STABILITY CALCULATION OF PILE-REINFORCED SLOPES BY THE LIMIT EQUILIBRIUM METHOD CONSIDERING SLIDING RESISTANCE OF PILES ACCORDING TO ITO-MATSUI METHOD

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Abstract

In the study, advantages and disadvantages of the calculation methods for overall sliding stability of pile-reinforced slopes considering sliding resistance of pile were summarized and analyzed. An automatic program for stability calculation of pile-reinforced slope was established based on the limit equilibrium method considering sliding resistance of piles according to Ito-Matsui method. The program was used to evaluate the effects of the piles, such as position and spacing of the piles on the stability of the slopes.

Keywords: *Pile-reinforced slope; the limit equilibrium method; stability of the slope; sliding resistance; Ito-Matsui method.*

1. Introduction

Nowadays, solution of using piles to reinforce slopes and improve stability of the slopes is widely applied in the world in construction fields such as river ports, seaports, transportation works, etc. Many successful cases have been reported [7, 8, 10, 12], and numerous methods have been developed for the analysis of pile-reinforced slopes [8, 9, 11, 12]. One of the main effects of the piles that can enhance the stability of the soil slope is through soil arching in which the interslice forces transmitted to the soil slice behind the piles being reduced [9]. In Vietnam, there were some technical standards and geotechnical documents that indicate the method of calculation of the overall sliding stability of construction foundation. However, the only standard so-called 22TCN 207-92 [5] that shows clearly and in detail how to determine the stability of the pile-reinforced slopes. Recently, some authors such as N. Q. V. Le et al. [1] and H. T. Tran et al. [6] , have mentioned the topic of stability of pile-reinforced slopes. Based on numerical tests, their work of research reaffirm the role of the piles in slope stability. The present paper will summarize and analyze the advantages and disadvantages of calculation methods for overall sliding stability of pile-reinforced slopes. From there, the suitable method is chosen which is combined with the limit equilibrium method to establish an automatic program for stability calculation of the pile-reinforced slope.

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2. Calculation methods for overall sliding stability of pile-reinforced slopes

2.1. Method of 22TCN 207-92

According to the guide of 22TCN 207-92 [5], with respect to riverports and seaports, calculation of overall sliding stability of the slopes reinforced with piles based on the ordinary method of slices considering sliding force of the pile expressed as follows:

$$Q_c = \frac{4M_c}{t_z.L} \tag{1}$$

where M_c is the bending moment of the part of the pile below the sliding surface that takes the minimum of the two values determined following two conditions:

a) Strength condition of steel reinforced concrete cross-section of the pile, M_{c1}, is calculated by the formula in TCVN 4116-85 [3].

b) Mounting condition of the pile underneath the sliding surface by a straight segment, $t_z = t_n/1.25$, M_{c2}

$$M_{c2} = \frac{(\sigma_p - \sigma_a).l_c.t_z^2}{8} \tag{2}$$

in which t_n is the length of the portion of the pile under the sliding surface to the tip of the pile; l_c is the length of the straight segment over which the active and passive earth pressures will be transferred to the pile; L is the distance from center to center between two piles; σ_a, σ_p are the active and passive earth pressures at the intersection between the sliding arc and reinforced pile.

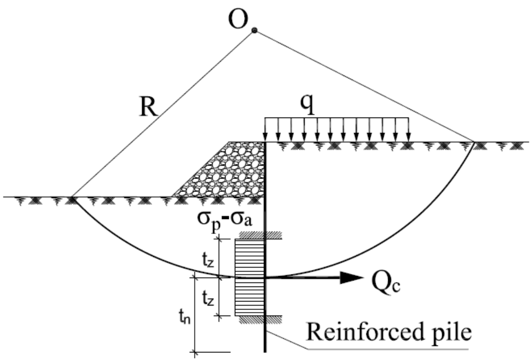

Fig. 1. Slope failure mechanism [5].

2.2. Broms' Method

According to this method, the calculation of overall sliding stability of slopes reinforced with piles is also based on the ordinary method of slices where the presence of the pile on the slope is expressed by a horizontal load capacity of a vertical pile, Q_c, with its length of $L_n = t_n$ (see Fig. 2). Herein, t_n is also the length of the portion of the pile from the sliding surface to the tip of the pile. Q_c is determined from graphs provided in [4] that were established for 2 types of piles, short and long piles.

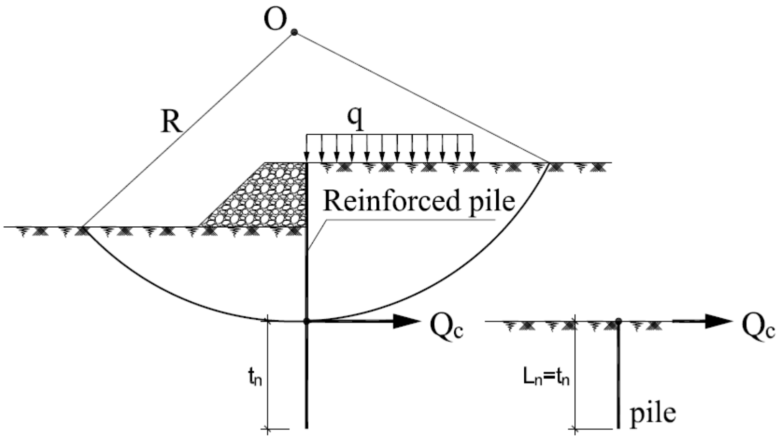

Fig. 2. Slope failure mechanism, Broms' method.

2.3. The ordinary method of slices combined with the finite element method

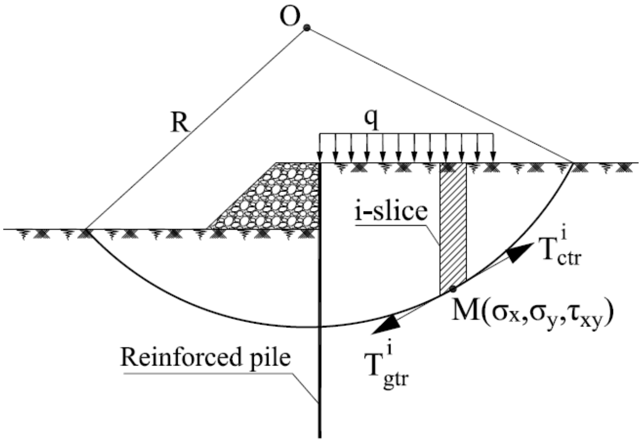

Fig. 3. Slope failure mechanism.

The process of the ordinary method of slices combined with the finite element method is divided into two steps:

Step 1: Analysis of the stress-strain state of the soil and reinforced pile;

Step 2: Assuming sliding arc (center *O*, radius *R*), based on the stress field found in step 1 to determine slip and anti-slip tangential forces at the bottom of soil column (Fig. 3).

The factor of safety is calculated by the following equation:

$$FS = \sum_{i=1}^{n} \frac{T_{ctr}^{i}}{T_{gtr}^{i}} \tag{3}$$

In order to apply this method, we can use the Geostudio software by combining two modules: module Sigma/W (for step 1) and module Slope/W (for step 2).

2.4. Strength Reduction Method (SRM)

The Strength Reduction Method (RSM) is based on the reduction of the sliding strength that is composed of two parameters, unit cohesion (c) and internal friction angle (φ) of the soil. The parameters are reduced in two steps until the soil mass fails. In some numerical softwares such as Plaxis, Geo 5, Phase 2 or Flac, a coefficient exhibits the reduction of the input parameters during the calculation at any stage is defined as follows:

$$FS = \frac{c'}{c} = \frac{\mathrm{tg}\varphi'}{\mathrm{tg}\varphi} \tag{4}$$

where tgφ’ and c’ are the input parameters of the soil, tgφ and c are the reduced parameters calculated by the program.

Calculation results are presented in the form of a graph where the graph shows the effect of the strength reduction coefficient (FS) on the displacement of the considered point (the nodes of the grid of finite elements). Failure criteria of the model are determined according to the Mohr-Coulomb. Once the solution for the final steady state is obtained, the line graph is in horizontal direction, and now the strength reduction coefficient corresponds to the stability coefficient. The sliding surface obtained from this method is formed during the calculation process.

2.5. The limit equilibrium method considering sliding resistance of piles according to Ito-Matsui method

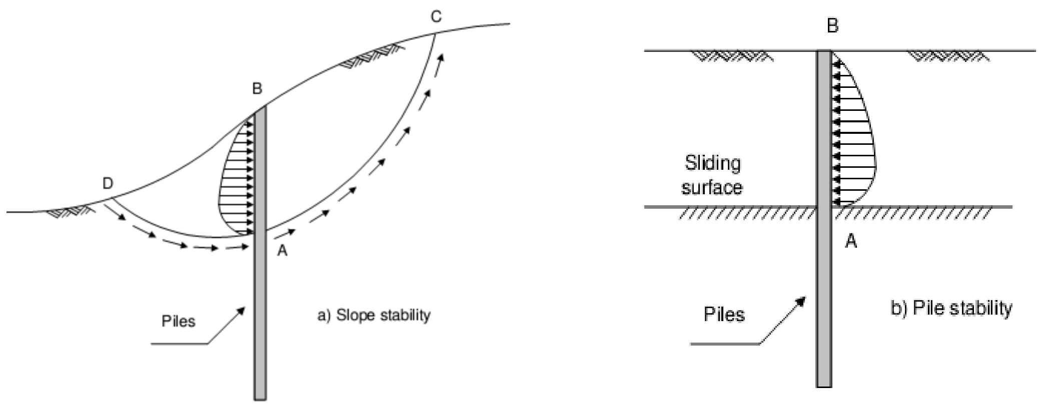

Fig. 4. Slope stability analysis containing piles in a row.

In this approach, calculating sliding stability of pile-reinforced slopes is still based on the limit equilibrium method. However, the presence of the pile on the slope is expressed by sliding resistance of pile that is proposed by T. Ito and T. Matsui.

a) Determine the sliding resistance of the pile according to the Ito-Matsui method

Ito and Matsui [12] analyzed piles placed in the soil based on the theory of plastic deformation (sliding mass area) and the pile against plastic deformation. Following that, the soil between two piles, the area of *ACDFF'D'C'A'*, is illustrated in Fig. 5. The adopted assumptions are expressed in following:

- When the soil deforms, two sliding surfaces occur along the lines *AEB* and A'E'B' in which the lines $\overline{\mathrm{EB}}$ and $\overline{\mathrm{E'B'}}$ make angles ($\pi/4 + \varphi/2$) with the x-axis;

- The plastic flow state of the soil occurs only in the AEBB'E'A' area just around the piles and satisfies the Mohr-Coulomb's yield criterion. Thereafter, the soil is characterized by two parameters: angle of internal friction φ and cohesion c;

- The problem satisfies the plane-strain conditions in the transversely plane;

- The friction on the surfaces *AEB* and *A'E'B'* is ignored in the analysis.

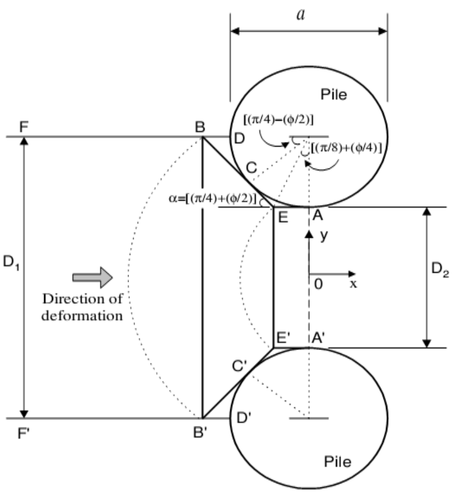

Fig. 5. State of plastic deformation in the ground just around piles [12].

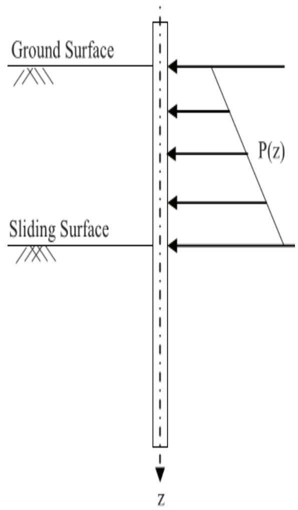

Fig. 6. Lateral pressure on piles due to horizontal soil.

Theoretically, the equation for the lateral force per unit length p is given as follows [11]:

$$\begin{aligned} p(z) = A.c.&\left[\frac{1}{N_\varphi.\mathrm{tg}\varphi}.\left\{\exp\left(\frac{D_1}{D_2}.N_\varphi.\mathrm{tg}\varphi.\mathrm{tg}\left(\frac{\pi}{8}+\frac{\varphi}{4}\right)\right)-2.N_\varphi^{1/2}.\mathrm{tg}\varphi-1\right\}\right.\\ &\left.+\frac{2.\mathrm{tg}\varphi+2.N_\varphi^{1/2}+N_\varphi^{-1/2}}{N_\varphi^{1/2}.\mathrm{tg}\varphi+N_\varphi-1}\right]-c.\left[D_1.\frac{2.\mathrm{tg}_\varphi+2.N_\varphi^{1/2}+N_\varphi^{-1/2}}{N_\varphi^{1/2}.\mathrm{tg}\varphi+N_\varphi-1}-2.D_2.N_\varphi^{-1/2}\right]\\ &+\frac{\gamma.z}{N_\varphi}.\left[D_1.\left(\frac{D_1}{D_2}\right)^{\left(N_\varphi^{1/2}.\mathrm{tg}\varphi+N_\varphi-1\right)}.\exp\left(\frac{D_1-D_2}{D_2}.N_\varphi.\mathrm{tg}\varphi.\mathrm{tg}\left(\frac{\pi}{8}+\frac{\varphi}{4}\right)\right)-D_2\right] \end{aligned} \tag{5}$$

where $N_\varphi = \mathrm{tg}^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)$; $A = D_1.\left(\frac{D_1}{D_2}\right)^{N_\varphi^{1/2}.\mathrm{tg}\varphi+N_\varphi-1}$; D_1 is center-to-center interval between piles in a row; D_2 is clear interval between piles; c is cohesion of the soils; φ is angle of internal friction of the soils.

The use of theory of plastic deformation for analysis of pile-reinforced slopes was first proposed by T. Ito and T. Matsui (1975) and later discussed by De Beer and Carpentier (1977) [11]. The latter authors developed comparable equation by modifying the one of Ito and Matsui and proposed the following equation as another way in order to obtain the lateral force per unit length p induced on the piles [11]:

$$p(z) = \frac{\gamma.z}{N_\varphi}.\left(1+\frac{\sin\varphi}{2}.N_\varphi\right)\left[D_1.\left(\frac{D_1}{D_2}\right)^{F_1(\varphi)}.e^{\frac{D_1-D_2}{D_2}.F_2(\varphi)} - D_2\right]$$
$$+\, c.\cot g\varphi.\left[D_1.\frac{\left(\frac{D_1}{D_2}\right)^{F_1(\varphi)}}{N_\varphi}.\left(1+\frac{\sin\varphi}{2}.N_\varphi\right).e^{\frac{D_1-D_2}{D_2}.F_2(\varphi)} - D_1 - D_2.\frac{1+\frac{\sin\varphi}{2}.N_\varphi}{N_\varphi} + D_2\right] \tag{6}$$

where $F_1(\varphi) = \frac{N_\varphi}{tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)}.(1-\sin\varphi).\mathrm{tg}\varphi + N_\varphi.(1-\sin\varphi)-1$;

$$F_2(\varphi) = \frac{1-\sin^2\varphi}{1+\sin^2\varphi}.\mathrm{tg}\varphi.\mathrm{tg}\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)$$

b) Analysis steps for slopes stability with reinforced piles

- Step 1: Determination of the soil pressure acting on the pile from the bottom of the sliding arc to the natural ground ($\sum p_z$). Soil pressure is calculated according to Ito and Matsui's method (or De Beer and Carpentier's modified formula);

- Step 2: Calculation of sliding stability of pile-reinforced slopes is based on the ordinary method of slices taking into account anti-sliding forces that is soil pressure acting on piles from the bottom of sliding arc;

- Step 3: Analysis of the stability of the piles in an elastic foundation (or nonlinear elastic foundation) subjected to horizontal load. This load is defined in step 1.

2.6. Discussion of calculation methods of stability of pile-reinforced slope

In order to compare the calculation methods aforementioned, the authors conducted numerical tests for graphs of soil pressure acting on the pile. Soil parameters are chosen as follows: γ = 15.6 kN/m^3; φ = 10°; c = 7.5 kPa.

Fig. 7 illustrates the pressure distributions along the depth of the piles. It is seen in the figure that the soil pressure calculated according to Ito-Matsui and De Beer-Carpentier has a just small difference. Meanwhile, the soil pressure obtained from Coulomb's theory is much larger than that of Ito-Matsui and De Beer-Carpentier methods. Note that, Coulomb's model is only appropriate when the piles and soils surrounding are hard. Moreover, it always exists a gap between the piles, so Coulomb's formula cannot be applied to calculate the soil pressure on piles as the one on the earth wall. Fig. 8 shows the effect of the gap between two piles to pressure distributions on the piles (depth of 5m). It is seen that the soil pressure acting on the piles decreases as the gap D_2 increases. The lateral force on the piles becomes infinite if $D_2 = 0$, i.e., the spacing between the piles reaches to zero. This does not reflect real phenomenon. For this reason, it is suggested that the gap (D_2) between two piles should be superior to $2D/3$ [11].

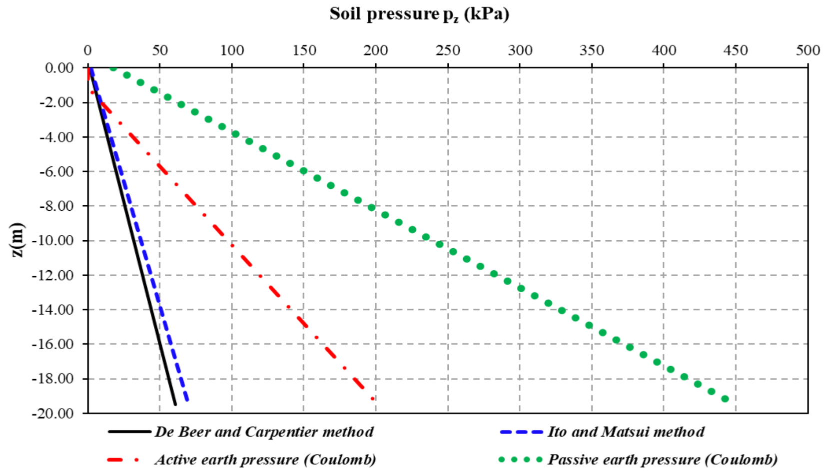

Fig. 7. Pressure distributions along the depth of piles.

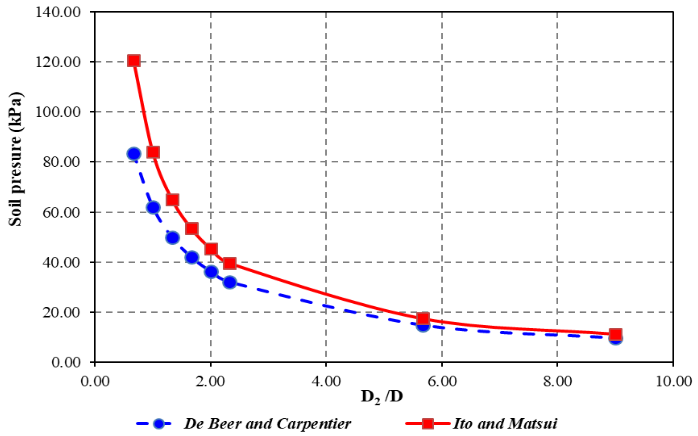

Fig. 8. Surveying the effect of gap between two piles to pressure distributions on piles.

It could give some comments for the methods to evaluate slope stability with reinforced piles aforementioned as follows:

- The method of TCN 207-92 [5] calculates the sliding resistance force (at the position of sliding arc) through the difference between the passive and active effective pressure within the conventional length (pile with 2 virtual fixed points). This method considers the pile row absolute hard and applies the theory of active or passive earth pressure. However, the piles could not be considered absolutely hard, and the reinforced piles are usually arranged with gaps, so the pressure on the pile depends on the spacing between the piles.

- The Broms' method of calculation is complicated. The determination of the horizontal load capacity of the pile is not yet clear because the soil from the bottom of the slide to the natural ground is not considered. In addition, the calculation method does not consider the influence of the distance between the piles and the response constants of the background are not determined experimentally but according to the lookup table with quite large errors.

- The ordinary method of slices combined with the finite element method is used in Geostudio software (combining Sigma/W and Slope/W modules) and the strength reduction method is used in specialized softwares such as Plaxis, Geo5, Phase 2, FLAC... These softwares often require many soil input parameters that are determined difficultly. These input parameters depend on the results of the triaxial compression test and the process of standardisation to appropriately select the behavior model of the soils (Mohr-Coulomb, Hardening soil, soft soil, etc.). To achieve high accuracy, this method requires an engineer to have deep geotechnical expertise, really knowledgeable in analyzing triaxial compression test data and the nature of soil models.

- The limit equilibrium method to determine the overall sliding stability of pile-reinforced slopes considering sliding resistance of pile according to Ito-Matsui method has all the advantage and disadvantage of the limit equilibrium method. The disadvantage is due to relying on the assumptions that make the problem simpler, so the obtained results of the stability coefficient could be overestimated whereas the advantage is simplification in use. Concretely, this method allows considering the effects of permeability, load conditions, and different soil conditions without additional calculations. T. Ito and T. Matsui conducted a series of experiments to validate the proposed analytical formulas [12]. In addition to the analysis of the slope stability taking into account sliding resistance, this method also considers the problem of the stability of the pile subjected to horizontal load.

From some reasons above, the authors chose the limit equilibrium method combined with the sliding resistance of the pile according to Ito-Matsui method to develop an automatic program to analyze stability of the pile-reinforced slope.

3. Establishment of automatic program for stability calculation of the pile-reinforced slope

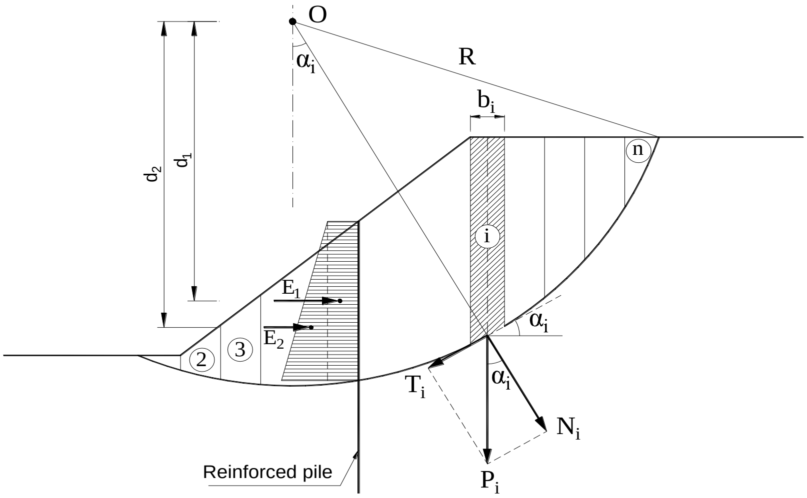

Fig. 9. Stability calculated diagram of slopes reinforced with piles.

To build the program, the authors rely on the limit equilibrium method such as Fellenius or Bishop method. In the calculation process the sliding resistance of piles is supplied, which is the soil pressure (according to T. Ito and T. Matsui or De Beer-Carpentier) calculated from the position of the pile-slide surface intersection to the top of the pile. The pressure diagram is in the form of a trapezoid. For the sake of simplification in determination of center of gravity, one divides the trapezoid into 2 sub-geometries, triangle and rectangle.

On the basis of such approach, the authors developed a calculation program that based on the source code of the program SSSV (Slope Stability Software of Vietnamese) [2]. Not only the sliding strength of the soil but also soil pressure due to the piles are supplied to the anti-sliding moment E_1 (rectangular diagram) and E_2 (triangle diagram). Assuming that interactions between slices are neglected as in Fellenius method. The algorithm diagram of program is shown in Fig. 10 and stability coefficient is calculated by the following equation:

$$K = \frac{E_1.d_1 + E_2.d_2 + \sum_{i=1}^{n}\left(P_i.\cos\alpha_i.\mathrm{tg}\varphi_i + C_i.l_i\right)}{\sum_{i=1}^{n} P_i.\sin\alpha_i} \tag{7}$$

where P_i is weight of slice; C_i is effective cohesion; φ_i is effective internal friction angle; b_i is width of slice; $l_i = b_i/\sin\alpha_i$ and α_i is the angle of base.

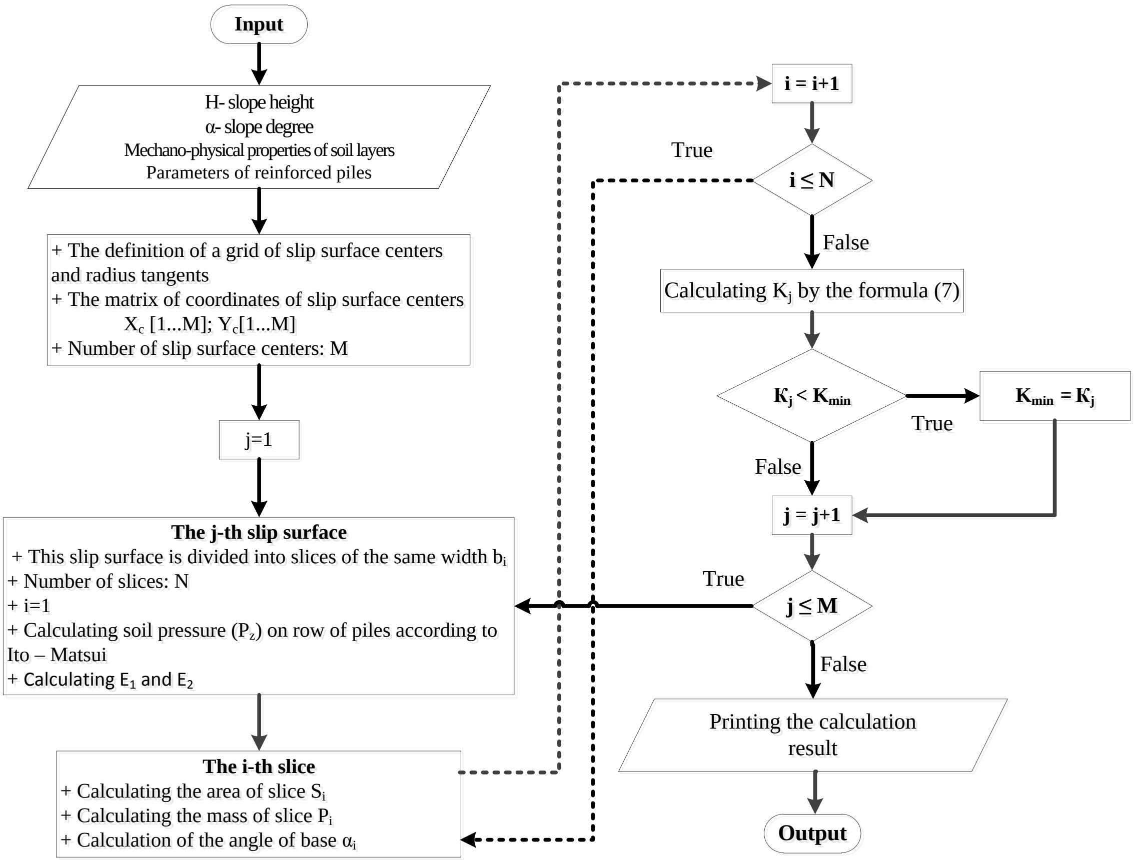

Fig. 10. Block diagram of program (Fellenius method).

In the next stage, the authors will use the established program to investigate the pile parameters affecting the slope stability. An estimated slope consists of 5 soil layers with the mechano-physical properties as shown in Table 1.

Table 1. Mechano-physical properties of soil layers.

Soil layer	Unit weight, γ	Internal friction angle, φ	Cohesion, c
	kN/m^3	degree	kPa
1	18.00	12	9.00
2	15.60	7	7.00
3	16.00	15	8.00
4	16.50	18	8.50
5	17.00	25	5.00

Parameters of reinforced pile are:

+ Pile diameter: D = 30 cm;

+ Pile length: L = 10 m;

+ Clear interval between piles: $D_2 = 3D = 0.9$ m;

Three problems were analyzed in following:

a) Problem 1: Evaluation of the stability level of the slope without or with a row of reinforced piles

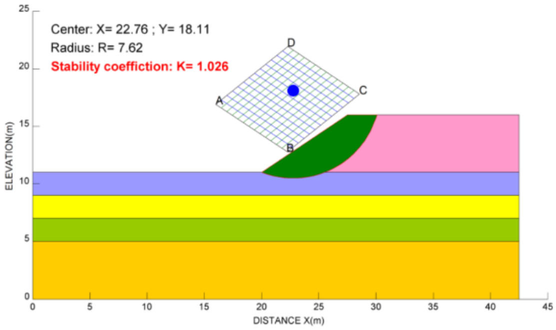

Fig. 11. Stability level of the slope without a row of reinforced piles.

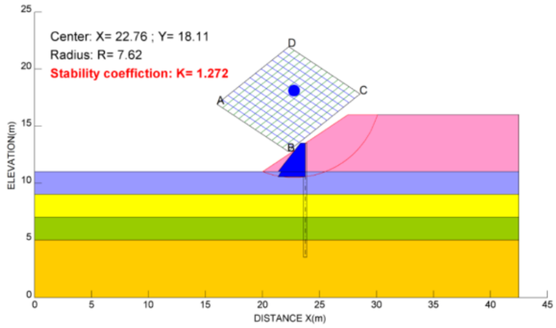

Fig. 12. Stability level of the pile-reinforced slope (a row of piles in the middle slope).

Stability levels of the slope without and with a row of reinforced piles are shown in Fig. 11 and Fig. 12 respectively. It can be seen in Fig. 11 and Fig. 12 that, the stability coefficient of the slope without pile-reinforced is K = 1.026, and in case of the slope reinforced with a row of piles in the middle part of the slope, the stability coefficient increases by the amount of 24%, i.e., K = 1.272.

b) Problem 2: Evaluation of the influence of the position of a row of piles on the stability coefficient of the slope

Table 2. Correlation of position of pile to stability coefficient of slope.

Case	Coordinates of pile top position		Stability coefficient
	X (m)	Y (m)	
1	20.97	11.65	1.069
2	21.74	12.16	1.121
3	22.72	12.81	1.196
4	23.19	13.13	1.232
5	23.75	13.50	1.272
6	24.21	13.80	1.301
7	24.95	14.30	1.339
8	25.71	14.81	1.332
9	26.33	15.22	1.244
10	26.88	15.59	1.182

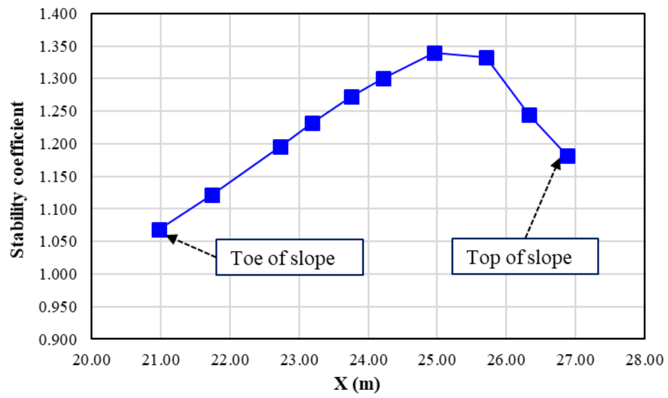

Fig. 13. Effect of the position of a row of piles on the stability coefficient of slope.

Table 2 and Figure 13 present the influence of the position of a row of piles on the stability coefficient of the slope. They show that, the position of piles affects the stability coefficient of the slope. Relationship between the position of piles and the stability coefficient is in a parabolic shape. As the row of the piles reaches to the vicinity of the toe of the slope, the stability coefficient decreases. The safety factor takes the maximum value corresponding to the row of the piles in the middle part of the slope.

c) Problem 3: Evaluation of the influence of spacing of piles on the stability coefficient of the slope

Table 3 and Figure 14 are the results of the evaluation of influence of spacing of piles on the stability coefficient of the slope. According to Ito-Matsui's formula, the soil pressure increases as the distance between of the piles decreases. Therefore, the distance between the piles also affects the stability coefficient of the slope. Concretely, Table 3 and Figure 14 show that, the stability coefficient increases as the spacing between of pile decreases.

Table 3. Correlation of spacing of piles to stability coefficient of slope.

Case	Coordinates of pile top position		Length of pile L (m)	Diameter of pile D (m)	Center-to-center interval between piles D_1 (m)	D_1/D	Stability coefficient
	X (m)	Y (m)					
1	23.75	13.50	10.00	0.30	0.60	2.00	1.468
2	23.75	13.50	10.00	0.30	0.66	2.20	1.406
3	23.75	13.50	10.00	0.30	0.72	2.40	1.360
4	23.75	13.50	10.00	0.30	0.78	2.60	1.324
5	23.75	13.50	10.00	0.30	0.84	2.80	1.296
6	23.75	13.50	10.00	0.30	0.90	3.00	1.272
7	23.75	13.50	10.00	0.30	0.96	3.20	1.253
8	23.75	13.50	10.00	0.30	1.02	3.40	1.236
9	23.75	13.50	10.00	0.30	1.08	3.60	1.222
10	23.75	13.50	10.00	0.30	1.20	4.00	1.199
11	23.75	13.50	10.00	0.30	2.20	7.33	1.130

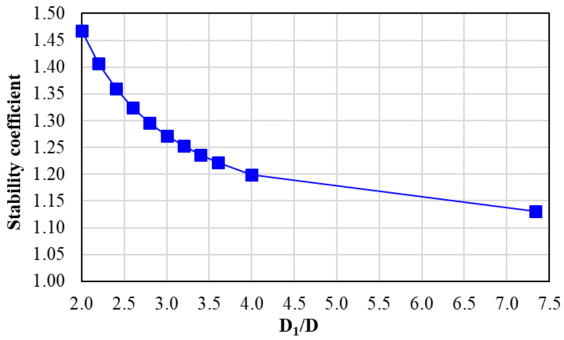

Fig. 14. Effect of spacing of piles on the stability coefficient of slope.

4. Conclusion

- The slopes of the river ports, seaports, and of the road can be reinforced by piles. In this case, the stability of the slope is significally improved.

- A automatic program for stability calculation of pile-reinforced slope was established based on the limit equilibrium method considering sliding resistance of piles according to Ito-Matsui method. The program could be useful for design and calculation.

- The greater the spacing between the piles the lower the stability coefficient obtained and vice versa. Choosing a suitable distance between the piles is an economic and technical problem.

- The position of the row of reinforced piles in the middle of the slope would give the highest stability coefficient.

References

1. Lê Nguyễn Quốc Việt, Đỗ Hữu Đạo, Bùi Phú Doanh (2011). Nghiên cứu ứng dụng phương pháp cọc kháng trượt trong ổn định mái ta-luy. *Tạp chí Khoa học và Công nghệ, Đại học Đà Nẵng, 3*(44), 102-110.
2. Phạm Đức Tiệp, Nguyễn Tương Lai (2011). *Xây dựng phần mềm tự động hóa tính toán ổn định trượt tổng thể nền móng công trình*. Đề tài Khoa học và công nghệ cấp Học viện (Học viện KTQS).
3. TCVN 4116-85. *Tiêu chuẩn thiết kế công trình thủy lợi - Kết cấu bê tông và bê tông cốt thép thủy công - Tiêu chuẩn thiết kế*.
4. TCXD 205:1998. *Móng cọc - Tiêu chuẩn thiết kế*.
5. Tiêu chuẩn ngành 22TCN 207-92. *Công trình bến cảng biển - Tiêu chuẩn thiết kế*.
6. Trần Huy Thanh (2010). Phân tích ổn định mái dốc có sử dụng cọc đứng. *Tạp chí Khoa học và Công nghệ Hàng hải*, *8*(23), 83-89.
7. E. E. De Beer and M. Wallays (1970). Stabilization of a slope in schist by means of bore piles reinforced with steel beams. *Proceedings of the 2nd International Conference on Rock Mechanics*, *3*, 361-369, Belgrade, Serbia.
8. H. G. Poulos (1995). Design of reinforcing piles to increase slope stability. *Canadian Geotechnical Journal*, *32*(5), 808-818.
9. R. Liang, S. Zeng (2002). Numerical study of soil arching mechanism in drilled shafts for slope stabilization. *Soils and Foundations*, *42*(2), 83-92.
10. R. Merriam (1960). Portuguese bend landslide, Palos Verdes Hills, California. *The Journal of Geology*, *68*(2), 140-153.
11. S. Firat (2009). Stability analysis of pile-slope system. *Scientific Research and Essay*, *4*(9), 842-852.
12. T. Ito and T. Matsui (1975). Methods to estimate lateral force acting on stabilizing piles. *Soils and Foundations*, *15(*4), 43-59.

TÍNH TOÁN ỔN ĐỊNH MÁI DỐC CÓ CỌC GIA CƯỜNG BẰNG PHƯƠNG PHÁP CÂN BẰNG GIỚI HẠN CÓ XÉT ĐẾN LỰC KHÁNG TRƯỢT CỦA CỌC THEO PHƯƠNG PHÁP ITO-MATSUI

Tóm tắt: *Bài báo tổng hợp, phân tích ưu nhược điểm của các phương pháp tính toán ổn định trượt tổng thể nền móng công trình có xét đến sự gia cường của cọc. Chương trình tính toán ổn định mái dốc có cọc gia cường được thiết lập trên cơ sở phương pháp cân bằng giới hạn kết hợp bổ sung lực kháng trượt của hàng cọc theo phương pháp Ito-Matsui. Sử dụng chương trình khảo sát các yếu tố của cọc gia cường ảnh hưởng đến hệ số ổn định của mái dốc như vị trí bố trí cọc trên mái dốc, khoảng cách giữa các cọc.*

Từ khóa: Mái dốc có cọc gia cường; phương pháp cân bằng giới hạn; ổn định của mái dốc; sức kháng trượt; phương pháp Ito-Matsui.

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