

# COMBINATION OF MONTE-CARLO AND WANG'S METHODS TO ESTIMATE THE SAFETY-PROBABILITY OF TUNNEL STRUCTURE DURING EARTHQUAKE

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## Abstract

The calculation and design of underground structures subjects to different types of loads, especially the effects of earthquake play an important role in choosing a structural design solution to ensure the safety of people and vehicles. Besides, the mechanical characteristics of the soil around the tunnel can change along the tunnel axis, so the calculation according to the reliability is necessary and important. This article presents the combination of the Monte-Carlo numerical simulation method and the predetermined solution proposed by Wang to calculate the reliability (non-failure probability) of the earthquake-resistant tunnel structure when it comes to randomness of the thickness of the tunnel structure and the characteristics of the surrounding medium.

**Keywords:** *Non-failure probability; reliability; earthquake-resistant underground structures; deformation of the free field.*

## 1. Introduction

In the recent period, the public transport network is becoming more and more important in the overall planning of central urban areas in our country, especially cities with a large population density like Hanoi. According to the master plan on construction of the capital by 2030 with a vision to 2050, the urban railway network in Hanoi will include 8 lines with a total length of about 300 km, [1] in which there will be many tunnels placed in the ground. The calculation and design of underground structures subject to different types of loads, especially the effects of earthquakes, is of great significance in choosing solutions to design the tunnel structure as well as human safety and vehicle.

Based on the point of view of modeling the effects of earthquakes, the methods of calculating earthquake-resistant underground structures are divided into two groups: Group of methods to model the effects of earthquakes through the deformation of the free field (method for imposing seismic ground deformation - ISGD) and the group of methods based on the ground acceleration. Among these methods, the analytical method that belongs to the group that imposes ground deformation proposed by Wang [2] allows for simple and effective calculations, this method is also included in the

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calculation guide, engineering design underground earthquakes of some countries [3-5]. In addition, because some input parameters of the computational model are random, in this article the author presents the use of Wang's analytical method combined with Monte-Carlo's numerical simulation method to investigate the probability of failure (reliability) of underground structures. The selected random variable is the geometrical parameter of the structure and the elastic modulus of the medium.

## 2. Theoretical basis

### 2.1. Wang's method for calculating the internal forces of tunnel structure with circular section in earthquake

In this method, the effect of earthquakes is shown through the shear deformation of the soil and rock medium  $\gamma_{\max}$ . The solution is obtained by analysis on the basis of elastic theory (Figure 1).

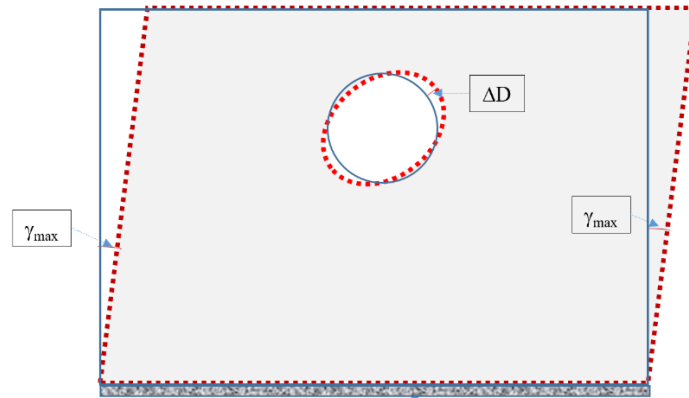


Figure 1. Deformation-based approach solution.

To determine the relative stiffness between the tunnel structure and the surrounding medium, we use two stress transfer ratios: compression ratio ( $C$ ) and bending ratio ( $F$ ):

$$C = \frac{E_m (1 - \nu_1^2) D_l}{2 E_s t (1 + \nu_m) (1 - 2 \nu_m)} \quad (1)$$

$$F = \frac{E_m (1 - \nu_1^2) D_l^3}{48 E_s I (1 + \nu_m)} \quad (2)$$

where  $D_l$  is the tunnel diameter (m);  $t$  is the thickness of the tunnel (m);  $\nu_s$  is the Poisson coefficient of the tunnel structure material;  $E_s$  is the elastic modulus of the structure material;  $E_m$  is the elastic modulus of the soil;  $\nu_m$  is Poisson coefficient of the soil;  $I$  is the moment of inertia of the tunnel cross-section.

Wang's analytic solution in full-slip condition [1] determine the internal forces of the tunnel structure such as maximum normal force  $T_{\max}$ ; maximum bending moment  $M_{\max}$  and the maximum strain in the diametric direction  $\Delta D/D_l$  is determined by formulas below:

$$M_{\max} = \pm K_1 \frac{E_m}{24(1+\nu_m)} D_l^2 \cdot \gamma_{\max} \quad (3)$$

$$T_{\max} = \pm K_2 \frac{E_m}{12(1+\nu_m)} D_l \cdot \gamma_{\max} \quad (4)$$

$$\frac{\Delta D_{\max}}{D_l} = \pm \frac{1}{3} K_1 F \cdot \gamma_{\max} \quad (5)$$

in there  $K_1$  and  $K_2$  are the stress transfer coefficients:

$$K_1 = \frac{12(1-\nu_m)}{2F+5-6\nu_m} \quad (6)$$

$$K_2 = 1 + \frac{F[(1-2\nu_m) - (1-2\nu_m)C] - \frac{1}{2}(1-2\nu_m)^2 C + 2}{F[(3-2\nu_m) + (1-2\nu_m)C] + (\frac{5}{2} - 8\nu_m + 6\nu_m^2) + 6 - 8\nu_m} \quad (7)$$

where the maximum shear deformation is determined by the formula (8):

$$\gamma_{\max} = \frac{v_{s,\max}}{C_s} \quad (8)$$

$$C_s = \sqrt{\frac{E_m}{2 \cdot \rho_m \cdot (1+\nu_m)}} \quad (9)$$

where  $v_{s,\max}$  is peak ground velocity on the ground level;  $C_s$  is the velocity of the shear wave in the ground.

## 2.2. The basis for calculating the probability of failure is based on the Monte-Carlo numerical simulation method

The reliability of a structure is understood in a broad sense as the degree to which the function of the structure can be completed under certain conditions, maintaining the established working parameters within a given limit, for a period of time. Therefore, in a narrow sense, the probability of failure is the reliability of the structure. Monte-Carlo simulation method has a history of development over half a century. Instead of analytic solutions for random variables, the Monte-Carlo method converts to numerical solutions to deterministic solution by generating instances of random variables. The procedure of the Monte-Carlo simulation method includes 3 steps [6]:

- *Step 1:* Numerical simulation of input random variables from their given density functions or probability distribution functions into predetermined values;
- *Step 2:* Compute many times on the predefined model of the system according to the input instances to get the output instances (predetermined);
- *Step 3:* Statistical processing of output instances to find its probabilistic features and test statistical hypotheses. The larger the number of instances (tests) produced, the more accurate the results.

### 2.3. Build the program automatically calculates the probability of non-failure of the tunnel

Investigate the problem of determining the internal forces occurring in the structure of a circular underground structure, subjected to earthquakes. The structure is placed in a homogeneous soft soil layer with a thickness of  $H(m)$  up to the bedrock. Let  $B$  be the load-carrying capacity of the structure, and  $U$  be the effect of the load.

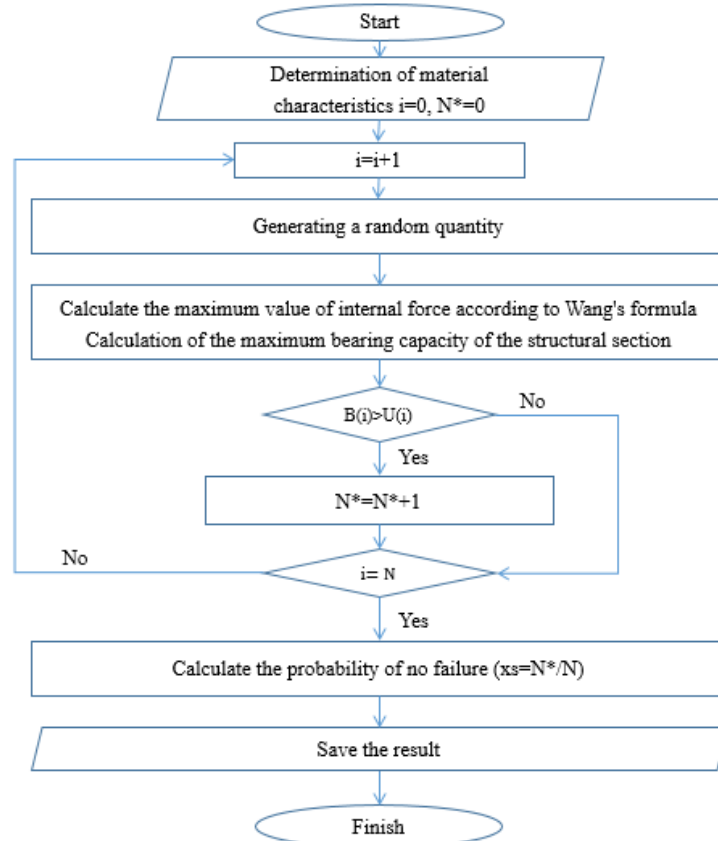


Figure 2. Program block diagram of program TinhtheoWang.

The effect of load  $U$  is the maximum stress appearing in the hazardous section, the maximum moment appearing in the tunnel structure, is determined according to the following parameters by formula (3). The problem of determining the probability of

non-failure according to the ultimate strength state (B) is performed on the cross section reaching the maximum moment. Based on the Monte-Carlo method and Matlab language, the author has built a program TinhtheoWang to automatically calculate the probability of failure of underground structures subjected to the earthquake, the block diagram is shown in Figure 2.

### 3. Result and discussion

The article investigates several numerical examples to show the influence of some parameters on the reliability (non-failure probability) of the tunnel structure is subjected to earthquake. Random parameters were determined including the thickness of the tunnel and the elastic modulus of the soil (original paramamters), assuming a normal distribution [7].

#### 3.1. Investigate the effect of the number of numerical trials

##### *Medium parameters*

Assuming a homogeneous layer on the bedrock, the thickness of this layer is 50 m. The parameters are assumed, including: the elastic modulus of the soil is assumed to be a random quantity with a normal distribution with the expected  $\sigma_{Em} = 1500$  kPa and the coefficient of variation is  $CV_{Em} = 10\%$ ; Poisson's coefficient:  $\nu_m = 0.35$ ; Specific gravity of soil:  $\rho_m = 16.5$  kN/m<sup>3</sup>.

##### *Structural parameters*

Tunnel structure made of reinforced concrete [1], circular cross-section with specific parameters: Inner diameter is  $D_1 = 6$  m; Structural thickness is assumed to be a random quantity with an expected normal distribution:  $\sigma_t = 0.3$  m and coefficient of variation  $CV_t = 10\%$ ; Elastic modulus of structural materials  $E_s = 2.5 \cdot 10^5$  kPa; Poisson's coefficient:  $\nu_s = 0.25$ ; Specific gravity of structural material  $\gamma_s = 25$  kN/m<sup>3</sup>; Yield strength of steel  $R_s = 2.80 \cdot 10^5$  kPa; Compressive strength of concrete  $R_b = 2.89 \cdot 10^4$  kPa.

##### *Earthquake parameters*

The peak ground velocity on the ground level  $v_{s,max} = 0.2$  m/s.

The results obtained by varying the number of trials N are below.

Table 1. Effect of the number of trials on the non-failure probability

N	100	500	1,000	5,000	10,000	50,000	100,000
Xs	0.9246	0.9174	0.9171	0.9125	0.9130	0.9132	0.9125

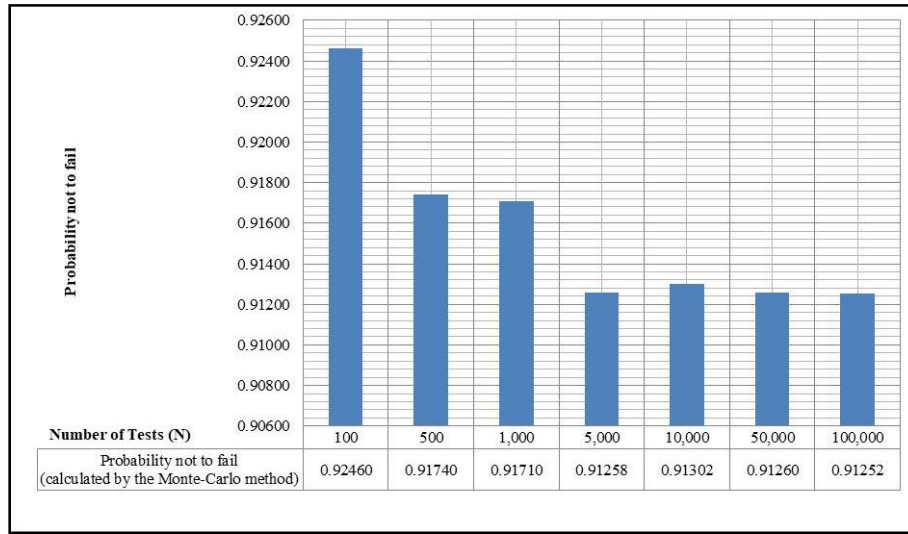


Figure 3. Effect of the number of tests on the non-failure probability.

Through the investigating results, it can be seen that, when the number of tests (N) is increased, the probability of non-failure probability does not change significantly (when the number of tests changes from 500 to 100,000 the difference is less than 1%) therefore, to reduce the calculation time, the number of trials can be chosen as  $N = 1,000$  (for a stable histogram plot, it is at least 1,000 samples [6]).

### 3.2. Evaluation of the effect of maximum velocity on the probability of failure of the tunnel structure

In this part, the author continues to evaluate the non-failure probability of the tunnel structure when changing the peak velocity  $v_{s,max}$  in the range from 0.05 m/s to 0.35 m/s. The coefficient of variation  $CVE_m$  takes the value 10% or 20%. The other input parameters of the soil and the tunnel structure are taken as in Section 3.1 above. The results of calculation are as shown in Table 2 and Figure 4 below.

Table 2. Investigating the effect of the maximum shear wave velocity

$V_{s,max}$	$X_s$ ( $CVE_m=10\%$ )	$X_s$ ( $CVE_m=20\%$ )	$V_{s,max}$	$X_s$ ( $CVE_m=10\%$ )	$X_s$ ( $CVE_m=20\%$ )
0.05	0.9973	0.9975	0.21	0.8830	0.8598
0.06	0.9965	0.9974	0.22	0.8396	0.8012
0.07	0.9961	0.9963	0.23	0.7600	0.7085
0.08	0.9947	0.9962	0.24	0.6434	0.5992
0.09	0.9948	0.9943	0.25	0.4755	0.4584
0.10	0.9941	0.9930	0.26	0.2586	0.2886
0.11	0.9926	0.9909	0.27	0.0721	0.1432

$V_{s,max}$	$X_s$ ( $CVE_m=10\%$ )	$X_s$ ( $CVE_m=20\%$ )
0.12	0.9903	0.9899
0.13	0.9868	0.9849
0.14	0.9846	0.9835
0.15	0.9800	0.9782
0.16	0.9764	0.9735
0.17	0.9642	0.9614
0.18	0.9526	0.9440
0.19	0.9396	0.9227
0.20	0.9171	0.9025

$V_{s,max}$	$X_s$ ( $CVE_m=10\%$ )	$X_s$ ( $CVE_m=20\%$ )
0.28	0.0044	0.0563
0.29	0.0001	0.0113
0.30	0.0000	0.0019
0.29	0.0001	0.0113
0.30	0.0000	0.0019
0.31	0.0000	0.0001
0.32	0.0000	0.0000
0.33	0.0000	0.0000
0.34	0.0000	0.0000

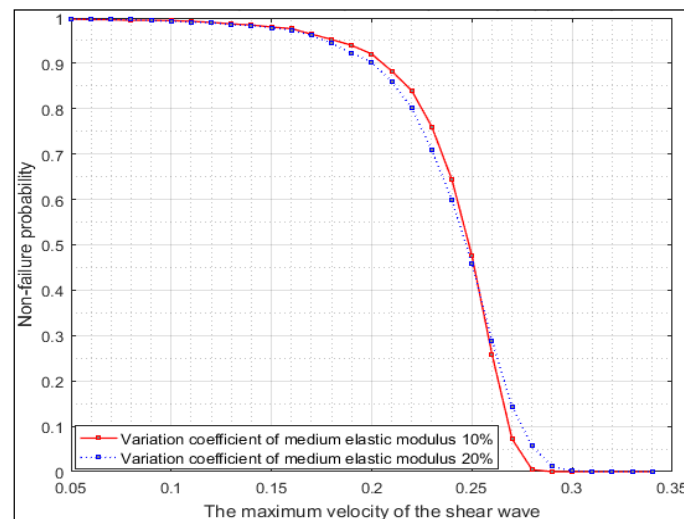


Figure 4. Results of the influence of the peak ground velocity on the probability of failure of the tunnel structure.

From the investigating results, it can be seen that the probability of non-failure of the structure decreased rapidly when the peak ground velocity changes in the range from 0.2 m/s to 0.30 m/s. Therefore, when designing tunnel seismic resistance, a preliminary assessment should be made of the range of  $v_{s,max}$  values that can pose a significant danger.

#### 4. Conclusions

The article presents the combination of the Monte-Carlo numerical simulation method and the predetermined solution proposed by Wang to calculate the non-failure probability of the earthquake-resistant tunnel structure. Over the investigating results, it can be seen that parameters such as the peak ground velocity, the deformation modulus of

soil... have a significant influence on the calculation results. Therefore, in the calculation process, the designer needs to strictly control the input data to minimize the risks.

In addition, the investigating results also show that using the Monte-Carlo numerical simulation method to calculate the non-failure probability of the structure when taking into account the stochastic properties of the structure can help to ensure safety for underground structures.

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## KẾT HỢP PHƯƠNG PHÁP MÔ PHỎNG SỐ MONTE-CARLO VÀ PHƯƠNG PHÁP CỦA WANG ĐỂ TÍNH TOÁN XÁC SUẤT KHÔNG HỎNG CỦA KẾT CẤU VỎ HẦM KHI CHỊU ĐỘNG ĐẤT

Vũ Ngọc Anh

**Tóm tắt:** Việc tính toán, thiết kế công trình ngầm chịu các dạng tải trọng khác nhau, đặc biệt là tác dụng của động đất có ý nghĩa quan trọng trong lựa chọn giải pháp thiết kế kết cấu vỏ hầm nhằm đảm bảo sự an toàn của con người và phương tiện trong đường hầm. Bên cạnh đó, các đặc trưng cơ lý của đất đá xung quanh khoang hầm có thể thay đổi dọc theo trục hầm nên việc tính toán theo độ tin cậy là cần thiết và quan trọng. Bài báo trình bày việc kết hợp phương pháp mô phỏng số Monte-Carlo và phương pháp tính do Wang đề xuất để tính toán độ tin cậy (xác suất không hỏng) của kết cấu vỏ hầm chịu động đất khi kể đến tính ngẫu nhiên của một số tham số tính toán của môi trường và kết cấu.

**Từ khóa:** Xác suất không hỏng; độ tin cậy; công trình ngầm chịu động đất; biến dạng của miền tự do.

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