

FREE VIBRATION ANALYSIS OF A MULTI-STOREY SPACE FRAME SYSTEM

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Abstract

This paper carries out the vibration analysis of a high-storey space frame system. The finite element formulations are based on classical space frame theory and the finite element method. Then the calculation program is established in the MATLAB environment. The computed results are compared with those of exact solutions to demonstrate the accuracy of the program. In addition, we use the artificial neural network to predict the natural frequencies of the structure. The parameter study is conducted to evaluate the effect of geometrical coefficients and boundary conditions on vibration responses of the structure. These numerical results are a good basis to develop many complicated problems dealing with artificial intelligence (AI) to predict mechanical responses of structures.

Keywords: *Vibration; space frame system; artificial neural network; artificial intelligence (AI); finite element method.*

1. Introduction

For many decades, space frame systems have been one type of common structures in engineering applications, especially in the construction area and marine technology. Due to the development of science and technology, buildings and structures can be higher than those existing today. Therefore, we need to control the mechanical responses of these structures better. That is why numerous works dealt with these problems. Kwong-Wing Chau et al. [1] examined how building height was determined in the absence of building height regulatory restrictions. A model was developed for determining optimal height using simple neo-classical economic analysis; this was then tested using empirical data from Hong Kong. Hasan and co-workers [2] investigated the effect of non-sway and sway methods for analysis and design of reinforced concrete frames for multi-storey building. Sultan and Peera [3] carried out a dynamic analysis of multi-storey building for different shapes. Treloar and his colleagues [4] introduced an analysis of the embodied energy of office buildings by height. Nguyen Thai Chung et al. [6] studied the dynamic response of a high building on an elastic foundation with cracks in the column of the building under earthquake loading using the finite element method (FEM). The authors used a 3D modeling frame - plate system and integration method

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developed by Newmark. In the work [5], Nguyen Chi Tho presented new numerical results from simulations of beams and space frame systems with a tuned mass damper. In their work, static and dynamic equations were formed through the finite element method. In addition, they also established artificial neural networks (ANNs) in order to predict the vibration response of the first frequencies of the structure. Numerical studies showed that the data set of the TMD device strongly affected the first frequencies of the mechanical system, and the proposed artificial intelligence (AI) model could predict exactly the vibration response of the first frequencies of the structure. Mehdi and his co-workers [7] employed an artificial neural network to model the frequency of the first mode, using the beam length, the moment of inertia, and the load applied on the beam as input parameters on a database of 100 samples. Three different heuristic optimization approaches were used to train the ANN: genetic algorithm (GA), particle swarm optimization algorithm and imperialist competitive algorithm. The suitability of these algorithms in training ANN was determined based on accuracy and runtime performance. To our knowledge, it seems to be not many works dealing with high space frame structures using AI to control mechanical responses. Thus this work firstly presents vibration analysis of a high-storey space frame system. Besides, we employ AI to predict natural frequencies of the structure. The calculation program and numerical results are an important basis for further works.

This paper is structured by 5 main sections. Finite element formulations and artificial neural networks (ANNs) are presented in section 2 and section 3, respectively. Verification problems and numerical results are introduced in section 4. Section 5 sums up some highlight works of this investigation.

2. Finite element formulations

Consider a space frame element of the mechanical system (Figure 1). At any point on the frame element, there is a local coordinate x that has an unknown displacement vector $\mathbf{u}(x, t)$.

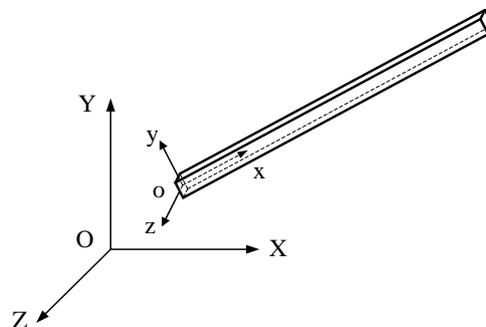


Figure 1. A model of the space frame element.

$$\mathbf{u} = [u \quad v \quad \theta_z \quad w \quad \theta_y \quad \varphi]^T \quad (1)$$

in which u , v , w , and φ are the displacements along the x , y , and z directions and the rotation angle of cross area around the x -axis, respectively. θ_z , θ_y are the angular components corresponding to the displacements u and w .

$$\theta_z = \frac{dv}{dx}; \theta_y = \frac{dw}{dx} \quad (2)$$

Consider \mathbf{q} is the nodal displacement of the frame element:

$$\mathbf{q}(t) = [\mathbf{q}_u(t) \quad \mathbf{q}_v(t) \quad \mathbf{q}_w(t) \quad \mathbf{q}_\varphi(t)]^T \quad (3)$$

where

$$\begin{cases} \mathbf{q}_u(t) = [u_1 \quad u_2]^T \\ \mathbf{q}_v(t) = [v_1 \quad \theta_{1z} \quad v_2 \quad \theta_{2z}]^T \\ \mathbf{q}_w(t) = [w_1 \quad \theta_{1y} \quad w_2 \quad \theta_{2y}]^T \\ \mathbf{q}_\varphi(t) = [\varphi_1 \quad \varphi_2]^T \end{cases} \quad (4)$$

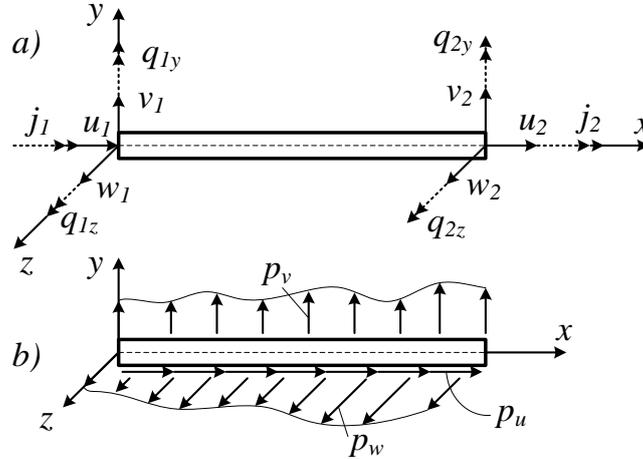


Figure 2. The displacement components at element nodes (a); the distributed load components on 3D beam element (b).

The displacement field of the frame element is approximated through the nodal displacement vector as follows:

$$\mathbf{u}(x,t) = \mathbf{N}(x)\mathbf{q}(t) \quad (5)$$

where \mathbf{N} are the shape functions

$$\mathbf{N}(x) = \begin{bmatrix} \mathbf{N}_u(x) & & & \\ & \mathbf{N}_v(x) & & \\ & & \mathbf{N}_w(x) & \\ & & & \mathbf{N}_\varphi(x) \end{bmatrix} \quad (6)$$

herein, the Hermite shape functions are employed and expressed as follows:

$$\mathbf{N}_u(x) = [\mathbf{N}_{1u}(x) \quad \mathbf{N}_{2u}(x)] \quad (7)$$

$$\mathbf{N}_v(x) = \begin{bmatrix} N_{1v}(x) & N_{2v}(x) & N_{3v}(x) & N_{4v}(x) \\ \frac{dN_{1v}(x)}{dx} & \frac{dN_{2v}(x)}{dx} & \frac{dN_{3v}(x)}{dx} & \frac{dN_{4v}(x)}{dx} \end{bmatrix} \quad (8)$$

$$\mathbf{N}_w(x) = \begin{bmatrix} N_{1w}(x) & N_{2w}(x) & N_{3w}(x) & N_{4w}(x) \\ \frac{dN_{1w}(x)}{dx} & \frac{dN_{2w}(x)}{dx} & \frac{dN_{3w}(x)}{dx} & \frac{dN_{4w}(x)}{dx} \end{bmatrix} \quad (9)$$

$$\mathbf{N}_\varphi(x) = [N_{1\varphi}(x) \quad N_{2\varphi}(x)] \quad (10)$$

in which a is the length of the frame element. These shape functions can be found in [5].

The elastic potential energy of the internal forces in the frame element is calculated as follows:

$$A = \frac{1}{2} \left\{ EF \int_0^a [u'(x,t)]^2 dx + EI_z \int_0^a [v''(x,t)]^2 dx + EI_y \int_0^a [w''(x,t)]^2 dx + GJ_p \int_0^a [\varphi''(x,t)]^2 dx \right\} \quad (11)$$

where u', v'', w'', φ' are the derivative components of $u, v, w,$ and φ displacements, correspondingly. E is the elastic modulus of the material. G is the shear modulus of the material. F is the area of the cross-section of the frame element. I_y and I_z are the moments of inertia of the area in the y - and z -directions in the local coordinate system. J_p is the polar moment of inertia of the area.

Note that Equations (5) and (11) can be rewritten in the matrix form as follows:

$$A = \frac{1}{2} \int_0^a \mathbf{D} [\mathbf{B}(x) \mathbf{q}(t)]^2 dx = \frac{1}{2} \mathbf{q}^T(t) \left[\int_0^a \mathbf{B}^T(x) \mathbf{D} \mathbf{B}(x) dx \right] \mathbf{q}(t) \quad (12)$$

where \mathbf{B} is the strain-displacement relation matrix of the 3D beam element.

$$\mathbf{B}(x) = \begin{bmatrix} \mathbf{N}'_u(x) & & & \\ & N''_v(x) & & \\ & & N''_w(x) & \\ & & & N'_\varphi(x) \end{bmatrix} \quad (13)$$

\mathbf{D} is the eslastic matrix of the frame element.

$$\mathbf{D} = \begin{bmatrix} EF & & & \\ & EI_z & & \\ & & EI_y & \\ & & & GJ_p \end{bmatrix} \quad (14)$$

The work done by external forces is expressed as follows:

$$A_e = - \left\{ \int_0^a p_u(x,t)u(x,t)dx + \int_0^a p_v(x,t)v(x,t)dx + \int_0^a p_w(x,t)\mathbf{w}(x,t)dx + \int_0^a p_\varphi(x,t)\varphi(x,t)dx \right\} +$$

$$- \frac{1}{2} \left\{ \int_0^a -m\ddot{u}(x,t)u(x,t)dx + \int_0^a -m\ddot{v}(x,t)v(x,t)dx + \int_0^a -m\ddot{\mathbf{w}}(x,t)\omega(x,t)dx + \int_0^a -m_p\ddot{\varphi}(x,t)\varphi(x,t)dx \right\} + \quad (15)$$

$$- \frac{1}{2} \left\{ \int_0^a -c\dot{u}(x,t)u(x,t)dx + \int_0^a -c\dot{v}(x,t)v(x,t)dx + \int_0^a -c\dot{\mathbf{w}}(x,t)\mathbf{w}(x,t)dx + \int_0^a -c_p\dot{\varphi}(x,t)\varphi(x,t)dx \right\}$$

Equation (15) can be rewritten in the matrix form as follows:

$$A_e = -\mathbf{q}^T(t) \int_0^a \mathbf{N}^T(x) \mathbf{p}(x,t) dx +$$

$$+ \frac{1}{2} \mathbf{q}^T(t) \int_0^a \mathbf{N}^T(x) \mathbf{N}(x) dx m_e \ddot{\mathbf{q}}(t) \quad (16)$$

$$+ \frac{1}{2} \mathbf{q}^T(t) \int_0^a \mathbf{N}^T(x) \mathbf{N}(x) dx c \dot{\mathbf{q}}(t)$$

where m is the mass per unit length. m_p is the moment of inertia of m per unit length, which is defined as follows:

$$m_p = mJ_p \quad (17)$$

c is the viscous drag coefficient per unit length. p_u , p_v , p_w , and p_φ are the distributed loads per unit length (Figure 2b). $\mathbf{p}(x,t)$ is the distributed load on the frame element:

$$\mathbf{p}(x,t) = [p_u(x,t) \quad p_v(x,t) \quad p_w(x,t) \quad p_\varphi(x,t)]^T \quad (18)$$

\mathbf{m}_e is the distributed mass matrix of the frame element:

$$\mathbf{m}_e = \begin{bmatrix} m_u & & & \\ & m_v & & \\ & & m_w & \\ & & & m_\varphi \end{bmatrix} \quad (19)$$

in which distributed mass matrix components can be found in [5].

According to the minimum of the potential energy, the equilibrium condition of the system has a form as follows:

$$\frac{\partial V}{\partial q_i} = \frac{\partial (A + A_e)}{\partial q_i} = 0 \quad \text{with } i = u, v, w, \varphi. \quad (20)$$

Substituting equations (12) and (16) into Equation (20) with generalized coordinates i are u , v , w , and φ , respectively, we obtain the finite element oscillation equation as follows:

$$\mathbf{m}\ddot{\mathbf{q}}(t) + \mathbf{c}\dot{\mathbf{q}}(t) + \mathbf{k}\mathbf{q}(t) = \mathbf{p}(t) \quad (21)$$

where \mathbf{k} , \mathbf{m} , and \mathbf{c} are the element stiffness matrix, the element mass matrix, and the element viscous drag matrix, respectively.

$$\begin{aligned} \mathbf{k} &= \int_0^a \mathbf{B}^T(x) \mathbf{D} \mathbf{B}(x) dx \\ \mathbf{m} &= \mathbf{m}_e^T \int_0^a \mathbf{N}^T(x) \mathbf{N}(x) dx \\ \mathbf{c} &= c \int_0^a \mathbf{N}^T(x) \mathbf{N}(x) dx \end{aligned} \quad (22)$$

\mathbf{p} is the element nodal force:

$$\mathbf{p}(t) = \int_0^a \mathbf{N}(x)^T \mathbf{p}(x,t) dx \quad (23)$$

By integrating equations (22) and (23), we obtain the finite element matrices of the space frame element. These element matrices can be found in [5].

By assembling the element matrices, vectors and eliminating boundary conditions, we obtain the forced oscillation equation of the structure as follows

$$M\ddot{Q} + C\dot{Q} + KQ = F \quad (24)$$

where M , C , and K are the global mass matrix, the global viscous drag matrix, and the global stiffness matrix of the structure, respectively, and Q and F are the global displacement vector and the global force vector of the structure, respectively.

For the free vibration without the viscous drag, Equation (24) becomes

$$M\ddot{Q} + KQ = 0 \quad (25)$$

We then obtain the equation to determine the natural frequencies and the mode shapes:

$$(K - \omega^2 M)Q = 0 \quad (26)$$

Using eigenvalue command in MATLAB to solve Equation (26) we get ω , which denotes the natural frequencies of the system, for each value of ω we have an eigenvector.

3. Artificial neural networks

Nowadays, the using of artificial intelligence (AI) in solving the complicated issues in science and technology as well as in mechanical problems is an interesting trend. The artificial neural network (ANN) model is one of the types of artificial intelligence based on the simulation of the massively parallel working process of the human brain and descriptions of how it works. It is obvious that neural network nodal functions can be done numerous works at the same time with less cost of time to give out the output results. In other words, the easier way to explain this new concept of AI is that a neural network is a black box [5], which can predict output results from particular input ones. On the other hand, after the training process, the neural network can be aware of similarities from the new input samples. For many ANN systems that have been investigated, the most commonly employed network system is the multi-layer feed-forward network, which is the system conducted in this work. The hinge system of an ANN model is usually comprised of three special layers, the input layer, where the data are introduced to the model, the hidden layer or layers, where data are computed, and the output layer, where the results of the ANN are exported (Figure 4). Each layer includes nodes referred to as neurons. In a feed-forward neural network, information moves only in one way, forward, from the input neurons, through the hidden nodes to the output neurons. Each neuron is linked to other neurons in the processing layer (the second layer). It is different from the neurons in the input layer, which only get and

forward the input signals to the other neurons in the hidden layer, each neuron in the other layers includes three main components; weights, bias, and an activation function, which can be continuous, linear or nonlinear. Standard activation functions include, nonlinear sigmoid functions (logsig, tansig) and linear functions (poslin, purelin). When the system of a feed-forward neural network has been defined completely (including number of layers, number of neurons in each layer, activation function for each layer), the weights and bias levels are the only free parameters that can be modified. Adjusting these parameters can change the output values of the network. In order to model a given real function or process, the weights and bias levels are modified to get the desired network output within an error margin. This adjustment procedure is referred to as a training process. Different training algorithms are conducted depending on the type of neural network implemented. The well-known ANN training algorithm is the backprop algorithm. This training algorithm distributes the network error in order to arrive at a best fit or minimum error. For details regarding the several types of ANN structures as well as training algorithms, we can find more information in [8-9].

For the mechanical applications, many mechanical systems need to obtain immediately natural frequencies of the structure such as the problem related to the resonance or predicting cracks by processing natural frequencies of the system. According to ANN models, we can predict mechanical responses (natural frequencies for example) of the mechanical structures through the input data. In addition, using the ANN model costs less time and memories, consequently, it can shorten the simulating time to export output parameters (in this work is the natural frequencies). This is one of the remarkable advantages of ANN models, and they can be fully applied to solving other complex problems in mechanics. The authors have the first scientific work deal with AI, which can be found in [5].

4. Numerical results

4.1. Verification problems

Consider a 15-storey space frame system as shown in Figure 3. Herein, we have two types of frame elements: 1) vertical frames where the diameter $d_0 = 0,3\text{m}$ (5 main columns); 2) the other ones where $d = 0,05\text{m}$. The mechanical properties of the steel are as follows: Young's modulus $E = 2 \times 10^{11} \text{ N/m}^2$, shear modulus $G = 0,8 \times 10^{11} \text{ N/m}^2$, and density $\rho = 7850 \text{ kg/m}^3$. The height between 2 adjacent floors is 1m, so the total height of the structure is 15m. In this problem, the system is clamped at A_0, B_0, C_0, D_0 , and E_0 . The results of the first three natural frequencies of this work, obtained from commercial software SAP2000 and ETABS, are presented in Table 1.

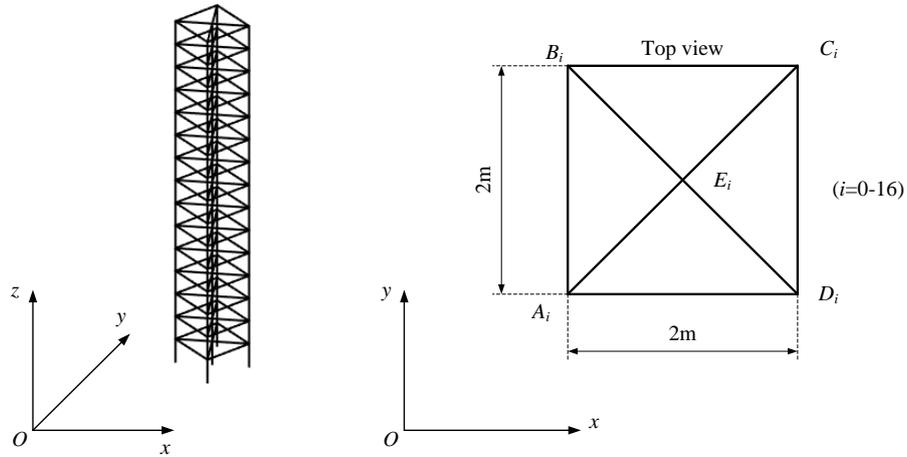


Figure 3. Model of a 15-storey space frame system

Table 1. The comparison of the first three natural frequencies of the space frame

Method	ω_i (rad/s)		
	ω_1	ω_2	ω_3
SAP2000	3,2241	3,2432	11,7688
ETABS	3,2123	3,2427	11,7542
This work	3,2420	3,2439	11,7646

From Table 1 we can see that the computed results of this work are in good agreement with the results obtained from SAP2000 and ETABS. Therefore, the proposed theory and calculation program are verified.

4.2. Parameter studies

Let us consider a 15-storey space frame system with the geometrical and material parameters mentioned above (Figure 3).

4.2.1. Effect of diameter d_0

In this section, the authors investigate the effect of the diameter d_0 of vertical frames. Let d_0 increases respectively by 3%, 5%, 7%, and 10% than original value (0,3 m).

Table 2 below presents the natural frequencies of the structure changes as a function of diameter d_0 .

Table 2. The natural frequencies of the structure change as a function of diameter d_0

The increase of d_0	Natural frequency (Hz)		
	f_1	f_2	f_3
3%	3,3900	3,3920	12,1143
5%	3,4922	3,4942	12,3389
7%	3,5971	3,5992	12,5568
10%	3,7596	3,7618	12,8717

From Table 2 we obtain that when d_0 increases, the natural frequency of the structure increases. The reason is that when d_0 increases, the structure becomes stiffer.

4.2.2. Effect of the height of the structure

Next, in order to investigate the effect of the height of the structure, we consider three cases as follows

- Case 1: The total height of the structure is $h = 5\text{m}$ (5-storey space frame system).
- Case 2: The total height of the structure is $h = 10\text{m}$ (10-storey space frame system).
- Case 3: The total height of the structure is $h = 15\text{m}$ (15-storey space frame system).

The structure is clamped at $A_0, B_0, C_0, D_0,$ and E_0 . All the parameters of the structure in three cases are the same as described in the first paragraph of section 4.2.

Table 3. The natural frequencies change as a function of height of the structure

Case	Natural frequency (Hz)		
	f_1	f_2	f_3
1	23,3747	23,3978	30,4518
2	6,6315	6,6349	15,1641
3	3,2420	3,2439	11,7646

4.2.3. Effect of boundary conditions

In this subsection, we examine the effect of boundary conditions. The model of the structure and its parameters are the same as described in the first paragraph of section 4.2. Herein, we consider 3 cases as follows: case 1 - the structure is clamped at $A_0, B_0, C_0, D_0,$ and E_0 (5C); case 2 - the structure is clamped at A_0, B_0, C_0, D_0 and simply supported at E_0 (4C1S); case 3 - the structure is clamped at E_0 and simply supported at A_0, B_0, C_0, D_0 (1C4S). Table 4 below presents the natural frequencies of the structure with different boundary conditions.

Table 4. The natural frequencies of the structure with different boundary conditions

Boundary conditions	Natural frequency (Hz)		
	f_1	f_2	f_3
5C	3,2420	3,2439	11,7646
4C1S	3,2016	3,2035	10,7467
4S1C	2,7554	2,7570	6,9809

From Table 4 we can see that the more degrees of freedoms are fixed, the stiffer the structure becomes. Thus, the natural frequency increases.

4.3. Predicting the natural frequencies with ANNs

In this section, we employ ANNs to predict the first natural frequencies of the structure. For this work, based on the computed results in Tables 2 and 3, we choose two input parameters d_0 and h ; the output parameters are the first natural frequencies of

the structure. These parameters are listed in detail in Table 4. The ANN model is shown in Figure 5. To discover the data of layers required to model the process, the hidden data layers are chosen from one to three. The number of nodes (neurons) is changed in a range of 4 to 80 neurons in each one of the hidden layers. To control the magnitude of weight and bias updates, the learning rate parameter is installed during the simulation process. Therefore, the training time of the ANN depends strongly on the selection of this value. The weights are automatically corrected after each case of the training data in the MATLAB environment. Besides, there is one parameter (the momentum value) is employed to reduce the likeliness of the simulation, which can be stuck in local optima. In addition, the learning rate is generally established the data set in a range of 0,0001 to 6,0, which depends on the simulation during the training process, and the momentum is remained at an average value of 0,8 for the ANN model.

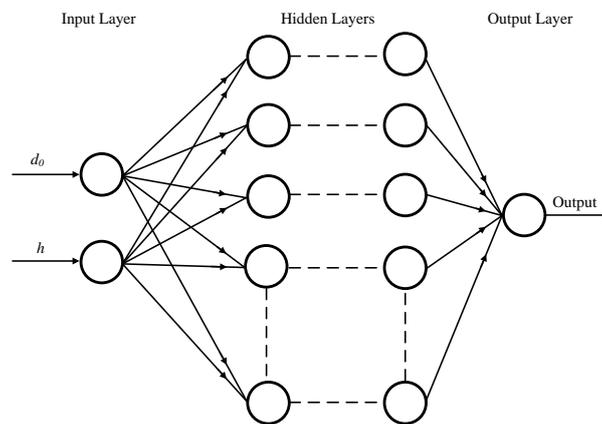


Figure 4. The ANN model with two inputs and one output.

Table 5. Predicting the natural frequencies of the structure with ANNs

The increase of d_0 (%)	h (m)	f_1 (Hz) Computed	f_1 (Hz) Predict	Error (%)
0	5	23,3747	23,3747	0
0	10	6,6315	6,6315	0
0	15	3,2420	3,2420	0
3	15	3,3900	3,3900	0
5	15	3,4922	3,5142	0,63
7	15	3,5971	3,5971	0
10	15	3,7596	3,7596	0

The predictive results using ANN model compared to the computed results show that the proposed ANN model can predict exactly the first natural frequencies of the

mechanical system, the largest error is only 0,63%. These numerical results are a very important premise to carry out many complicated problems dealing with AI, where it does not cost much time for calculations.

5. Conclusions

Based on the finite element method and classical beam theory, the finite element formulations are derived. The computed results are compared with those of exact solutions to demonstrate the accuracy of the program. This paper also introduces ANN model to predict first natural frequencies of the mechanical system. The predictive results are very close to computed results. This is very meaningful in engineering practice, because the ANN model can solve almost output parameters immediately, which does not require high computer hardware, and it is convenient for the field calculations, especially, in the area related to the construction engineering diagnostics.

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PHÂN TÍCH DAO ĐỘNG CỦA HỆ KHUNG KHÔNG GIAN CAO TẦNG

Tóm tắt: Bài báo tiến hành phân tích dao động riêng của hệ khung không gian cao tầng. Các công thức phân tử hữu hạn được thành lập dựa trên lý thuyết khung không gian cổ điển và phương pháp phân tử hữu hạn. Chương trình tính toán được lập trình trong môi trường MATLAB. Kết quả số được so sánh với các kết quả chính xác khác đã được công bố để xác minh độ tin cậy của chương trình tính toán. Thêm vào đó, tác giả sử dụng mạng trí tuệ nhân tạo để tiên đoán tần số dao động tự nhiên của kết cấu. Ảnh hưởng của các thông số hình học và điều kiện biên đến dao động riêng của kết cấu được nghiên cứu. Các kết quả số là cơ sở quan trọng để tiến hành phát triển tính toán các bài toán phức tạp liên quan đến trí tuệ nhân tạo để tiên đoán các ứng xử cơ học của kết cấu.

Từ khóa: Dao động tự do; hệ khung không gian; mạng thần kinh nhân tạo; trí tuệ nhân tạo; phân tử hữu hạn.

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