction as the z-axis, so u_1 is always positive:

$$u_1 = \sqrt{\frac{-u_5^2 + m^2l^2\left(\tilde{u}_1{}^2 + \left(\tilde{u}_3 + \mathrm{g}\right)^2 + \tilde{u}_2{}^2\right)}{l^2}} \tag{30}$$

Replace Eq. (30) into Eq. (27), we can find u_2 and into Eq. (23), X_{5d} is determined. From Eq. (23), we divide the two sides of the equation for $\sqrt{u_1^2\cos^2\left(X_{5d}\right) + u_2^2}$, we have:

$$\begin{aligned}\frac{m\tilde{u}_2}{\sqrt{u_1^2\cos^2\left(X_{5d}\right) + u_2^2}} = \cos\left(X_{4d}\right)&\frac{u_2}{\sqrt{u_1^2\cos^2\left(X_{5d}\right) + u_2^2}} - \\ -\sin\left(X_{4d}\right)&\frac{u_1}{\sqrt{u_1^2\cos^2\left(X_{5d}\right) + u_2^2}}\end{aligned} \tag{31}$$

It is easy to know $\frac{m^2\tilde{u}_2^{\,2}}{u_1^2\cos^2\left(X_{5d}\right)+u_2^2}\le 1$ so Eq. (32) has solutions. We set $\sin\left(\delta\right)=\frac{u_2}{\sqrt{u_1^2\cos^2\left(X_{5d}\right)+u_2^2}}$; $\cos\left(\delta\right)=\frac{u_1}{\sqrt{u_1^2\cos^2\left(X_{5d}\right)+u_2^2}}$. Rewrite (31), we have:

$$\frac{-m\tilde{u}_2}{\sqrt{u_1^2\cos^2\left(X_{5d}\right)+u_2^2}}=\sin\left(X_{4d}-\delta\right) \tag{32}$$

Because $\frac{-\pi}{2}\le X_{4d}\le\frac{\pi}{2}$, from Eq. (32), we can find X_{4d} in the following:

$$X_{4d}=\arcsin\left(\frac{-m\tilde{u}_2}{\sqrt{u_1^2\cos^2\left(X_{5d}\right)+u_2^2}}\right)+\delta \tag{33}$$

3.3. Synthesis of position control system C3

The design of position controller C3 is implemented after the inner-loop controllers are synthesized. The way to design the position controller of three channels is the same, so in this section we synthesize the controller for altitude channel Z. Simplifying synthesis, we assume that the transfer function of the velocity loop which is synthesized above is second order. Therefore, the transfer function of Z channel has a form:

$$W_{pz}=\frac{Z}{U_{cz}}=\frac{K_z}{s(T_{z1}s+1)(T_{z2}s+1)} \tag{34}$$

Using the module optimization method [8] the transfer function of the Z channel controller is in the form - the Proportional - Derivative controller (PD):

$$W_{cz}=\frac{T_{z2}s+1}{2K_zT_{z1}} \tag{35}$$

4. Simulation of control system

In order to implement the derived control system, a simulation model has been developed. The tri-rotor model (6) using the parameters of Tab. 1 is then implemented in MATLAB/Simulink for a simulation, which is shown in Fig. 3.

Tab. 1. Parameters of tri-rotor

Parameter	Value	Units
K_f	$2.92\cdot10^{-6}$	kg.m
K_t	$1.1\cdot10^{-7}$	kg.m^2
m	0.5	kg
l	0.3	m

Parameter	Value	Units
I_{xx}	0.3105	kg.m^2
I_{yy}	0.2112	kg.m^2
I_{zz}	0.2215	kg.m^2
g	9,81	m.s^{-2}

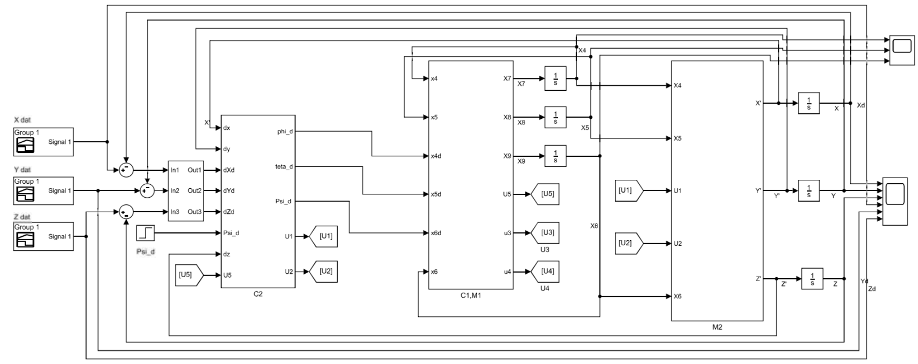
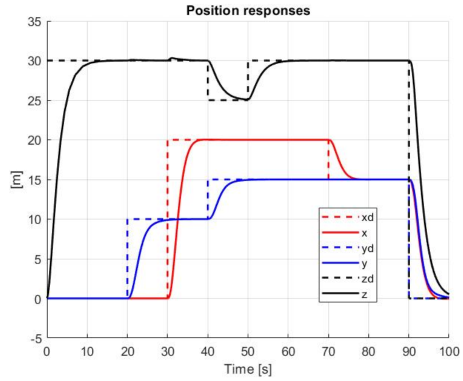
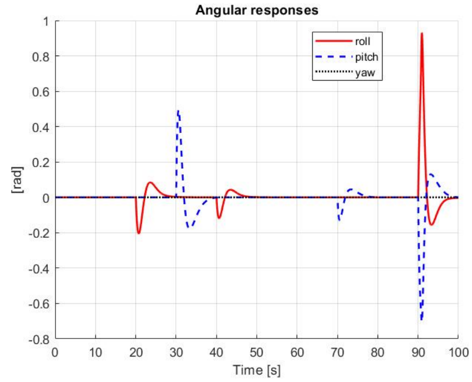


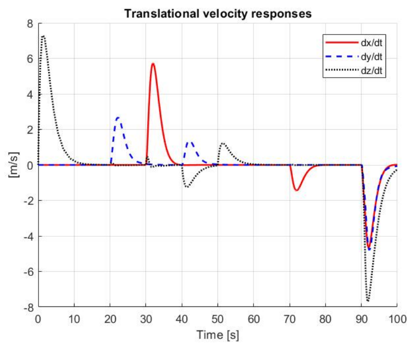
Fig. 3. Diagram simulating the tri-rotor control system



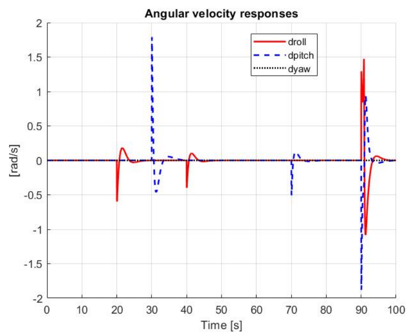
a) Position responses



b) Angular responses



c) Translational velocity responses



d) Angular velocity responses

Fig. 4. The simulation results of the tri-rotor control system with synthesized controllers

The parameters of the velocity controllers are chosen as $b_1 = 1;\ b_2 = 1;\ b_3 = 1$ while the design parameters of the inner loop attitude controllers are $K_3 = K_4 = K_5 = -80$ and $\varpi_3 = \varpi_4 = 40$, $\varpi_5 = 30$; the coefficients of position controllers PD are $K_{PX} = 0.25$; $K_{PY} = 0.25$; $K_{PZ} = 0.25$; $K_{DX} = K_{DY} = K_{DZ} = 0.1$. In simulation, we will implement with some steps: At the first time, the tri-rotor vertical takes off to height Zd = 30, after

20 seconds, UAV moves following the Y direction with Yd = 10; after the next 10 seconds, flies to the X direction with Xd = 20. At the time equal 40s, tri-rotor reduces the altitude from 30m to 25m and flies to the Y direction more 5mm, simultaneously. At the time point 60s, tri-rotor implements from 25m to 30m in Z direction. At the time point 70, tri-rotor moves back 5m in the X direction. At the time point 90, tri-rotor lands at the start point.

The obtained control result is shown in Fig. 4. In general, the controller shows good performance with tracking in all channels for take off, hovering and landing state. The controller succeeds to maintain the stability of the vehicle and follow the reference trajectory. The settling time of the system is about 5-8s with overshot is approximately zero. Both translational and rotational velocities converge to zero when tri-rotor at hovering state. At the time when X, Y channels change which also cause changing a little in Z channel. This is reflected from dynamic model.

5. Conclusion

In this paper, non-linear control systems for tri-rotor UAV is designed based on the dynamic model decomposition into a nested structure with the constraint that the responses of inner loops is much faster than the responses of outer loops. The controllers of the attitude loop, the translational loop and the position loop are synthesized using feedback linearization, modulus optimum. Stability of attitude loop is proved by Lyapunov theory. The simulation model has built on Matlab/Simulink from the tri-rotor dynamics and the synthesized controllers. The simulation results have shown the good performance of control system in take off, hover and landing modes with control parameters such as the settling time is about 5-8s and overshot is approximately equal zero.

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ĐIỀU KHIỂN PHI TUYẾN TRI-ROTOR DỰA TRÊN PHÂN CHIA MÔ HÌNH ĐỘNG LỰC HỌC VÀ TUYẾN TÍNH HÓA PHẢN HỒI

Tóm tắt: *Bài báo trình bày việc xây dựng mô hình động lực học UAV tri-rotor và phân chia mô hình động lực học thành cấu trúc các vòng điều khiển lệ thuộc với điều kiện ràng buộc rằng đáp ứng của các vòng trong nhanh hơn đáp ứng của những vòng ngoài. Từ sơ đồ các vòng điều khiển, các bộ điều khiển tư thế, vận tốc và vị trí đã được tổng hợp bằng phương pháp tuyến tính hóa phản hồi và tối ưu hóa mô đun. Tính ổn định của vòng điều khiển tư thế đã được chứng minh bằng lý thuyết Lyapunov. Cuối cùng, các kết quả mô phỏng trên MATLAB/Simulink khẳng định rằng các bộ điều khiển đã tổng hợp làm việc được trong tất cả các chế độ bay với các thông số điều khiển như thời gian quá độ từ 5-8s và độ quá điều chỉnh xấp xỉ bằng không.*

Từ khóa: Tri-rotor UAV; tuyến tính hóa phản hồi; mô hình động lực học; điều khiển phi tuyến; tối ưu hóa mô đun.

Received: 02/4/2019; *Revised:* 04/4/2020; *Accepted for publication:* 06/4/2020

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