# EFFECT OF PIEZOELECTRIC AND STIFFENER ON DISPLACEMENT OF THE SHALLOW CYLINDRICAL COMPOSITE SHELL SUBJECTED TO SHOCK WAVE

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**Abstract**

The vibration of the composite stiffened cylindrical shell with piezoelectric layers subjected to shock wave is presented in this paper. The differential equations which describe the nonlinear oscillation dynamic of the shell are solved by the Newton-Raphson iterative method in combination with the Newmark direct integration method. Numerical results are solved for the structures subjected to the effect of shock wave in MATLAB software. On the basis of the program, the effect of piezoelectric and stiffener on the nonlinear oscillation of the cylindrical composite shell are considered.

***Keywords:*** *Composite shell; stiffener; piezoelectric.*

## 1. Introduction

Piezoelectric composite shell structures are made of composite material with piezoelectric layers or piezoelectric pieces. Piezoelectric material works as a machine that directly converts energy into electricity and vice versa by taking advantage of the direct and converse piezoelectric effects. Because of these properties, when combining piezoelectric materials into the structure with the corresponding current direction, we can create vibrating in the same direction and in the opposite direction with mechanical oscillations. So the vibration of the structure would be controlled. The piezoelectric composite structures in practice are often in the form of bars, plates and shells. In the process of calculating these structures, the problem is in the quest for lightweight flexible structures with self-controlling or self-monitoring capabilities and still ensure structure work reliably as required. One of the most popular and effective solutions is adding reinforcing ribs instead of making thicker textures, thus, optimizing the use of materials while ensuring effective use in practice. In the world, the calculation of piezoelectric structures, especially structural beams and piezoelectric composite plates, so far published and achieved many remarkable results. For composite shell structures with piezoelectric layers or pieces, the application needs are very large, but the calculation under the effect of different types of loads is complicated. The static and free vibration

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analysis of composite piezoelectric shells have been published in many works, but the forced vibration analysis for the structure of composite piezoelectric shells is a problem that still needs to be solved, especially in building algorithms, calculating programs, and mastering computer programs. Stemming from that fact and on the basis of inheriting and developing previous studies (Nguyen et al., 2015, 2017, 2018; Hoang et al., 2013; Truong, 2014; Alijani& Amabili, 2014; Shen and Yang, 2014), the paper studies finite element algorithm to solve and observe the effect of some factors on the vibration of the nonlinear vibration of the shallow cylindrical stiffened composite shell.

## 2. Mathematical modeling and assumptions

Consider a composite shell with piezoelectric layers and stiffener in the coordinate system OXYZ. The shallow cylindrical composite shell has the ratio of $\frac{f_0}{\ell_{\min}} \le \frac{1}{5}$, where $f_0$ - curvature of the shell; $\ell_{\min}$ - minimum (a, b); a, b - the projection dimensions of the shell (Fig. 1).

This study is solved with the following assumptions:

- Composite shell and stiffener have absolute adhesion, composite layers and piezoelectric layers are ideally linked to each other.

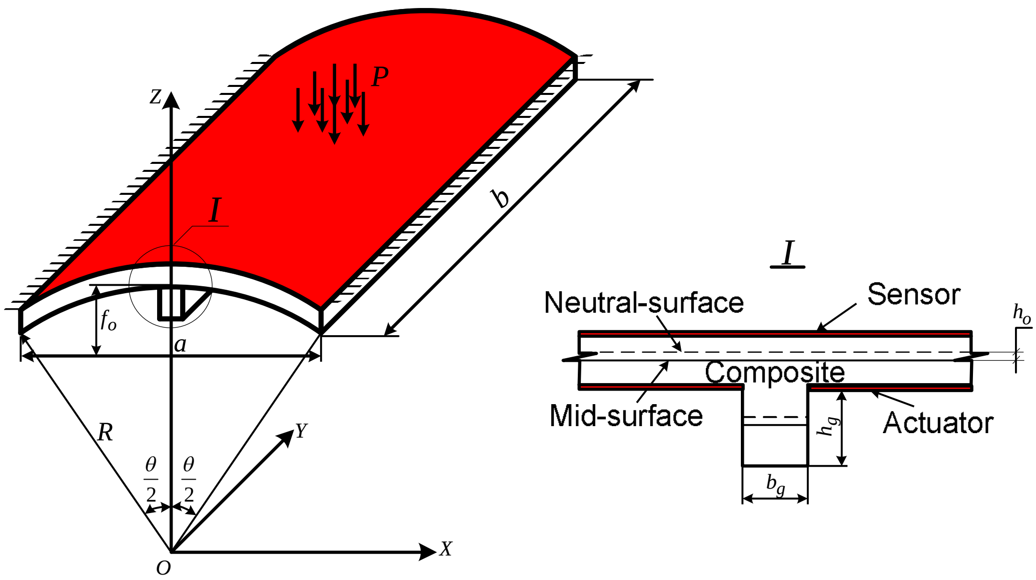


*Fig. 1. The shallow cylindrical piezoelectric composite stiffened shell*

- Shell and beam stiffener are used Reissner Mindlin's theory.

- The charge is a constant on the piezoelectric layer and a linear variation according to the piezoelectric layer thickness.

- The stiffeners are arranged to the inside of the shell and parallel to the straight boundary of the cylindrical shell.

## 3. Statement of the problem and governing equations

The displacement at a point (x, y, z) of the shell can be expressed as (Reddy, 2004):

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\theta_y(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) - z\theta_x(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{1}$$

where *u*, *v* and *w* are the displacements along the x, y and z directions at any point (x, y, z) of the shell, respectively; $u_0$, $v_0$ and $w_0$ are the associated mid-plane displacements; $\theta_x$, $\theta_y$ are the rotations about the y and x axes, respectively.

For the nonlinear problem, the components of the strain vector in relation to the displacement field Eq. (1) are represented as follows (Bathe, 1996):

$$\{\varepsilon\} = \{\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz}\}^T = \begin{Bmatrix} \{\varepsilon_b\} \\ \{\varepsilon_s\} \end{Bmatrix} = \begin{Bmatrix} \{\varepsilon_b^L\} \\ \{\varepsilon^N\} \end{Bmatrix} \tag{2}$$

where $\{\varepsilon_b^L\} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u_0 \\ v_0 \end{Bmatrix} + z \begin{bmatrix} 0 & \frac{\partial}{\partial x} \\ -\frac{\partial}{\partial y} & 0 \\ -\frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix} \begin{Bmatrix} \theta_x \\ \theta_y \end{Bmatrix} = \{\varepsilon_0\} + z\{\kappa\}$ - the linear strain vector,

$\{\varepsilon^N\} = \{\varepsilon_x^N \quad \varepsilon_y^N \quad \gamma_{xy}^N\}^T = \begin{Bmatrix} \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \end{Bmatrix}$ - the nonlinear strain vector,

$\{\varepsilon_s\} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 1 \\ \frac{\partial}{\partial y} & -1 & 0 \end{bmatrix} \begin{Bmatrix} \mathrm{w}_0 \\ \theta_x \\ \theta_y \end{Bmatrix}$ - the shear strain vector.

Stress-deformation relation and the expression which represents the charge equilibrium in the $k-th$ piezoelectric layer of the element can be written in the following form (Reddy, 2004; Volmia, 1972; Truong, 2014; Alijani and Amabili, 2014):

$$\begin{aligned} \{\sigma\}_k &= \left[\overline{C}_{ij}\right]_k \{\varepsilon\}_k - [e]_k^T \{E\}_k \\ \{D\}_k &= [e]_k \{\varepsilon\}_k + [p]_k \{E\}_k \end{aligned} \tag{3}$$

where $\{E\}$ - vector of electric field; $\{D\}$ - vector of electric displacement; $\left[\overline{C}_{ij}\right]$ ($i,j$ = 1, 2, 4, 5, 6) - elastic constants matrix; $[e]$ - piezoelectric constants matrix; $[p]$ - dielectric constants matrix.

Vector of electric field $\{E\}$ can be expressed as (Hernandes and Melim, 2012; Reddy, 2004):

$$\{E\}_{3\times1} = \{0 \quad 0 \quad E_z\}^T = \left\{0 \quad 0 \quad -\frac{\phi}{t}\right\}^T \tag{4}$$

The membrane forces, torsion moments and shear forces components in the shell element which has $n$ composite layers and $m$ piezoelectric layers can be defined (Reddy, 2004; Shen and Yang, 2014):

$$\begin{Bmatrix} \{N\} \\ \{M\} \\ \{Q\} \\ \{D_k^P\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] & [0] & [\bar{e}]_k \\ [B] & [D] & [0] & [\bar{\bar{e}}]_k \\ [0] & [0] & [F] & [\tilde{e}]_k \\ [\bar{e}] & [\bar{\bar{e}}] & [\tilde{e}] & [\bar{p}]_k \end{bmatrix} = \begin{Bmatrix} \{e_0\} + \{e^N\} \\ \{K\} \\ \{e_s\} \\ \{-E\}_k \end{Bmatrix} \tag{5}$$

where $[A] = \left[A_{ij}\right]$ with $\left[A_{ij}\right]_{3\times3} = \sum_{k=1}^{n}(h_k - h_{k-1})\left[Q'_{ij}\right]_k + \sum_{k=1}^{m}(h_k - h_{k-1})\left[C_{ij}\right]_k$

$[B] = \left[B_{ij}\right]$ with $\left[B_{ij}\right]_{3\times3} = \frac{1}{2}\sum_{k=1}^{n}\left(h_k^2 - h_{k-1}^2\right)\left[Q'_{ij}\right]_k + \frac{1}{2}\sum_{k=1}^{m}\left(h_k^2 - h_{k-1}^2\right)\left[C_{ij}\right]_k$

$[D] = \left[D_{ij}\right]$ with $\left[D_{ij}\right]_{3\times3} = \frac{1}{3}\sum_{k=1}^{n}\left(h_k^3 - h_{k-1}^3\right)\left[Q'_{ij}\right]_k + \frac{1}{3}\sum_{k=1}^{m}\left(h_k^3 - h_{k-1}^3\right)\left[C_{ij}\right]_k$

$[F] = \left[F_{ij}\right]$ with $[F]_{2\times2} = \sum_{k=1}^{n} f\left(h_k - h_{k-1}\right)\left[C'_{ij}\right]_k + \sum_{k=1}^{m} f\left(h_k - h_{k-1}\right)\left[C_{ij}\right]_k$

$\left[Q'_{ij}\right]$ and $\left[C'_{ij}\right]$ - compact stiffness matrix (Truong, 2014)

$[\bar{p}]_k = \sum_{k=1}^{m} f\left(h_k - h_{k-1}\right)\left[p_{(3\times3)}\right]_k$

$$\left[\overline{e}\right]_k = \sum_{k=1}^{m}\left(h_k - h_{k-1}\right)\left[e_{(3\times3)}\right]_k ;$$

$$\left[\overline{\overline{e}}\right]_k = \frac{1}{2}\sum_{k=1}^{m}\left(h_k^2 - h_{k-1}^2\right)\left[e_{(3\times3)}\right]_k , \left[\tilde{e}\right]_k = \sum_{k=1}^{m}\left(h_k - h_{k-1}\right)\left[e_{(2\times2)}\right]_k .$$

# 4. Finite element formulation of the piezoelectric composite stiffened cylindrical shell

### *4.1. Shell element and stiffener element*

*Shell element:*

The shallow shell can be discrete by nine-node flat shell elements, which each node has 6 degrees of freedom ($u_i$, $v_i$, $w_i$, $\theta_{xi}$, $\theta_{yi}$, $\theta_{zi}$) as shown in Fig. 2 (Reddy, 2004):

The displacement node vector of shell element:

$$\left\{q_e^u\right\}_{54\times1} = \left\{u_1 \quad v_1 \quad \mathrm{w}_1 \quad \theta_{x1} \quad \theta_{y1} \quad \theta_{z1} \quad u_2 \quad v_2 \quad \ldots \quad \theta_{x9} \quad \theta_{y9} \quad \theta_{z9}\right\} \tag{6}$$

The voltage node vector of piezoelectric layer element:

$$\left\{q_e^\phi\right\}_{9\times1} = \left\{\phi_1 \quad \phi_2 \quad \ldots \quad \phi_9\right\}^T \tag{7}$$

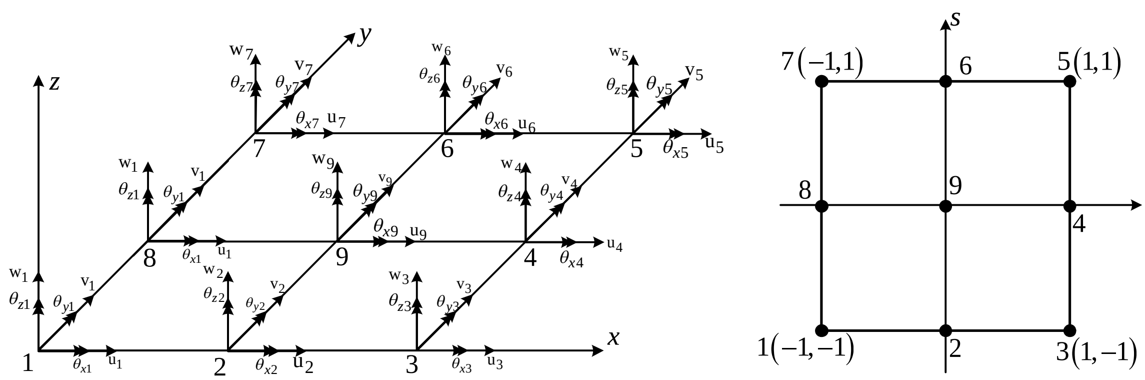


*Fig. 2. Nine-node flat shell element (a) and the reference element (b)*

The displacement of the point belongs to the element is discretized by the shape function and can be written, in terms of the degrees-of-freedom nodal of shell structure, by:

$$\left\{u_e\left(r,s\right)\right\}_{6\times1} = \left[N_i^u\right]_{6\times54}\left\{q_e^u\right\}_{54\times1} \tag{8}$$

where $\left[N_i^u\right]_{6\times54}$ are the shape function described detail in (Reddy, 2004).

The electric potential element is discretized by the shape function and the electric potential terms:

$$\{\phi(r,s)\} = \left[N_i^\phi\right]_{1\times 9}\{q_e^\phi\}_{9\times 1} \tag{9}$$

where $\left[N_i^\phi\right]_{1\times 9}$ is the potential shape function (Reddy, 2004; Shen and Yang, 2014).

*Stiffener element:*

The three-point beam element which each node has six displacement components is used in this paper. The node displacement vector of the stiffened beam element parallel to the y-axis (Fig. 3):

$$\{q\}_{e\ (18\times 1)}^{gy} = \left\{u_{o1}^{gy}\quad v_{o1}^{gy}\quad w_{o1}^{gy}\quad \theta_{x1}^{gy}\quad \theta_{y1}^{gy}\quad \theta_{z1}^{gy}\quad \ldots\quad \theta_{x3}^{gy}\quad \theta_{y3}^{gy}\quad \theta_{z3}^{gy}\right\}^T \tag{10}$$

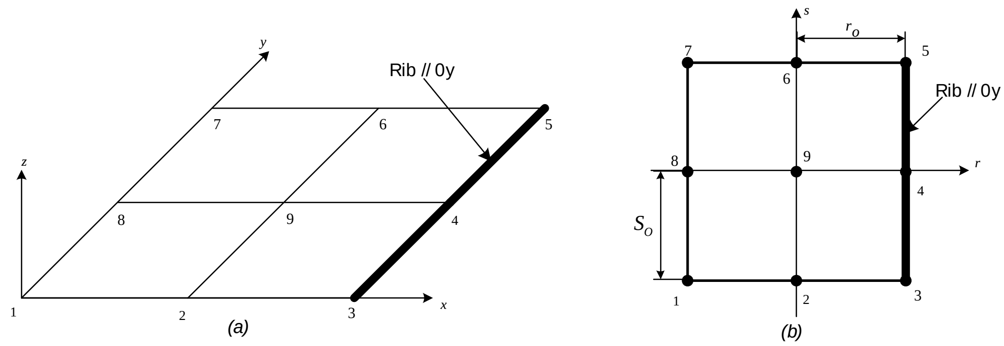


*Fig. 3. 9-node flat shell element and 3-point node beam element*

Displacement at any point of the stiffened beam element according to node displacement and shape function:

$$\{u\}_{e\ (6\times 1)}^{gy} = [N]_{6\times 18}^{gy}\{q\}_{e\ (18\times 1)}^{gy} \tag{11}$$

where $[N]^{gy}$ obtained by replacing $r = r_0$ into the shape functions of the shell element.

***4.2. The nonlinear differential equation of the composite piezoelectric stiffened cylindrical shell***

The basic equation can be written using Hamilton's principle. The kinetic energy, the potential of elastic deformation, the power of the electric field force and the power caused by external force and external charge. Then change and shorten the expression, the differential equation of the element without a feedback circuit (Reddy, 2004; Shen and Yang, 2014).

$$\begin{aligned}&\left[M_{uu}^e\right]\{\ddot{q}_e^u\} + \left[C_R^e\right]\{\dot{q}_e^u\} + \left(\left[K_{uu}^e\right] + \left[K_{u\phi}^e\right]\left[K_{\phi\phi}^e\right]^{-1}\left[K_{\phi u}^e\right] + \left[K_g^e\right]\right)\{q_u^e\}\\ &\qquad = \{F_M^e\} + \left[K_{u\phi}^e\right]\left[K_{\phi\phi}^e\right]^{-1}\{Q_c^e\}\end{aligned} \tag{12}$$

Using the transformation matrix method, the differential equation of the shell in the global coordinate system can be written. Summing up the contributions from Eq. (12) for all elements, yields the system equations of motion without the feedback circuit (Reddy, 2004; Shen and Yang, 2014):

$$\begin{aligned}&\left[M_{uu}\right]\left\{\ddot{q}^{u}\right\}+\left[C_{R}\right]\left\{\dot{q}^{u}\right\}+\left(\left[K_{uu}\right]+\left[K_{u\phi}\right]\left[K_{\phi\phi}\right]^{-1}\left[K_{\phi u}\right]+\left[K_{g}\right]\right)\left\{q_{u}\right\}\\&\quad=\left\{F_{M}\right\}+\left[K_{u\phi}\right]\left[K_{\phi\phi}\right]^{-1}\left\{Q_{c}\right\}\end{aligned}\tag{13}$$

where $\left[M_{uu}\right]$ - the mass matrix; $\left[C_{R}\right]$ - the mechanical damping matrix of the structure, respectively (considering as the effect of mechanical interaction - electricity to the structure is negligible, so $\left[C_{R}\right]=\alpha\left[M_{uu}\right]+\beta\left[K_{uu}\right]$ where Rayleigh's damping coefficients $\alpha$, $\beta$ are determined by the ratio of damping and the natural frequency of the shell); $\left[K_{\phi u}\right]$, $\left[K_{u\phi}\right]$ - the mechanical-electrical, electric-mechanical interaction matrices, respectively; $\left[K_{\phi\phi}\right]$ - the dielectric stiffness matrix; $\left[K_{uu}\right]$- the mechanical stiffness matrix; $\left\{F_{M}\right\}$ - the shock wave force vector; $\left\{Q_{C}\right\}$ - the external electrical force vector; $\left[K_{g}\right]$ - the stiffened matrix (Truong, 2014 ).

These nonlinear equations were solved by the direct integration Newmark method combined with the Newton-Raphson iterative method. Based on the method described previously, the MATLAB program has been developed and the results show good agreement when compared with works published (Truong, 2014). The present method can be used to analyze nonlinear transient of the shallow composite piezoelectric stiffened cylindrical shell.

## 5. Numerical results and discussion

### *5.1. Numerical verification*

To evaluate the reliability of the present formulation, we have made a comparison the results of the study (Prusty, 2003). The centrally stiffened and cross-stiffened cylindrical shell shown in Fig. 4 is considered to have the geometry as $a=1\text{m}$, $b=1\text{m}$, $h_g=50\times10^{-3}\text{m}$, the loading considered on the structure is uniformly distributed $p=10^{6}\,\text{N/m}^2$ and the material properties $E_1=175.78\times10^{9}\,\text{Pa}$; $E_2=E_3=7.031\times10^{9}\,\text{Pa}$; $G_{12}=3.516\times10^{9}\,\text{Pa}$, $G_{23}=1.406\times10^{9}\,\text{Pa}$, $b_g=8.6\times10^{-3}\text{m}$, $\nu=0.25$.

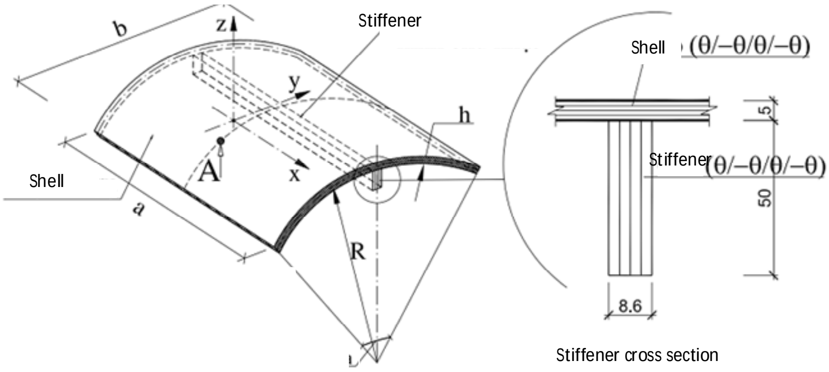


*Fig. 4. Laminated centrally stiffened shell (Prusty, 2003)*

The displacement variation at point A in the composite centrally stiffened shell with hat shaped stiffener (Fig. 4) is calculated and compared in Tab. 1. The results show good agreement when compared with previously published data.

*Tab. 1. The displacements along z directions at point A [m]*
*(a/h = 200; R/a = 5)*

| Laminate | Prusty [4] | This paper | Error (%) |
|---|---|---|---|
| $[0^o/90^o]_2$ | 0.0677 | 0.0648 | 4.28 |
| $[30^o/-30^o]_2$ | 0.0587 | 0.0573 | 2.39 |
| $[45^o/-45^o]_2$ | 0.1264 | 0.1219 | 3.56 |

### *5.2. The starting problem*

Let us consider the composite graphite/epoxy T300/976 shell with the thickness $h$ = 0.015 m, the radius of curvature $R$ = 1.0 m, the length $L$ = 0.3 m, span angle $\theta = 30^o$. The composite shell with 4 layers which are the same thickness, laminate $[45^o/-45^o]_2$. The following material properties are: $E_{11} = 150\times10^9$ N/m$^2$; $E_{22} = E_{33} = 9\times10^9$ N/m$^2$; $G_{12} = G_{13} = 7.1\times10^9$ N/m$^2$; $G_{23} = 2.5\times10^9$ N/m$^2$; $\nu_{12} = 0.3$; $\nu_{13} = 0.3$; $\nu_{23} = 0.3$; $\rho = 1600$ kg/m$^3$. Three laminated rectangular stiffeners are arranged parallel to the straight boundary of the shell (are shown in Fig. 5), and made from the same material with the composite layers, stiffened width $w_g$ = 0.015 m, stiffened height $h_g$ = 0.015 m. The PZT G1195N piezoelectric coating covers the entire surface on the shell (where does not have tiffeners),with the height $t$ = 0.001m, material properties: $E_p = 63\times10^9$ N/m$^2$; $G_p = 24\times8.10^9$N/m$^2$; $\nu = 0.28$; $\rho_{pzt} = 7600$ kg/m$^3$; $d_{31} = d_{32} = 2.54\times10^{-10}$ m/V; $p_{11} = p_{22} = p_{33} = 15\times10^{-9}$ F/m.

Mechanical load: A shock wave, evenly distributed on the upper surface of the shell with the load rule:

$$p(t) = p_{\max} F(t)$$
$$F(t) = \begin{cases} 1 - \dfrac{t}{\tau_\theta}: & 0 \le t \le \tau_\theta \\ 0: & t > \tau_q \end{cases} \qquad (14)$$

where $p_{\max} = 1 \times 10^5\,\mathrm{N/m^2}$, $\tau_\theta = 0.025\,\mathrm{s}$.

Potential load $V_p = 50\,\mathrm{V}$;

The shell was clamped along two straight edges: u = 0, v = 0, w = 0, $\theta_x = 0$, $\theta_y = 0$, $\theta_z = 0$ at x = 0 and x = a. The center point on the upper surface of the shell (point A) was considered.

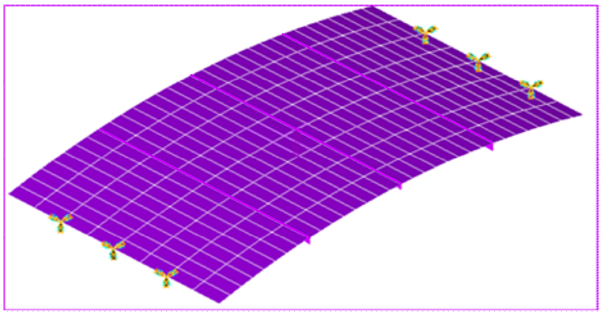

*Fig. 5. Finite element model of the problem*

Based on the program, the influence of some factors on the nonlinear dynamic of the shallow composite piezoelectric stiffened cylindrical shell.

### *5.3. The influence of some factors on the nonlinear vibration of the shell*

#### *5.3.1. Effect of nonlinear properties*

To examine the effect of nonlinearity, the linear problem was compared with the solved nonlinear problem. Fig. 6 and Fig. 7 show the displacement and stress variations at point A in two cases.

The time response of displacement, stress at the calculation point of the linear problem is different from the nonlinear problem both in amplitude and cycle. In particular, the response values of the nonlinear problem are much larger than that of the linear problem, which shows that the calculation by the nonlinear method is more stable and safer. According to the authors, this is the advantage of solving nonlinear problems for this particular case.

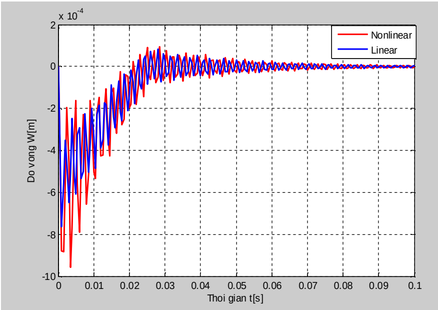


*Fig. 6. Time history response of vertical displacement W at point A*

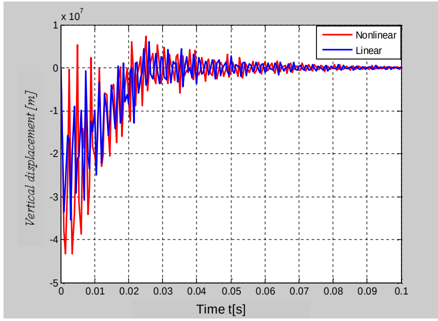


*Fig. 7. Time history response of stress $\sigma_x$ at point A*

### *5.3.2. Effect of imposed voltage*

We consider the potential effect of the piezoelectric layer surface on the response of the shell, in this section, the problem with $V_p$ voltage varying from 0V to 300V was solved. The vertical displacement at point A is shown in Fig. 8.

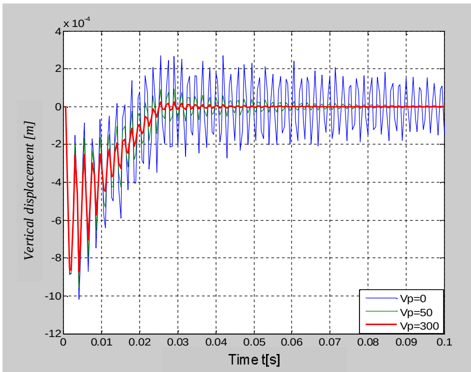


*Fig. 8. Time history response of vertical displacement W at point A ($V_P$ = 0V, 50V, 300V)*

When the voltage increases, in general, the maximum displacement and stress at point A are reduced. Comparing to the voltage increasing from 0V to 300V, the reduction of these quantities is not large, but when the voltage increases, the extinguishing effect is very fast, especially after the time of applying force - this is one of the outstanding advantages of piezoelectric materials in the shell structures.

### *5.3.3. Effect of the stiffened beam*

To consider the effect of stiffener on the response of the shell, in this section, the authors solved the problem with 3 cases: There are 3 stiffeners (as in the starting problem); there is one along the apical line of the shell, and the case that has no stiffener. Results of vertical displacement at point A are shown in Fig. 9.

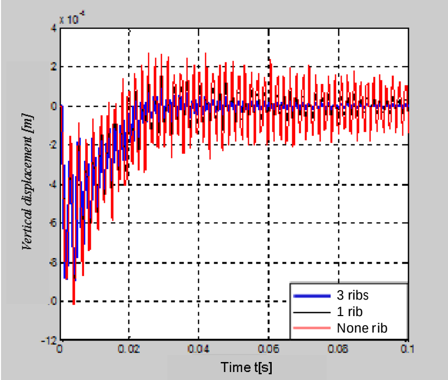


*Fig. 9. Vertical displacement variation at point A with different stiffened arrangements*

As the number of the stiffener increases, the maximum displacement and stress are reduced, which proves that the stiffness of the structure significantly increases, the reinforcement effect of the stiffener for the shell structure is evident. Thus, increasing the number of stiffener is an effective measure to increase the hardness and durability of the shell.

## 6. Conclusions

The nonlinear vibration of the piezoelectric composite stiffened cylindrical shell with stiffener subjected to shock wave is investigated here using the finite element method:

- Investigate the effects of nonlinear problem, piezoelectric properties, the number of stiffeners to the dynamic of the composite cylindrical shell.

- The numerical results provide some valuable conclusions for increasing rigidity, durability and oscillation control in the piezoelectric structure with stiffener.

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# KHẢO SÁT ẢNH HƯỞNG CỦA ÁP ĐIỆN VÀ GÂN GIA CƯỜNG ĐẾN CHUYỂN VỊ CỦA VỎ TRỤ THOẢI COMPOSITE LỚP CHỊU TẢI TRỌNG SÓNG XUNG KÍCH

***Tóm tắt:*** *Bài báo trình bày kết quả nghiên cứu dao động của vỏ trụ thoải composite lớp áp điện có gân gia cường. Phương trình vi phân mô tả dao động phi tuyến của vỏ trụ thoải composite lớp áp điện có gân gia cường được tác giả giải trên cơ sở kết hợp phương pháp tích phân trực tiếp Newmark với phương pháp lặp Newton-Raphson. Chương trình tính cụ thể hóa thuật toán, phân tích bài toán được các tác giả viết bằng phần mềm MATLAB. Trên cơ sở chương trình đã lập, các tác giả khảo sát ảnh hưởng của áp điện và gân gia cường đến dao động phi tuyến của vỏ trụ thoải composite lớp.*

**Từ khóa:** Vỏ composite; gân gia cường; áp điện.

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