IDENTIFY THE NATURAL FREQUENCIES OF STRUCTURES BY FREQUENCY DOMAIN DECOMPOSITION METHOD

Tran Trung Duc[1,*], Le Anh Tuan[1], Vu Dinh Huong[1], Nguyen Cong Nghi[1]

[1]*Le Quy Don Technical University*

Abstract

Natural frequencies are important dynamic characters of building structures and can be determined by analytical or experimental methods. Over time, under the effect of loads, environment, random factors..., the characteristics of the building structure are changed, leading to a change in the dynamic characteristics. The paper presents how to determine the natural frequencies of the building structures by the frequency domain decomposition (FDD) method. This method belongs to the group of Operational Modal Analysis (OMA) method, which only uses vibration measurement data of structures to determine the natural frequencies without knowing forces acting on the structure.

Keywords: *Natural frequencies; EMA; OMA; FDD.*

1. Introduction

When designing or testing any structure, it's important to determine dynamic parameters and must be done first. In particular, the natural frequencies of the structure is an important parameter in the analysis, design, and testing of the project. There are many methods of identifying the dynamic characteristics of structural structures. According to measurement data characteristics, there is time domain method [6] and frequency domain method [4]. The time domain methods often require prolonged measurement time and are sensitive to noise, so frequency domain methods are more common. According to the measurement data source, there are groups of identification methods based on input stimulation and measurement of the dynamic response of the structure (Input-Output), also known as experimental modal analysis (EMA) [5], this method requires knowing the factors affecting the structure and measuring the dynamic response of the structure to construct a descriptive transfer function for the structure. Group of identification methods on the basis of using only the dynamic response measurement data of the structure (Output-Only) also known as the method of Operational Modal Analysis (OMA) [3, 4, 7]. In OMA, the excitation forces are indeterminate or impossible to measure; the only information is the measure of the dynamic response of the structure. However, if the acting forces are assumed to be in the form of white noise and are randomly distributed over the space surrounding the

* Email: ducmta93@gmail.com
https://doi.org/10.56651/lqdtu.jst.v3.n02.155.sce

structure, then structural dynamic response measurements will contain all the necessary information to describe the building structure. This is obviously a big advantage of the OMA method because we do not use any expensive stimulants, but sometimes artificial devices can damage the building during the experiment. Instead, we take advantage of vibrations caused by environmental loads on the structure, such as the effect of wind, movement of traffic, or other effects, and can identify the dynamic parameters of the building without interrupting the operation of the building. Frequency domain decomposition (FDD) is one of the techniques of the OMA identification method.

The paper conducts vibration measurements and determines the natural frequencies of steel beam structures by frequency domain decomposition method (FDD).

2. Frequency Domain Decomposition (FDD) method

Frequency domain decomposition is proposed by Brincker et al. [4]. This method decomposes the spectral density matrix at each frequency into singularity values and singularity vectors by the singular value decomposition (SVD). Frequency domain decomposition is an extension of the basic frequency domain technique or commonly known as the Pick Peaking technique, in which natural frequencies is identified by finding peaks in the spectral density matrix.

2.1. Theoretical basis

The relationship between unknown input x(t) and measured response output y(t) can be expressed as follows:

$$[G_{yy}(\omega)] = [H(\omega)]^{*}[G_{xx}(\omega)][H(\omega)]^{T} \tag{1}$$

where $[G_{xx}(\omega)]$ is the Power Spectral Density (PSD) matrix of the input; $[G_{yy}(\omega)]$ is the PSD matrix of the responses; $[H(\omega)]^{*}$ is the complex conjugate matrix of Frequency Response Function (FRF); $[H(\omega)]^{T}$ is the transpose matrix of FRF.

The FRF can be written in prutial fraction

$$\left[H(\omega)\right] = \sum_{1}^{N} \frac{[\mathrm{R}_k]}{j\omega - \lambda_k} + \frac{[\mathrm{R}_k]^{*}}{j\omega - \lambda_k^{*}} \tag{2}$$

$$\lambda_k = -\sigma_k + j\omega_{dk} \tag{3}$$

where n is the number of modes, λ_k is the pole of the k^{th} mode shape, σ_k is minus the real part of the pole and ω_{dk} is the damped natural frequencies of the k^{th} mode shape.

$[\mathrm{R}_k]$ is the residue expressed as follows:

$$[\mathrm{R}_k] = \phi_k . \gamma_k^{T} \tag{4}$$

where ϕ_k is the mode shape vector, γ_k the modal participation vector.

Suppose the input is white noise, its power spectral density is constant or $[G_{xx}(\omega)] = C$, (C is constant). Formula (1) is rewritten as follows:

$$[G_{yy}(\omega)] = \sum_{1}^{N}\sum_{1}^{N}\left[\frac{[\mathrm{R}_k]}{j\omega-\lambda_k}+\frac{[\mathrm{R}_k]^*}{j\omega-\lambda_k^*}\right].C.\left[\frac{[\mathrm{R}_k]}{j\omega-\lambda_k}+\frac{[\mathrm{R}_k]^*}{j\omega-\lambda_k^*}\right]^T \tag{5}$$

Multiplying the two partial fraction factors and making use of the Heaviside partial fraction theorem, after some mathematical manipulations, the output PSD can be reduced to a pole/residue form as follows:

$$[G_{yy}(\omega)] = \sum_{1}^{N}\frac{[A_k]}{j\omega-\lambda_k}+\frac{[A_k^*]}{j\omega-\lambda_k^*}+\frac{[B_k]}{-j\omega-\lambda_k}+\frac{[B_k^*]}{-j\omega-\lambda_k^*} \tag{6}$$

where $[A_k]$ is the k^{th} residue matrix of the output PSD.

At a certain frequency ω only a limited number of modes will contribute significantly, typically one or two modes. Thus, in the case of a lightly damped structure, the response spectral density can always be written:

$$[G_{yy}(\omega)] = \sum_{k\in Sub(\omega)}\frac{d_k\phi_k\phi_k^T}{j\omega-\lambda_k}+\frac{d_k^*\phi_k^*\phi_k^{*T}}{j\omega-\lambda_k^*} \tag{7}$$

where $k \in$ Sub(ω) is the set of modes be denoted at a specific frequency, ϕ_k is the mode shape vector and λ_k is the pole of the k^{th} mode shape.

The Frequency domain decomposition technique is based on the singular value decomposition of the Hermitian response spectral density matrix.

$$[G_{yy}(\omega)] = [U][S][U]^H \tag{8}$$

where [S] is a diagonal matrix holding the scalar singular values, [U] is a unitary matrix holding the singular vectors and $[U]^H$ is a Hermitian matrix.

From vibration measurement data of the structure (acceleration), we calculate the spectral density matrix $[G_{yy}(\omega)]$ and decompose the singular value according to formula (8) to determine the natural frequencies of the structure.

3. Determination of natural frequencies by experiment

3.1. Test objectives

The test to obtain dynamic responses (acceleration) of steel beam structures at nodes over time. The result of vibration measurement is used to identify the natural frequencies of the structure.

3.2. Test model

Test structure is a steel beam. The physical parameters of the structure are shown in Table 1.

Table 1. The physical parameters of the test structure

No.	Parameter	Value	Unit
1	Length	710	mm
2	Density weight	7850	Kg/m^3
3	Modulus of elasticity	$2.03 \cdot 10^5$	MPa
4	Width	60	mm
5	Height	8	mm

3.3. Test equipment

The equipment used in the test is listed in Table 2.

Table 2. Test equipment

No.	Equipment name	Code	Company	Measuring range	Quantity
1	Vibration measurement equipment	NI cDAQ-9137	National Instrument	Multi - channel	01
2	Accelerometer	PCB 352C68	PCB Group	±50g (100 mV/g)	01
3	Accelerometer	PCB 353B33	PCB Group	±50g (100 mV/g)	01

3.4. Test layout

The test layout for determining the natural frequencies of the steel beam is arranged as shown in Figure 1. In which, using two accelerometer sensors to measure the vibration of the beam, the position of the sensors is shown in Figure 2, the NI cDAQ-9137 Connected with accelerometer sensors and display. Accelerometer measurements are collected and displayed through the NI Signal Express software pre-installed.

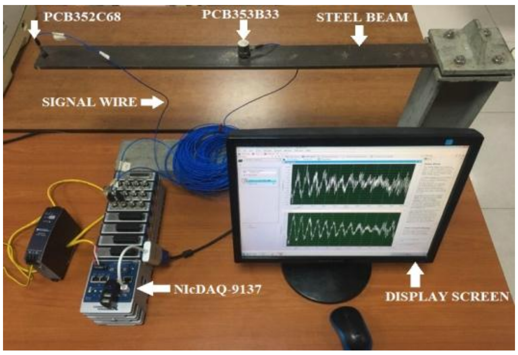

Figure 1. Experiment setup of the structure

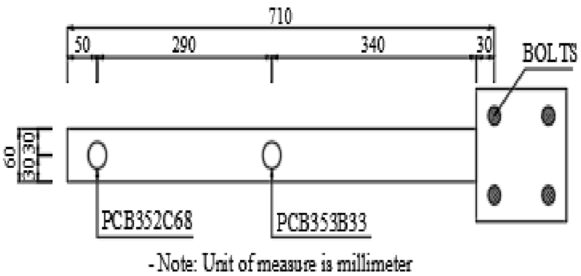

Figure 2. The position of the sensors

3.5. Test methods

Proceed with the installation and install parameters for measuring equipment, Create vibration for the structure by any stimulus is large enough for the structure to work in the elastic stage. The measured data is recorded as the value of the acceleration overtime at the location where the acceleration is mounted.

4. Test results

After measuring the vibration of the structure, acceleration at the nodes on the steel girder structure is obtained over time. The data of one measurement is shown in Figure 3, Figure 4.

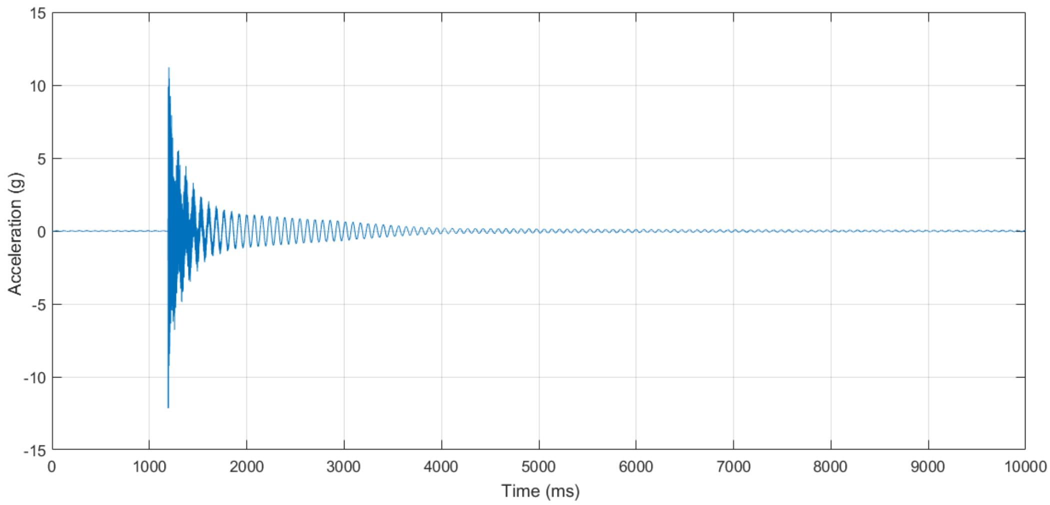

Figure 3. Results of acceleration at the middle of the beam

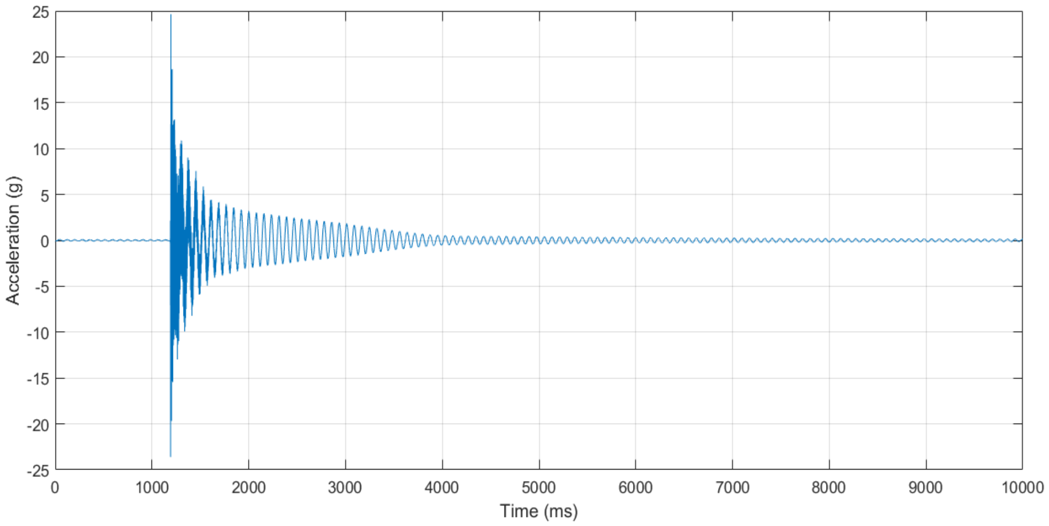

Figure 4. Results of acceleration at the free position of the beam

With the acceleration data obtained from the experiment, calculate and estimate the power spectral density according to Welch's estimation method and resolve the singularity values by SVD algorithm according to formula (8). We determine the natural frequencies of the structure corresponding to the positions of the maximum power spectral density function. Results of identifying the five natural frequencies are shown in Figure 5.

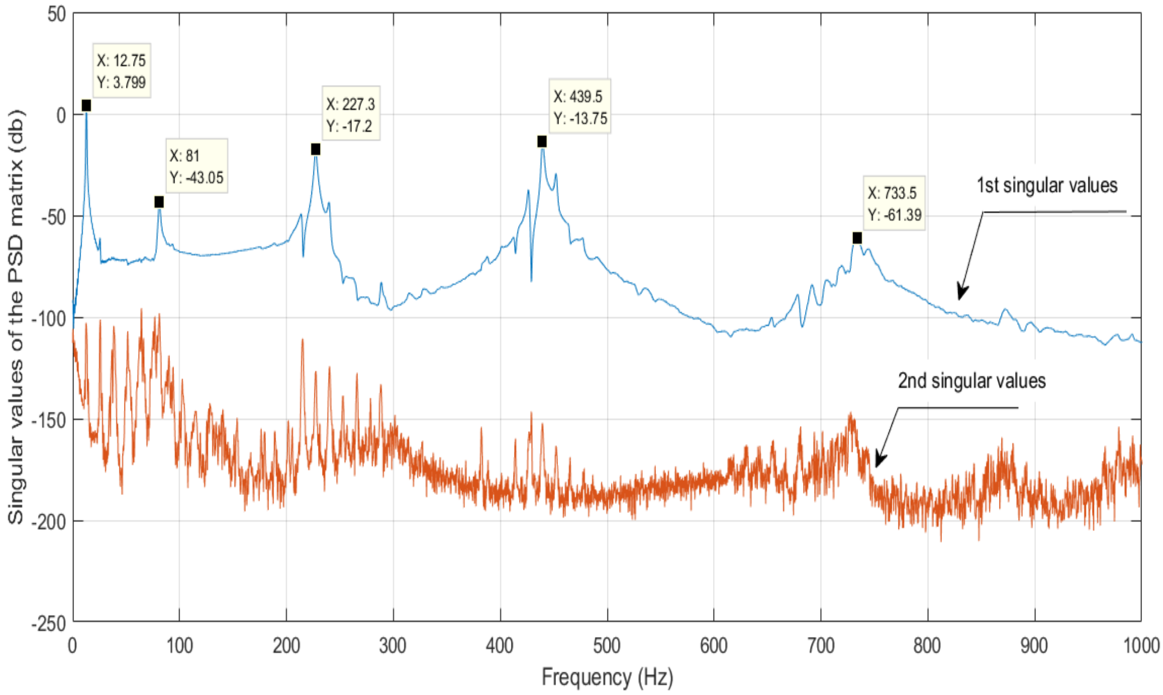

Figure 5. Power spectral density (PSD)

Comparing the natural frequencies obtained by the FDD method and the results of the calculation of the natural frequencies by the experimental modal analysis (EMA) method [2] and according to theory [1] are shown in the Table 3.

The deviation in the results of identifying natural frequencies of the structure by the FDD method compared with other calculation methods is shown in the following formula.

$$\Delta = \frac{f_{FDD} - f_K}{f_K} \times 100\ (\%) \tag{9}$$

where f_{FDD} is the natural frequencies of the structure determined by FDD method; f_K is the natural frequencies of the structure determined by other methods.

Table 3. Comparison of natural frequencies between methods

No.	Mode	FDD (Hz)	EMA (Hz)	Error (%)	Theory (Hz)	Error (%)
1	1	12.75	12.8	0.4	12.9	1.2
2	2	81.0	79.8	1.5	80.9	0.1
3	3	227.3	228.6	0.6	226.6	0.3
4	4	439.5	446.1	1.5	444	1.01
5	5	733.5	735.6	0.3	734	0.07

From the comparison results in Table 3, it shows that the results of identification by the frequency domain decomposition method (FDD) are very close to the results calculated by the experimental modal analysis (EMA) and the theory method. FDD gives highly accurate results.

5. Conclusion

The paper presents the content of the experiment measures frequencies of the steel beam structure and uses the frequency domain decomposition method to identify the natural frequencies of the structure.

The results of the identification of the natural frequencies by the frequency domain decomposition method are consistent with the natural frequencies obtained by the experimental modal analysis method and calculated theoretically, with small errors. This shows the reliability of the experimental method and the identification method.

The frequency domain decomposition method (FDD) can be used to identify other vibration characteristics such as mode shape, damping ratios and can be used in monitoring the technical state of the structure during the process of work.

References

1. Phạm Đình Ba, Nguyễn Tài Trung (2010). *Động lực học công trình*. Nxb Xây dựng, Hà Nội.
2. Tạ Đức Tuân, Lê Anh Tuấn, Vũ Đình Hương (2017). Nhận dạng tần số dao động riêng của kết cấu bằng phương pháp kích động cưỡng bức. *Tạp chí Khoa học công nghệ Xây dựng*, số 1/2017, tr. 27-31.
3. Brincker R., Ventura C. (2015). *Introduction to Operational Modal Analysis*, 1st edn., New York: Wiley.
4. Brincker R., Zhang L., Andersen P. (2001). Modal identification of output-only systems using frequency domain decomposition. *Smart Mater. Struct., 10*, 441-445.

5. Ewins, D. J. (2000). *Modal testing, theory practice and application,* 2nd edition. Research Studies Press, Ltd.
6. Ljung, L. (1987). *System Identification, theory for the user*. Prentice-Hall, Inc.
7. Zhang, L., Brincker, R. and Andersen, P. (2005). An overview of operational modal analysis: Major developments and issues. *Proceedings of the 1st International Operational Modal Analysis Conference (IOMAC)*, Apr. 26-27, Copenhagen.

XÁC ĐỊNH TẦN SỐ DAO ĐỘNG RIÊNG CỦA KẾT CẤU BẰNG PHƯƠNG PHÁP PHÂN RÃ MIỀN TẦN SỐ

Tóm tắt: *Tần số dao động riêng là một đặc trưng động lực học quan trọng của kết cấu công trình và có thể xác định bằng phương pháp giải tích hoặc thực nghiệm. Theo thời gian, dưới tác dụng của tải trọng, môi trường, các yếu tố ngẫu nhiên..., các đặc trưng của kết cấu công trình bị thay đổi dẫn tới sự thay đổi các đặc trưng động lực học. Bài báo trình bày cách xác định tần số dao động riêng của kết cấu bằng phương pháp phân rã miền tần số (FDD). Phương pháp này thuộc nhóm các phương pháp phân tích Model hoạt động (OMA), chỉ sử dụng dữ liệu đo rung động của kết cấu để xác định tần số dao động riêng tức là không cần biết lực kích thích tác động lên kết cấu.*

Từ khóa: Tần số dao động riêng; EMA; OMA; FDD.

Received: 15/10/2020; *Revised:* 25/11/2020; *Accepted for publication:* 23/12/2020

❑