

# Environmental Policy, Health and Growth

**Hoang Khac Lich**

*University of the Thai Chamber of Commerce, Thailand*

## **Abstract**

*This paper develops a standard neoclassical model of growth in which pollution affects individuals' health and the government can influence the quality of the environment via a tax on emissions. In such an economy, we analyze the effects of a change in this policy on the trade-offs between the resources allocated to abatement, health and consumption (or savings). We demonstrate that less pollution lowers healthcare spending, and shows the existence of an inverted U-shaped relationship between the pollution tax and (i) the level of health; (ii) consumption; and (iii) welfare. Results are analyzed both at steady state and along a transition path.*

**Keywords:** Pollution, Health, Welfare, Environmental Policy, Growth.

**JEL Classification:** O23, O41.

## **1. Introduction**

It is well established that pollution emissions which deteriorate the quality of the environment also affect individuals' health. For instance, Prüss-Üstün and Corvalán (2007) stated that 24% of global disease and 23% of all deaths could be attributed to environment factors. In particular, 42% of chronic obstructive pulmonary diseases can be attributed to occupational exposures to dust and chemicals, as well as indoor air pollution from household solid fuel use.

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Studying the relationship between pollution, economic development and health is important not only because health is a factor of production (affecting the productivity of individuals) but also because it affects individuals' welfare (see e.g. the seminal article by Grossman, 1972). The environmental literature (e.g., Gradus and Smulders, 1993, Brock and Taylor, 2005, and Xepapadeas, 2005), however, overlook this issue<sup>1</sup>. More precisely, the authors do not explicitly take into account the costly aspect of health and thus the resulting trade-offs between health, capital investment, and individuals' consumption.

In this paper, in contrast, we explicitly formulize a health sector to analyze these potential trade-offs. We should mention that Grossman (1972), van Zon and Muysken (2001) and Aísa and Pueyo (2006) also analyze some of these trade-offs, but they do not discuss those related to pollution. In that sense, this paper builds on two strands of the literature in a unified framework: it takes the environmental effects on health into account in an analysis of the relation between growth and the environment.

Another contribution of this paper is the analysis of individuals behavior along a transitional path. It analyzes the changes in healthcare spending, consumption and savings when the level of pollution varies. This issue, surprisingly, has also been overlooked in most of the articles cited above.

To conduct the analysis and analyze how a pollution tax affects individual behavior and economic development, we develop a Ramsey-Cass-Koopmans model. Although the model is standard, it is rich enough to capture a number of important features. As stated by Barro and Sala-i-Martin (2004), the key element of this model is that it provides a useful benchmark to analyze individuals' behavior. Moreover, it helps us to discuss welfare issues in a clear-cut manner.

In our model, firms produce an output which is polluting. Pollution emissions (by product of production) affect the level of health (productivity) of individuals and their welfare. Government can reduce emissions by using a pollution tax whose proceeds are invested in abatement technologies. Thereby, the aim of this paper is to analyze the effects of a change in the pollution tax both in the long run (steady state) and in the short run (along the transition).

Our main findings can be summarized as follows. First, we demonstrate that an increase in abatement spending (a higher pollution tax) reduces healthcare spending both in the short run and in the long run. It means that the health benefit of an improved environmental quality allows people to spend fewer resources on healthcare services. Interestingly, this finding is supported by empirical data. Romley *et al.* (2010), for instance, shows that improved air quality

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has reduced total spending on hospital care by \$193,100,184 in California in 2005-2007.

Second, we show the existence of three inverted U-shaped relationships resulting from the trade-offs discussed above: (i) the first one between the pollution tax and the level of health, (ii) the second one between the pollution tax and consumption, and (iii) the third one between the pollution tax and welfare. This result suggests that if the productivity of abatements is relatively high, a tighter environmental policy makes people better-off because this policy increases both the level of consumption and health. However, if abatements become less productive, people have to sacrifice consumption in exchange for a higher quality of the environment as well as a higher level of health. Herein, we have two possible outcomes on welfare: People are better-off if the welfare gain from a better health is greater than the welfare loss from a lower level of consumption; otherwise, people are worse-off. Finally, if the productivity of abatements is very low, a better environmental quality requires an amount of resources so that an increase in the pollution tax reduces the level of consumption, health and welfare.

Turning to the analysis of the short-run dynamics of the model, we show that individuals face an instantaneous loss in consumption at the time of the policy change. Then, along the transitional path, the outcome depends on the productivity of abatements. If it is high, an increase in environmental care leads to great improvement in environmental quality, labor productivity and growth. That is, both consumption and physical capital increase along the transition. However, if the productivity of abatements is low, it crowds out the investments which are necessary to promote growth. As a result, consumption and physical capital decrease.

The remaining part of this paper is organized as follows. Section 2 presents the model. In Section 3, we investigate the long-run and short-run effects of a pollution tax on individuals' behaviors and welfare, and then we construct comparative studies to examine the responses of the economy following a change in some structural parameters. Section 4 concludes.

## 2 Model

Consider a closed economy in continuous time. Time, denoted by  $t$ , goes from zero to infinity:  $t \in [0, \infty)$ . At each instant, there is a representative household owning  $K_t$  units of wealth and comprising  $L_t$  identical members growing at an exogenous rate,  $n > 0$ :  $\dot{L}_t = nL_t$ . Each individual has one unit of labor that is supplied inelastically to output production,  $Y_t$ . Output is distributed to consumption, healthcare spending, abatement and saving. Let us denote  $C_t$  consumption;  $Z_t$  healthcare spending,  $Q_t$  abatement spending and  $K_t$  physical capital. For simplicity, we assume that depreciation of physical capital is zero. The resource constraint is then given by:

$$Y_t = C_t + Z_t + Q_t + \dot{K}_t. \quad (1)$$

The technology of the output production is assumed to be given by:

$$Y_t = B(K_t)^\alpha (h_t A_t L_t)^{1-\alpha}, \quad (2)$$

where  $0 < \alpha < 1$ ,  $B > 0$  is a constant productivity parameter,  $A_t$  is technical progress evolving at an exogenous rate of growth,  $g_A > 0$  (i.e.,  $\dot{A}_t = g_A A_t$ ) and  $h_t$  is the level of health of an individual. A notable feature of the production function is that it clearly distinguishes between the standard efficient units of labor (i.e., the combination of raw labor and knowledge,  $A_t L_t$ ) and healthy units of effective labor,  $h_t A_t L_t$ .

The novelty of this paper is the introduction of a health production sector whereby individuals choose how much resources to spend on healthcare services. The interesting feature of the health technology is that it is negatively affected by pollution emissions. For simplicity, following Aloï and Tournemaine (2011), we set:

$$h_t = \phi(\eta_t)^\gamma (P_t)^{-\chi}, \quad (3)$$

where  $\phi > 0$ ,  $\gamma > 0$ ,  $\chi > 0$ ,  $P_t$  denotes pollution emissions,  $\eta_t \equiv Z_t / Y_t$  is the fraction of output devoted to healthcare services. Note that in the above technology, we assume that healthcare spending is the percentage of output rather than its level. This formalization, which is taken from Aïsa and Pueyo (2006), allows us to simplify the analysis and to avoid the problem of exploding growth paths.

To keep the analysis simple, we focus on the immediate effects of emissions, such as air pollution, whose implications on health are for the most part direct and are drastically reduced when addressed (see, e.g., Künzli, 2002). It should be noted, however, that none of the results we derive in this paper hinge on this assumption. In line with Gradus and Smulders (1993) and Brock and Taylor (2005), we assume that pollution is a by-product of output production. As mentioned, these emissions can be reduced through abatements that consume output; in so doing the flow of pollution does not grow without bound and is constant at steady-state. Formally, we have

$$P_t = \left( \frac{Y_t}{Q_t} \right)^\beta, \quad (4)$$

where  $\beta > 0$ .

Turning to the specification of preferences, in line with Grossman (1972), van Zon and Muysken (2001) and Aloï and Tournemaine (2011), among others, it is assumed that individuals derive utility from consumption and health:

$$U = \int_0^{\infty} \frac{[\bar{c}_t(h_t)^{\theta}]^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \quad (5)$$

where  $\sigma > 0$  is the inverse of the elasticity of substitution,  $\rho > 0$  is the rate of time preference,  $\theta > 0$  measures the relative contribution of health to utility, and  $\bar{c}_t \equiv C_t / L_t$  represents per capita consumption.

### 3 Equilibrium

In this section, we characterize the equilibrium of the model. For simplicity, we assume that abatements are public activities. These are funded by a constant pollution tax rate,  $\tau$ , levied on output, so that  $Q_t = \tau Y_t$  at each moment. Two comments are in order here. First, as mentioned by Aloï and Tournemaine (2011), we can rationalize our approach, i.e. abatements are public activities, by appealing to the fact that governments may promote the adoption of technologies that reduce pollution originating from the use of resources, such as coal or fuel, impairing air or water quality. For example, governments may promote: renewable energies to replace fossil fuel (Künzli, 2002), "green" buses for public transport, "green" power stations for energy, or water purification systems removing contaminants and other harmful micro organisms from rivers and water sources. Second, some might argue that the ratio  $Q_t / Y_t$  is time varying, and so might be the pollution tax rate. As we will see below, the noteworthy feature here, however, is that this ratio is in a constant steady state. As the pollution tax rate level is set by the government, we make the simplifying assumption that it is constant over time. This will allow us to simplify the computations and the analysis of both the steady state and the transitional dynamics.

As well know, to analyze the model, it is easier to transform the model in order to obtain variables in unit per effective labor. We thus define  $y_t \equiv Y_t / (A_t L_t)$ ,  $k_t \equiv K_t / (A_t L_t)$  and  $c_t \equiv C_t / (A_t L_t)$ . The law of motion of physical capital becomes:

$$\dot{k}_t = (1 - \eta_t - \tau) B(k_t)^{\alpha} [\phi(\eta_t)^{\gamma} (\tau)^{\beta\chi}]^{1-\alpha} - c_t - (g_A + n)k_t. \quad (6)$$

The output technology becomes

$$y_t = B(k_t)^{\alpha} (h_t)^{1-\alpha}. \quad (7)$$

Lastly, the preferences are rewritten as

$$U = \int_0^{\infty} \frac{[A_t c_t (h_t)^{\theta}]^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \quad (8)$$



We now turn to the characterization and analysis of the equilibrium. First, we characterize the first order conditions. We then analyze the solution of the model in steady state and along the transition. Finally, we construct some comparative studies to examine the responses of the economy following a change in some structural parameters.

### 3.1 The first order conditions

The problem of the representative individual is to choose the level of consumption per unit of effective labor,  $c_t$ , and the fraction of income (output) devoted to healthcare services,  $\eta_t$ , that maximize lifetime utility (8) subject to the law of motion of physical capital per unit of effective labor (6) and the initial condition  $k_0 > 0$ . The Current-Value Hamiltonians to this problem is

$$CVH = \frac{\left\{ A_t c_t \left[ \phi(\eta_t)^\gamma (\tau)^{\beta_X} \right]^\theta \right\}^{1-\sigma} - 1}{1-\sigma} + \lambda_t \left\{ (1-\eta_t - \tau) B(k_t)^\alpha \left[ \phi(\eta_t)^\gamma (\tau)^{\beta_X} \right]^{1-\alpha} - c_t - (g_A + n) k_t \right\}$$

where  $\lambda_t$  is the co-state variable associated to the law of motion of capital. The solution to this problem is defined by the first order conditions:  $\partial CVH / \partial c_t = 0$ ;  $\partial CVH / \partial \eta_t = 0$ ;  $\partial CVH / \partial k_t = -\dot{\lambda}_t + \rho \lambda_t$ ; and the transversality condition:  $\lim_{t \rightarrow \infty} \lambda_t k_t e^{-\rho t} = 0$ .

After some manipulations, we get:

$$\frac{\left\{ A_t c_t \left[ \phi(\eta_t)^\gamma (\tau)^{\beta_X} \right]^\theta \right\}^{1-\sigma}}{c_t} = \lambda_t, \quad (9)$$

$$\frac{\gamma \theta \left\{ A_t c_t \left[ \phi(\eta_t)^\gamma (\tau)^{\beta_X} \right]^\theta \right\}^{1-\sigma}}{\eta_t} + \lambda_t \gamma (1-\alpha) (1-\eta_t - \tau) \frac{y_t}{\eta_t} = \lambda_t y_t, \quad (10)$$

$$\alpha (1-\eta_t - \tau) \frac{y_t}{k_t} - (g_A + n) + \frac{\dot{\lambda}_t}{\lambda_t} = \rho. \quad (11)$$

Equation (9) states that the marginal utility of consumption equals the marginal cost of wealth. Equation (10) ensures that the marginal benefit of an additional fraction of output spent on healthcare services (measured by the utility gain plus the production gains) equals its marginal cost (measured by output losses). Equation (11) shows that the rate of return on wealth

(capital) equals the discount rate. The rate of return on wealth is given by the marginal productivity of capital net of the marginal cost to maintain capital at its existing level (break-even investment) plus the change in the shadow price.

Before proceeding, we derive two intermediate equations that will be useful for the characterization of steady state and transitional dynamics. Differentiating (9) with respect to time yields:  $(1-\sigma)g_A - \sigma \dot{c}_t / c_t + \gamma\theta(1-\sigma)\dot{\eta}_t / \eta_t = \dot{\lambda}_t / \lambda$ . Combining this with (11) we have:

$$\sigma \frac{\dot{c}_t}{c_t} - \gamma\theta(1-\sigma) \frac{\dot{\eta}_t}{\eta_t} = \alpha(1-\eta_t-\tau) \frac{y_t}{k_t} - \sigma g_A - \rho - n. \quad (12)$$

From (9) and (10), we obtain:

$$c_t = \frac{[1+\gamma(1-\alpha)]\eta_t - \gamma(1-\alpha)(1-\tau)}{\gamma\theta} B(k_t)^\alpha [\phi(\eta_t)^\gamma (\tau)^{\beta\chi}]^{1-\alpha}. \quad (13)$$

### 3.2 Steady State

In this subsection, we focus on the steady state. By construction, we know that the growth rates of variables per unit of effective labor are zero:  $\dot{y}_t = \dot{k}_t = \dot{c}_t = \dot{h}_t = \dot{\eta}_t = 0$ . Using (12) and (13), we have:

$$c_{ss} = \frac{\{[1+\gamma(1-\alpha)]\eta_{ss} - \gamma(1-\alpha)(1-\tau)\}(\sigma g_A + \rho + n)}{\gamma\theta\alpha} k_{ss}, \quad (14)$$

where the symbol "ss" is used to denote any variable in steady state.

Using (6) and (12), we get the relationship between consumption and physical capital per unit of effective labor:

$$c_{ss} = \frac{\sigma g_A + \rho + n - \alpha g_A - \alpha n}{\alpha} k_{ss}. \quad (15)$$

Combining (14) and (15), we obtain the fraction of output devoted to healthcare services:

$$\eta_{ss} = \frac{\gamma[\theta(\sigma g_A + \rho + n - \alpha g_A - \alpha n) + (1-\alpha)(\sigma g_A + \rho + n)](1-\tau)}{(1+\gamma-\gamma\alpha)(\sigma g_A + \rho + n) + \gamma\theta(\sigma g_A + \rho + n - \alpha g_A - \alpha n)}. \quad (16)$$

Plugging (16) to (3), we get the level of health:

$$h_{ss} = \phi(\eta_{ss})^\gamma (\tau)^{\beta\chi}. \quad (17)$$

Plugging (16) to (12), we obtain the level of physical capital per unit of effective labor:

$$k_{ss} = \left\{ \frac{B\alpha(1-\eta_{ss}-\tau) \left[ \phi(\eta_{ss})^\gamma (\tau)^{\beta\chi} \right]^{1-\alpha}}{\sigma g_A + \rho + n} \right\}^{\frac{1}{1-\alpha}}. \quad (18)$$

Equations (15)-(18) define a 4x4 system in  $c_{ss}$ ,  $k_{ss}$ ,  $\eta_{ss}$  and  $h_{ss}$ . Straightforward examination of this system shows that an interior solution where  $c_{ss} > 0$ ,  $k_{ss} > 0$ ,  $\eta_{ss} > 0$  and  $h_{ss} > 0$  exists and is unique.

In the remaining part of this subsection, we analyze: (i) the influences of a tighter environmental policy on healthcare spending,  $\eta_{ss}$ , the level of health,  $h_{ss}$ , capital,  $k_{ss}$ , and consumption,  $c_{ss}$ ; (ii) the effects of a tighter environmental policy on welfare.

### 3.2.1 Policy implication

We now study how a tighter environmental policy affects  $\eta_{ss}$ ,  $h_{ss}$ ,  $k_{ss}$ , and  $c_{ss}$ . Examination of equations (15)-(18) reveals that there is a level of pollution tax,  $\tau_{c,k}^{\max}$ , that maximizes both of  $c_{ss}$  and  $k_{ss}$ , and another level,  $\tau_h^{\max}$ , that maximizes  $h_{ss}$ <sup>2</sup>:

$$\tau_{c,k}^{\max} = \frac{\beta\chi(1-\alpha)}{(\beta\chi + \gamma)(1-\alpha) + 1}, \quad (19)$$

$$\tau_h^{\max} = \frac{\beta\chi}{\beta\chi + \gamma}, \quad (20)$$

where

$$\tau_{c,k}^{\max} < \tau_h^{\max}.$$

The following table shows the sign of the first-order derivative of  $\eta_{ss}$ ,  $h_{ss}$ ,  $k_{ss}$ , and  $c_{ss}$  with respect to  $\tau$ .

**Table 1: The effects of a tighter environmental policy on  $\eta_{ss}$ ,  $h_{ss}$ ,  $k_{ss}$ , and  $c_{ss}$ .**

	$0 < \tau \leq \tau_{c,k}^{\max}$	$\tau_{c,k}^{\max} < \tau \leq \tau_h^{\max}$	$\tau_h^{\max} < \tau < 1$
$\partial \eta_{ss} / \partial \tau$	$< 0$	$< 0$	$< 0$
$\partial h_{ss} / \partial \tau$	$> 0$	$\geq 0$	$< 0$
$\partial k_{ss} / \partial \tau = \partial c_{ss} / \partial \tau$	$\geq 0$	$< 0$	$< 0$



Table 1 shows that  $\partial \eta_{ss} / \partial \tau < 0$  for any level of pollution tax. This result means that a tighter environmental policy reduces the fraction of output devoted to healthcare services. In other words, the health benefit of improved environmental quality allows individuals to spend a lower fraction of output on healthcare services.

An important result from Table 1 is that the effect of a tighter environmental policy on  $h_{ss}$ ,  $k_{ss}$ , and  $c_{ss}$  crucially depends on its tax level. We will come back on this issue in Section 3.2.2 which discusses welfare. As a preamble, we can observe that for a low level of the pollution tax ( $0 < \tau \leq \tau_{c,k}^{\max}$ ), an increase in the tax has a positive effect on the level of health. This is because, in this case, the productivity of abatements is high. Interestingly, this effect carries on consumption and capital: that is, the health benefits of improved environmental quality allow individuals to save an amount of resources which can be used for consumption and capital accumulation. When the pollution-tax rate takes higher values ( $\tau_{c,k}^{\max} < \tau \leq \tau_h^{\max}$ ), the productivity of abatement is still sufficient to increase environmental quality which, in turn, leads to a higher level of health. However, as abatements become more costly, a tighter environmental policy now reduces consumption and capital accumulation.

Lastly, for high values of the pollution-tax rate ( $\tau_h^{\max} < \tau < 1$ ), the productivity of abatements becomes so small that an increase in abatement spending would reduce the level of health, consumption and capital. That is, individuals must sacrifice their present and future consumption (savings) as well as their level of health to maintain the environmental quality at its steady level. The following proposition summarizes our main results.

**Proposition 1** *On the long-run effects of a tighter environmental policy,  $\tau$ :*

- (a) *A tighter environmental policy reduces fraction of output devoted to healthcare services;*
- (b) *If  $0 < \tau \leq \tau_{c,k}^{\max}$ , a tighter environmental policy increases both consumption and the level of health;*
- (c) *If  $\tau_{c,k}^{\max} < \tau \leq \tau_h^{\max}$ , a tighter environmental policy improves the level of health but reduces consumption;*
- (d) *If  $\tau_h^{\max} < \tau < 1$ , a tighter environmental policy reduces both consumption and the level of health.*

### 3.2.2 Welfare analysis

In this part, we characterize the effects of an increase in the tax level on welfare. We assume that the economy is initially in a steady state. After some manipulations, we obtain the following lifetime utility function:

$$U = \frac{[A_0 c_0 \phi^\theta (\eta_0)^{\gamma\theta} (\tau)^{\beta\chi\theta}]^{1-\sigma}}{(1-\sigma)[(\sigma-1)g_A + \rho]} \quad (21)$$

Straightforward manipulations of (21) allow us to establish the following:

**Proposition 2:** Define  $\tau^w$  as the welfare-maximizing tax rate. Then, we have:

$$\tau^w = \frac{\beta\chi(1-\alpha)[\alpha + \theta(\sigma g_A + \rho + n - \alpha g_A - \alpha n)]}{(\beta\chi + \gamma)(1-\alpha)[\alpha + \theta(\sigma g_A + \rho + n - \alpha g_A - \alpha n)] + \alpha},$$

where:

$$\tau_{c,k}^{\max} < \tau^w < \tau_h^{\max}.$$

This proposition is important because it allows us to refine our analysis of the effects of pollution tax on individuals' behaviors in respect to consumption, healthcare spending and savings. From this condition, we can study four main cases: (i)  $(0; \tau_{c,k}^{\max}]$ ; (ii)  $(\tau_{c,k}^{\max}; \tau^w]$ ; (iii)  $(\tau^w; \tau_h^{\max}]$ ; and (iv)  $(\tau_h^{\max}; 1)$ .

**Case 1:**  $0 < \tau \leq \tau_{c,k}^{\max}$

When the economy devotes a low fraction of output to abatements, a greater amount of abatement spending increases both consumption and the level of health in the long run (Proposition 1). Hence, people are better-off. In other words, this result implies that the government can increase welfare by shifting up the level of the pollution tax. This is known as double dividends of the environmental care<sup>3</sup>.

**Case 2:**  $\tau_{c,k}^{\max} < \tau \leq \tau^w$

When the pollution-tax rate takes a value in the set  $(\tau_{c,k}^{\max}, \tau^w]$ , a tighter environmental policy leads to a higher level of health and less consumption (Proposition 1). The welfare gain from the increase in the level of health is greater than the welfare loss due to the decrease in the level of consumption. Thus, this policy makes individuals better-off. Note that maximum welfare is attained if  $\tau = \tau^w$ .

**Case 3:**  $\tau^w < \tau \leq \tau_h^{\max}$

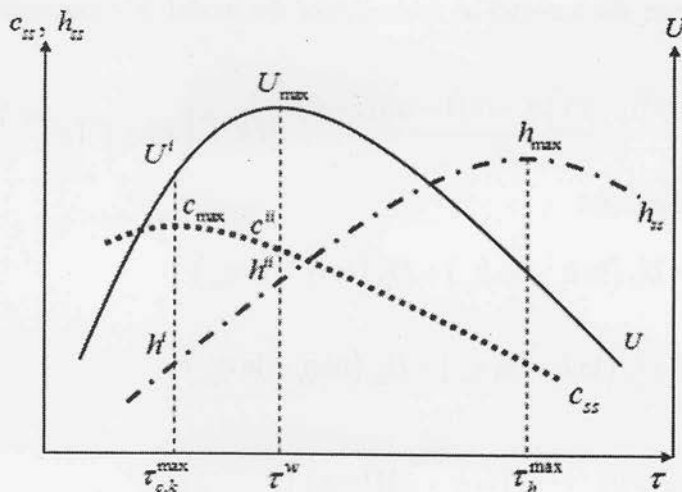
In this case, a greater amount of abatement spending improves the level of health of individuals but reduces consumption (Proposition 1). In contrast with Case 2, the welfare gain from the increase in the level of health is lower than the welfare loss due to the decrease in consumption. Consequently, individuals are worse-off.

**Case 4:**  $\tau_h^{\max} < \tau < 1$

Here, a tighter environmental policy reduces both the level of health of individuals and consumption (Proposition 1). Hence, it makes people worse-off. By cutting down the pollution-tax rate, the government reduces the tax burden on individuals, thus shifting up consumption, and the level of health and welfare.

To summarize, as depicted in Case 1-4, we obtain three inverted U-shaped relationships between the pollution tax and (i) the level of health; (ii) consumption; and (iii) welfare. They are graphically represented in Figure 1.

**Figure 1: The relationship between  $c_{ss}$ ,  $h_{ss}$ ,  $U$  and pollution tax.**



As pollution tax increases from 0 to  $\tau_{c,k}^{\max}$ , consumption per unit of effective labor increases to  $c_{\max}$ , the level of health is improved to  $h^i$ , and welfare rises to  $U^i$  (Case 1). Along with tax level from  $\tau_{c,k}^{\max}$  to  $\tau^w$ , while consumption decreases from  $c_{\max}$  to  $c^{ii}$ , the level of health

increases from  $h^i$  to  $h^{ii}$ , and welfare goes up from  $U^i$  to its maximum level,  $U_{\max}$  (Case 2). As pollution taxes increase from  $\tau^w$  to 1, the level of consumption and welfare decline continuously. The level of health, however, increases from  $h^{ii}$  to its maximum level,  $h_{\max}$ , when pollution tax rises from  $\tau^w$  to  $\tau_h^{\max}$  (Case 3); then it decreases when pollution tax increases from  $\tau_h^{\max}$  to 1 (Case 4).

An issue arisen from the above discussion is the following: which situations are we likely to observe in the real world? In fact, Case 3 and Case 4 can be excluded because there is no reason to keep that level of pollution tax. For the other cases, however, we can assume that developed countries usually impose a tighter policy than do developing countries (see, e.g., Mukhopadhyay, 2006). Hence, it is possible to take the level of pollution tax as a proxy for the development level of a country. Accordingly, Case 1 would represent the developing countries while Case 2 would represent developed countries. That is, developing countries can increase both consumption and the level of health while developed countries face a trade-off between the two.

### 3.3 Transitional Dynamics

We now characterize the transitional dynamics of the model. For convenience, we rewrite the equation (13).

$$c_t = \frac{[1 + \gamma(1 - \alpha)]\eta_t - \gamma(1 - \alpha)(1 - \tau)}{\gamma\theta} B(k_t)^\alpha [\phi(\eta_t)^\gamma (\tau)^{\beta\chi}]^{1-\alpha}. \quad (22)$$

In Appendix, we show that

$$\frac{d \ln k_t}{dt} = D_{11} (\ln k_t - \ln k_{ss}) + D_{12} (\ln \eta_t - \ln \eta_{ss}), \quad (23)$$

$$\frac{d \ln \eta_t}{dt} = D_{21} (\ln k_t - \ln k_{ss}) + D_{22} (\ln \eta_t - \ln \eta_{ss}), \quad (24)$$

where

$$D_{11} = \left[ \frac{1 + \gamma(1 - \alpha + \theta)}{\gamma\theta} \eta_{ss} - \frac{(1 - \alpha + \theta)(1 - \tau)}{\theta} \right] (1 - \alpha) \frac{y_{ss}}{k_{ss}},$$

$$D_{12} = \left[ \frac{(1 - \alpha + \theta)(1 - \tau)\gamma(1 - \alpha)}{\theta} - \frac{[1 + \gamma(1 - \alpha + \theta)][\gamma(1 - \alpha) + 1]}{\gamma\theta} \eta_{ss} \right] \frac{y_{ss}}{k_{ss}},$$

$$D_{21} = \frac{\left[ \left( 1 - \sigma \frac{1 + \gamma(1 - \alpha + \theta)}{\gamma\theta} \right) \eta_{ss} - 1 + \tau + \sigma \frac{(1 - \alpha + \theta)(1 - \tau)}{\theta} \right] \alpha (1 - \alpha) \frac{y_{ss}}{k_{ss}}}{\sigma [1 + \gamma(1 - \alpha)] \eta_{ss} + \sigma \gamma (1 - \alpha) - \gamma \theta (1 - \sigma)},$$

$$D_{22} = \frac{\left\{ \frac{\theta - \sigma(1 - \alpha + \theta)}{\theta} (1 - \tau) \gamma (1 - \alpha) + \frac{[\gamma(1 - \alpha) + 1]^2}{\gamma\theta} \eta_{ss} \right\} \alpha \frac{y_{ss}}{k_{ss}}}{\sigma [1 + \gamma(1 - \alpha)] \eta_{ss} + \sigma \gamma (1 - \alpha) - \gamma \theta (1 - \sigma)}.$$

Using matrix notations, equation (23) and (24) are rewritten as

$$\begin{pmatrix} \frac{d \ln k_t}{dt} \\ \frac{d \ln \eta_t}{dt} \end{pmatrix} = D \begin{pmatrix} \ln k_t - \ln k_{ss} \\ \ln \eta_t - \ln \eta_{ss} \end{pmatrix},$$

where

$$D = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}.$$

In what follows, we first show how to draw a phase diagram of the economy. Then, we characterize the short-run effects of a tighter environmental policy on  $c_t$ ,  $\eta_t$  and  $k_t$ .

### 3.3.1 The phase diagram

To analyze how the economy converges to the steady state in a simple way, we draw the phase diagram in the  $(k, \eta)$  space. For simplicity, we apply numerical methods. The benchmark value of parameters used to conduct the analysis is gathered in Table 2.

**Table 2: Baseline parameter values**

Description	Parameter	Base value	Range	Source
Population growth rate	$n$	0.0125	[0,0.025]	OECD in figures 2009
Growth rate of technical progress	$g_A$	0.02	[0.01,0.03]	Barro and Sala-i-Martin (2004)
Rate of time preferences	$\rho$	0.04	[0.02,0.07]	
Weight of health in utility	$\theta$	0.2		
Capital share in the output sector	$\alpha$	0.35		Brock and Taylor (2005)
Inverse elasticity of pollution with respect to abatements	$\beta$	0.3		Pautrel (2008)

Description	Parameter	Base value	Range	Source
Elasticity of health with respect to healthcare spending	$\gamma$	0.15		
Elasticity of health with respect to environmental quality	$\chi$	0.1		
Inverse of the elasticity of substitution	$\sigma$	1.75		Barro and Sala-i-Martin (2004)
Constant productivity parameter of the health sector	$\phi$	0.3		
Constant productivity parameter of the output sector	$B$	0.4		
Pollution tax	$\tau$	0.01	[0.01,0.02]	Brock and Taylor (2005)

We assume that the pollution tax can vary within [0%, 5%]<sup>4</sup>. After computation, for any level of pollution tax within this range we have:

$$D_{11} < 0, D_{12} < 0, D_{21} < 0, D_{22} > 0, \quad (25)$$

$$\left. \frac{d\eta_t}{dk_t} \right|_{\dot{k}_t=0} = -\frac{D_{11}}{D_{12}} < 0, \text{ and } \left. \frac{d\eta_t}{dk_t} \right|_{\dot{\eta}_t=0} = -\frac{D_{21}}{D_{22}} > 0. \quad (26)$$

To determine the stability properties of the model, we need to determine the sign of the eigenvalues ( $\Lambda_1, \Lambda_2$ ) of matrix  $D$ . As the determinant of  $D$  is given by:

$$\det(D) = \Lambda_1 \Lambda_2 = D_{11} D_{22} - D_{21} D_{12} < 0, \quad (27)$$

the two eigenvalues have opposite signs. Hence, the unique steady state is saddle point stable.

We now characterize the slope of the saddle path. As the dynamic system is log-linearized, to compute the slope of the balanced growth path we can infer that:

$$\ln \eta_t - \ln \eta_{ss} = \Omega (\ln k_t - \ln k_{ss}), \quad (28)$$

where  $\Omega$  is the slope of the transitional path,

$$\Omega > 0. \quad (29)$$

The exact value of  $\Omega$  is computed as follows. Plugging (28) to (23) yields

$$\frac{d \ln k_t}{dt} = (D_{11} + \Omega D_{12}) (\ln k_t - \ln k_{ss}). \quad (30)$$

Then, plugging (28) to (24), we get



$$\Omega \frac{d \ln k_t}{dt} = (D_{21} + \Omega D_{22})(\ln k_t - \ln k_{ss}) . \quad (31)$$

Equations (30) and (31) imply that:

$$D_{11} + \Omega D_{22} = \frac{D_{21} + \Omega D_{22}}{\Omega} .$$

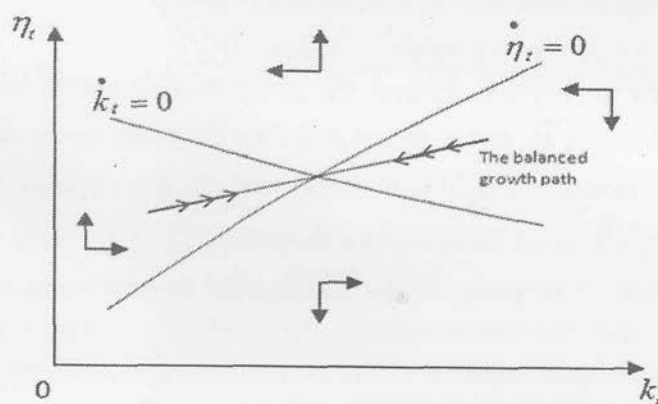
Solving this equation for  $\Omega$ , we obtain

$$\Omega = \frac{D_{22} - D_{11} \pm \sqrt{(D_{11} - D_{22})^2 + 4D_{12}D_{21}}}{2D_{12}} > 0 . \quad (32)$$

It is important to keep this result in mind when we turn to the construction of the phase diagram, to the analysis of the effects of a tighter environmental policy in Section 3.3.2 and to the comparative studies in Section 3.4.

From (25), (26) and (32), the phase diagram can be drawn as in Figure 2:

**Figure 2: The phase diagram**



The phase diagram displays the changes in physical capital per unit of effective labor and the fraction of output devoted to healthcare services around the steady state. The upward-sloping curve corresponds to combinations of  $(k_t, \eta_t)$  for which  $\dot{\eta}_t = 0$ , whereas the downward-sloping curve corresponds to combinations of  $(k_t, \eta_t)$  for which  $\dot{k}_t = 0$ . The steady state of the economy is depicted by the intersection of these two loci  $\dot{\eta}_t = 0$  and  $\dot{k}_t = 0$ . Then, we must see how the variables behave outside the steady state. From equation (23), we see that  $k_t$  increases when  $\eta_t$  is lower than its steady-state value. Thus, arrows must point East on the area below the locus  $\dot{k}_t = 0$ . Similarly,  $k_t$  decreases when  $\eta_t$  is higher than its steady state value.

The corresponding arrows must point West on the area above the locus  $\dot{k}_t = 0$ . We proceed in the same way for abatement spending. When  $k_t$  is lower than the value for which  $\dot{\eta}_t = 0$ ,  $\eta_t$  increases. Thus, arrows must point North on the left hand side of the locus  $\dot{\eta}_t = 0$ . Finally, if  $k_t$  exceeds the value that yields  $\dot{k}_t = 0$ ,  $\eta_t$  decreases. The corresponding arrows must point South on the right hand side of the locus  $\dot{\eta}_t = 0$ .

The direction of the arrows shows that the balanced growth path has a positive slope. Let us assume that the economy is initially on the balanced growth path. If the starting location is above the locus  $\dot{\eta}_t = 0$  and below the locus  $\dot{k}_t = 0$ , both  $k_t$  and  $\eta_t$  increase over time to the steady state. Conversely, if the starting location is below the locus  $\dot{\eta}_t = 0$  and above the locus  $\dot{k}_t = 0$ , both  $k_t$  and  $\eta_t$  decrease over time to the steady state.

### 3.3.2 The effects of a tighter environmental policy

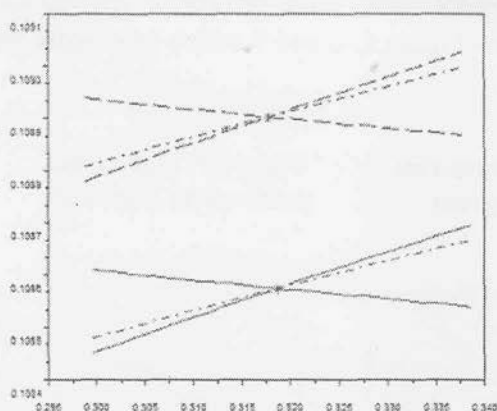
Assuming that the government tightens the environmental policy, we now analyze the effects of this change on  $k_t$ ,  $\eta_t$  and  $c_t$  in two cases: (i) the pollution-tax rate, before and after changing, is low ( $\tau$  increases from 1% to 1.3%); and (ii) the pollution-tax rate is high ( $\tau$  increases from 1.75% to 1.81%). Computations show that  $\tau_{c,k}^{\max} = 1.746\%$  and  $\tau^w = 1.813\%$ . Following the definition of the proxy for the development level of a country mentioned in the welfare analysis, the first case would represent the developing countries and the second case would represent the developed countries. Given the benchmark value of the parameters in Table 2, we draw Figure 3 for the first case and Figure 4 for the second one.

In each case, the two dashed lines denote the loci  $\dot{\eta}_t = 0$  and  $\dot{k}_t = 0$  for the initial tax rate. The intersection point of these two lines indicates the old steady state  $(k_{ss}^{old}, \eta_{ss}^{old})$ . Similarly, the solid lines denote the loci determined after the policy changes, and the intersection represent the new steady state  $(k_{ss}^{new}, \eta_{ss}^{new})$ . The balanced growth path is the dash-dotted line going through the steady state.

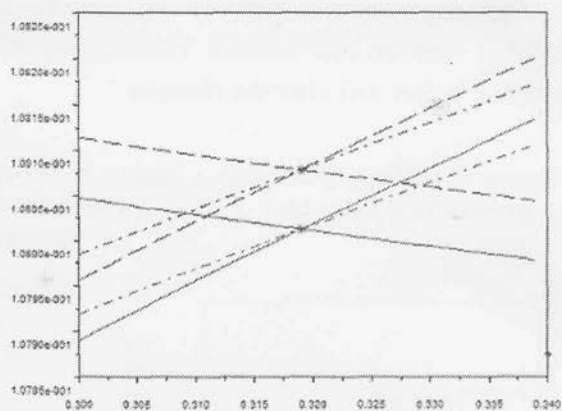
Let us assume that the amount of physical capital per unit of effective labor at the time of the change is given by  $k_t = k_{ss}^{old}$ . As physical capital per unit of effective labor is a predetermined variable, it cannot jump freely. By contrast, the fraction of output devoted to healthcare services,  $\eta_t$ , is a control variable and thus free to jump. In order to converge into the

new steady state,  $\eta_t$  must be set on the new balanced growth path at the time of the change in the policy instruments. However, to know whether  $\eta_t$  jumps up or down, we must determine the position of the new balanced growth path: if it locates above the starting point of the economy,  $\eta_t$  must increase; conversely, if it locates below the starting point,  $\eta_t$  must decrease.

**Figure 3: Pollution tax increases from 1% to 3%**



**Figure 4: Pollution tax increases from 1.75% to 1.81%**



In Figure 3, the new balanced growth path locates below the old steady state and  $k_{ss}^{old} < k_{ss}^{new}$  (the distance from  $k_{ss}^{old}$  to  $k_{ss}^{new}$  is graphically tiny). Hence,  $\eta_t$  must jump down at first, and then increases together with  $k_t$  to the new steady state. Equation (22) states that consumption,  $c_t$ , must jump down at the time of the change in the environmental policy, and then increases to the new steady state. The reason behind this result is following. At the time of the change, the tax burden reduces consumption and healthcare spending. However, because the government spends a low fraction of output on abatements, the productivity of abatements is high. The health benefit from the improvement of environmental quality increases labor productivity, thus, boosting growth. This allows individuals to spend more resources on consumption and healthcare services.

In Figure 4, the new balanced growth path also locates below the old steady state, but  $k_{ss}^{old} > k_{ss}^{new}$  (again, the distance from  $k_{ss}^{old}$  to  $k_{ss}^{new}$  is graphically tiny). Hence,  $\eta_t$  must jump down at first, and then decreases together with  $k_t$  to the new steady state. Equation (22) states that  $c_t$  must jump down at the time of the change, and then decreases to the new steady state.

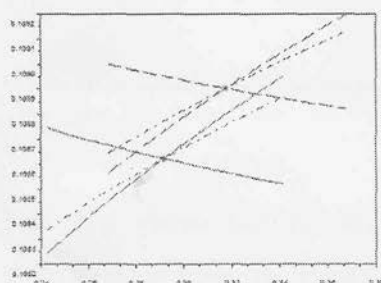
That is, the economy must substitute consumption and healthcare spending for abatements in the short run. Moreover, the productivity of abatements is so small that an increase in abatement spending crowds out capital investments leading to a decline in growth. It leads to a decrease in consumption and healthcare spending in the long run.

### 3.4 Comparative study

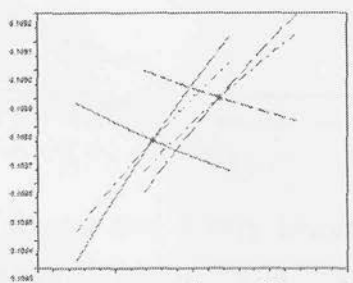
In this section, we construct some comparative studies to examine the short-run and long-run effects of an increase in the population growth rate,  $n$ , the growth rate of technical progress,  $g_A$ , and the rate of time preferences,  $\rho$ , on the variables  $\eta_t$ ,  $c_t$ , and  $k_t$ .

Applying numerical methods and using the benchmark values given in Table 2, we find that (25) and (26) are still verified. Thereby, we can draw Figure 5, 6 and 7 which represents the economy before and after the changes.

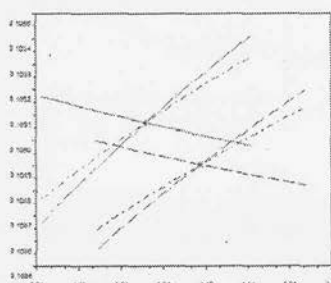
**Figure 5: If the population growth rate increases**



**Figure 6: If the growth rate of technical progress increases**



**Figure 7: If the time preferences increases.**



The values of  $\eta_{ss}$ ,  $c_{ss}$ , and  $k_{ss}$  before and after the changes, are shown in Table 3, where a sign "-" indicates a negative change and correspondingly a sign "+" indicates a positive change.

**Table 3: The effects of a change in  $n$ ,  $g_A$ , and  $\rho$  on  $\eta_{ss}$ ,  $c_{ss}$ , and  $k_{ss}$ .**

	Base values (see Table 5)	$n$ increases (from 1.25% to 1.75%)	$g_A$ increases (from 0.02% to 0.025%)	$\rho$ increases (from 0.04% to 0.045%)
$\eta_{ss}$	10.89%	-0.03%	-0.01%	+0.02%
$c_{ss}$	0.0691	-0.0029	-0.0039	-0.0015
$k_{ss}$	0.3178	-0.26	-0.433	-0.261

As shown in Figure 5, 6, 7 and Table 3, at the time of the change in population growth,  $\eta_t$  jumps down on the new balanced growth path. Then, both  $k_t$  and  $\eta_t$  decrease continuously until the economy reaches the new steady state. Equation (22) shows that  $c_t$  jumps down when  $n$  changes, then it decreases to the new steady state. Moreover, in the long run, the value of  $\eta_{ss}$ ,  $c_{ss}$ , and  $k_{ss}$  are lower. The explanation for this result is following. As effective labor grows at rate  $n + g_A$ , the quantity of available capital must be shared among an increasing number of more skilled individuals (dilution effect). That is, when  $n$  increases, individuals have fewer resources for consumption, healthcare spending and capital accumulation.

Second, if the growth rate of technical progress,  $g_A$ , increases from 0.02 to 0.025, Figure 5 shows that  $\eta_t$  jumps up on the new balanced growth path. It then decreases together with  $k_t$  until the economy reaches the new steady state. Equation (22) reveals that  $c_t$  jumps down when  $n$  changes, then it decreases to the new steady state. Finally, in the long run, the value of  $\eta_{ss}$ ,  $c_{ss}$ , and  $k_{ss}$  are lower. The reason behind this result is similar to the previous case: there is a dilution effect caused by the increase in the growth rate of technical progress.

Third, if the rate of time preference,  $\rho$ , increases from 0.04 to 0.045, individuals put a higher weight on current consumption and health relative to the future. Figure 6 shows that  $\eta_t$  jumps up on the new balanced growth path. Then it decreases together with  $k_t$  until the economy reaches the new steady state. Equation (22) shows that  $c_t$  jumps up at first, and then it decreases. In the long run,  $c_{ss}$  and  $k_{ss}$  are higher but  $\eta_{ss}$  is lower. This is because individuals are willing to sacrifice future consumption and wealth in exchange for more current consumption and healthcare spending. Facing the decline in physical capital per unit of effective labor in the long run, individuals increase healthcare spending.

#### 4 Conclusion

This paper developed a growth model in which we set out the complexity of the relations among pollution, health and economic growth. In our model, health has a direct effect on individuals' welfare and is a proxy of their productivity. Health can be improved by spending private resources on healthcare services, while it is reduced by pollution emissions coming from output production. These emissions can be reduced via abatement technologies funded by a pollution tax on output.

We investigated the effects of a pollution tax on individuals' decisions with respect to consumption, healthcare spending and savings in both the short run and the long run. We found

that an increase in the pollution tax reduces the long-run level of healthcare spending. This is because both elements are treated as imperfect substitutes as suggested by empirical evidences. We then showed the existence of three inverted U-shaped relationships reminiscent from the trade-offs we analyzed in this paper. They include the relationship between: (i) the pollution tax and the level of health; (ii) the pollution tax and consumption; (iii) the pollution tax and welfare. Finally, we analyzed the responses of the economy to a change in some structural parameters such as the growth rate of population, the growth rate of technical progress and the rate of time preference, with some numerical applications.

To keep the analysis simple and to focus on the key features of the problem, we presented a simple framework. Several extensions are possible. Future research could, for instance, consider an endogenous growth model to analyze the effects of pollution on long-run growth. Second, heterogeneity between individuals could also be introduced to study the effects of an environmental policy on economic development and inequality. Finally, empirical assessments would be interesting to see the relations among the variables in our model. These issues are on our agenda for future work. ■

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1. See Rici (2007) for a recent comprehensive survey of the environmental literature.
2. See the detailed calculation in the Appendix.
3. In Smulders and Gradus (1996), double dividends mean that the benefits of environmental improvement extend to other fields. In Glomm (2004), the double dividends include an efficiency dividend (a higher consumption) and a green dividend (a better environmental quality).
4. Brock and Taylor (2004) show that abatement spending in the order of 1-2% of GDP seems to be the norm in OECD countries. In this chapter, we allow abatement spending to vary within the range [0%, 5%] to cover more possibilities.

## Appendix

### *Pollution tax for maximum consumption, physical capital and health*

Note that we should assume  $\sigma > 1$  because this is the case which is empirically supported (see Hall, 1988). This condition is very useful in simplifying our computations. At first, we find the level of pollution tax that maximizes consumption and the level of health. Let us define  $\tau_c^{\max} = \text{agr max } c_{ss}$ ; and  $\tau_k^{\max} = \text{agr max } k_{ss}$ . Direct inspection on (15) yields:  $\partial k_{ss} / \partial \tau = \partial c_{ss} / \partial \tau$ . Thus, if there is a level of tax that maximizes physical capital, then it maximizes consumption. We denote this tax level by  $\tau_c^{\max} = \tau_k^{\max} = \tau_{c,k}^{\max}$ . Differentiating (18) with respect to  $\tau$ , we have:



$$\frac{\partial k_{ss}}{\partial \tau} = \frac{k_{ss}}{(1-\alpha)(1-\eta_{ss}-\tau)} \left[ \frac{(\gamma-\gamma\alpha)(1-\tau) - (\gamma-\alpha\gamma+1)\eta_{ss}}{\eta_{ss}} \frac{\partial \eta_{ss}}{\partial \tau} - \frac{(\beta\chi - \alpha\beta\chi + 1)\tau - \beta\chi(1-\alpha)(1-\eta_{ss})}{\tau} \right]. \quad (33)$$

Simple computations applied on (16) give:  $\partial \eta_{ss} / \partial \tau = -\eta_{ss} / (1-\tau) < 0$ . Using this result and equation (16) to solve  $\partial k_{ss} / \partial \tau = 0$ , we get:

$$\tau_{c,k}^{\max} = \frac{\beta\chi(1-\alpha)}{(\beta\chi + \gamma)(1-\alpha) + 1}.$$

Since  $c_t > 0$ , (14) reveals that  $\eta_{ss} > \gamma(1-\alpha)(1-\tau) / [1 + \gamma(1-\alpha)]$ . From (33), we see that the term outside the square bracket is positive. Thus the sign of  $\partial k_{ss} / \partial \tau$  depends on how high the value of the second term inside the square bracket is. After some manipulations we can show that:  $\partial k_{ss} / \partial \tau > 0$  if  $\tau < \tau_{c,k}^{\max}$  and  $\partial k_{ss} / \partial \tau < 0$  if  $\tau > \tau_{c,k}^{\max}$ .

We now find the tax rate that maximizes the level of health. Differentiating (17) with respect to  $\tau$  yields:

$$\frac{\partial h_{ss}}{\partial \tau} = \gamma \frac{h_{ss}}{\eta_{ss}} \frac{\partial \eta_{ss}}{\partial \tau} + \beta\chi \frac{h_{ss}}{\tau}.$$

We define  $\tau_h^{\max} = \arg \max h_{ss}$ . Solving  $\partial h_{ss} / \partial \tau = 0$  and using  $\partial \eta_{ss} / \partial \tau = -\eta_{ss} / (1-\tau)$ , we obtain:

$$\tau_h^{\max} = \frac{\beta\chi}{\beta\chi + \gamma}.$$

Simple comparisons show that:  $\partial h_{ss} / \partial \tau > 0$  if  $\tau < \tau_h^{\max}$ , and  $\partial h_{ss} / \partial \tau < 0$  if  $\tau > \tau_h^{\max}$ .

### The dynamic system

Taking the log-linearization (13), we obtain:

$$\ln c_t = \ln \left( \frac{[1 + \gamma(1-\alpha)] e^{\ln \eta_t} - \gamma(1-\alpha)(1-\tau)}{\gamma\theta} \right) + \ln B \left( \phi(\tau)^{\beta\chi} \right)^{1-\alpha} + \ln(k_t)^\alpha + \ln(\eta_t)^{\gamma(1-\alpha)}. \quad (34)$$

Then, differentiating both sides of (34) with respect to time around the steady state, we have:

$$\frac{d \ln c_t}{dt} = \left[ \frac{[1 + \gamma(1 - \alpha)] \eta_{ss}}{[1 + \gamma(1 - \alpha)] \eta_{ss} - \gamma(1 - \alpha)(1 - \tau)} + \gamma(1 - \alpha) \right] \frac{d \ln \eta_t}{dt} + \alpha \frac{d \ln k_t}{dt} \quad (35)$$

Let us rewrite (12) as

$$\sigma \frac{d \ln c_t}{dt} - \gamma \theta (1 - \sigma) \frac{d \ln \eta_t}{dt} = \alpha (1 - \tau - \eta_t) \frac{B(k_t)^\alpha [\phi(\eta_t)^\gamma (\tau)^{\beta \chi}]^{1-\alpha}}{k_t} - \sigma g_A - \rho - n. \quad (36)$$

Combining (35) and (36) yields:

$$\begin{aligned} & \left[ \frac{\sigma [1 + \gamma(1 - \alpha)] \eta_{ss}}{[1 + \gamma(1 - \alpha)] \eta_{ss} - \gamma(1 - \alpha)(1 - \tau)} + \sigma \gamma (1 - \alpha) - \gamma \theta (1 - \sigma) \right] \frac{d \ln \eta_t}{dt} + \sigma \alpha \frac{d \ln k_t}{dt} \\ &= \alpha (1 - \tau - e^{\ln \eta_t}) B[\phi(\tau)^{\beta \chi}]^{1-\alpha} e^{(\alpha-1) \ln k_t + \gamma(1-\alpha) \ln \eta_t} - \sigma g_A - \rho - n. \end{aligned} \quad (37)$$

Taking the first order of Taylor's expansion for the right hand side of (37), we have:

$$\begin{aligned} & \left[ \frac{\sigma [1 + \gamma(1 - \alpha)] \eta_{ss}}{[1 + \gamma(1 - \alpha)] \eta_{ss} - \gamma(1 - \alpha)(1 - \tau)} + \sigma \gamma (1 - \alpha) - \gamma \theta (1 - \sigma) \right] \frac{d \ln \eta_t}{dt} + \sigma \alpha \frac{d \ln k_t}{dt} \\ &= \alpha (1 - \tau - \eta_{ss}) (\alpha - 1) \frac{y_{ss}}{k_{ss}} (\ln k_t - \ln k_{ss}) + [(1 - \tau - \eta_{ss}) \gamma (1 - \alpha) - \eta_{ss}] \alpha \frac{y_{ss}}{k_{ss}} (\ln \eta_t - \ln \eta_{ss}). \end{aligned} \quad (38)$$

Dividing both sides of (6) by  $k_t$ , and then taking the first order of Taylor's expansion for the right hand side, we obtain:

$$\begin{aligned} \frac{d \ln k_t}{dt} &= \left[ \frac{1 + \gamma(1 - \alpha + \theta)}{\gamma \theta} \eta_{ss} - \frac{(1 - \alpha + \theta)(1 - \tau)}{\theta} \right] (1 - \alpha) \frac{y_{ss}}{k_{ss}} (\ln k_t - \ln k_{ss}) \\ &+ \left[ \frac{(1 - \alpha + \theta)(1 - \tau) \gamma (1 - \alpha)}{\theta} - \frac{[1 + \gamma(1 - \alpha + \theta)] [\gamma(1 - \alpha) + 1]}{\gamma \theta} \eta_{ss} \right] \frac{y_{ss}}{k_{ss}} (\ln \eta_t - \ln \eta_{ss}). \end{aligned} \quad (39)$$

Plugging (39) into (38) yields:

$$\begin{aligned} \frac{d \ln \eta_t}{dt} = & \frac{\left[ \left( 1 - \sigma \frac{1 + \gamma(1 - \alpha + \theta)}{\gamma \theta} \right) \eta_{ss} - 1 + \tau + \sigma \frac{(1 - \alpha + \theta)(1 - \tau)}{\theta} \right] \alpha(1 - \alpha) \frac{y_{ss}}{k_{ss}}}{\sigma [1 + \gamma(1 - \alpha)] \eta_{ss} + \sigma \gamma(1 - \alpha) - \gamma \theta(1 - \sigma)} (\ln k_t - \ln k_{ss}) \\ & + \frac{\left\{ \frac{\theta - \sigma(1 - \alpha + \theta)}{\theta} (1 - \tau) \gamma(1 - \alpha) + \frac{[\gamma(1 - \alpha) + 1]^2}{\gamma \theta} \eta_{ss} \right\} \alpha \frac{y_{ss}}{k_{ss}}}{\sigma [1 + \gamma(1 - \alpha)] \eta_{ss} + \sigma \gamma(1 - \alpha) - \gamma \theta(1 - \sigma)} (\ln \eta_t - \ln \eta_{ss}). \end{aligned} \quad (40)$$

Equations (39) and (40) are the equations (23) and (24) respectively.

### References:

- Aísa, R. and F. Pueyo (2006), 'Government health spending and growth in a model of endogenous longevity', *Economics Letters*, Elsevier, vol 90(2), pp. 249-253.
- Aloi, M. and F. Tournemaine (2011), 'Growth effects of environmental policy when pollution affects health', *Economic Modelling*, Elsevier, vol.28(4), pages 1683-1695, July.
- Barro, R.J. and X. Sala-i-Martin (2004), '*Economic Growth*', 2nd edition, MIT press.
- Brock, W. A. and M. S. Taylor (2005), 'Economic growth and the environment: A Review of Theory and Empirics', *Handbook of Economic Growth*, in: Philippe Aghion & Steven Durlauf (ed.), *Handbook of Economic Growth*, edition 1, vol 1, chapter 28, pp. 1749-1821.
- Chakraborty, S. 2004, 'Endogenous lifetime and economic growth', *Journal of Economic Theory*, Elsevier, vol 116(1), pp. 119-137.
- DeVol, R. and A. Bedroussian (2007), 'An unhealthy America: The economic burden of chronic disease charting a new course to save lives and increase productivity and economic growth', *Milken Institute*.
- Fullerton, D. and S. R. Kim (2008), 'Environmental investment and policy with distortionary taxes, and endogenous growth', *Journal of Environmental Economics and Management*, Elsevier, vol 56(2), pp. 141-154.
- Glomm, G., D. Kawaguchi, and F. Sepulveda (2006), 'Green Taxes and Double Dividends in a Dynamic Economy', *Caepr Working Papers 2006-017*, Center for Applied Economics and Policy Research, Economics Department, Indiana University Bloomington.
- Gradus, R. and S. Smulders (1993), 'The trade-off between environmental care and long-term growth - pollution in three prototype growth models', *Journal of Economics*, vol 58, pp. 25-51.

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- Grossman, M (1972), 'On the concept of health capital and demand for health', *Journal of Political Economy*, vol 80, pp. 223-255.
- Hall, R. E (1988), 'Intertemporal substitution in consumption', *Journal of Political Economy*, vol 96, pp. 330-357.
- Kunzli, N (2002), 'The public health relevance of air pollution abatement', *European Respiratory Journal*, vol 20, pp. 198-209.
- Mukhopadhyay, K (2006), 'Impact on the Environment of Thailand's Trade with OECD Countries', *Asia Pacific Trade and Investment Review*, vol 2, issue 1, pp. 1-22.
- OECD in Figures 2009.
- ONeill, M. S., M. Jerrett, I. Kawachi, J. I. Levy, A. J. Cohen, N. Gouveia, P. Wilkinson, T. Fletcher, L. Cifuentes, J. Schwartz and P. Wilkinson (2003), 'Health, wealth, and air pollution: Advancing theory and methods', *Environmental Health Perspectives*, vol 111, pp. 1861-1870.
- Pautrel, X 2008, 'Reconsidering the impact of the environment on long-run growth when pollution influences health and agents have a finite-lifetime', *Environmental Resource Economics*, vol 40, pp. 37-52.
- Prüss-Üstün, A. and C. Corvalán (2006), 'Preventing disease through healthy environments', The World Health Organization.
- Ricci, F. (2007), 'Channels of transmission of environmental policy to economic growth: A survey of the theory', *Ecological Economics*, vol 60, pp. 688-699.
- Romley, J. A., A. Hackbarth and D. P. Goldman (2010), 'The Impact of Air Quality on Hospital Spending', *Santa Monica, CA: RAND Corporation*.
- Smulders, S. and R. Gradus (1996), 'Pollution abatement and long-term growth', *European Journal of Political Economy*, vol 12, pp. 505-532.
- van Ewijk, C. and S. van Wijnbergen (1994), 'Can abatement overcome the conflict between environment and economic growth?', *De Economist*, vol 143, pp. 197-216.
- van Zon, A. and J. Muysken (2001), 'Health and endogenous growth', *Journal of Health Economics*, vol 20, pp. 169-185.
- Williams, R. III (2002), 'Environmental tax interactions when pollution affects health or productivity', *Journal of Environmental Economics and Management*, Elsevier, vol 44(2), pp. 261-270.
- Xepapadeas, A (1997), 'Economic development and environmental pollution: traps and growth', *Structural Change and Economic Dynamics*, vol 8, pp. 327-350.
- Xepapadeas, A (2005), 'Economic growth and the environment', *Handbook of Environmental Economics*, in: *Maler, K.G. & Vincent, J.R. (ed.), Handbook of Environmental Economics*, edition 1, vol 3, chapter 23, pp. 1219-1271.