# NOVEL EXPONENTIAL STABILITY CRITERION OF NONLINEAR SYSTEMS WITH INTERVAL TIME-VARYING DELAYS

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**Abstract:** *This paper presents a new result on delay-dependent exponential stability for nonlinear linear systems with interval time-varying delay. By constructing a set of improved Lyapunov-Krasovskii functionals combined with Wirtinger-based integral inequality, a new delay-dependent condition is established in terms of linear matrix inequality (LMI) which guarantees that the system is exponential stability.*

**Keywords:** *Exponential stability, nonlinear systems, Wirtinger-based integral inequality, Interval time-varying delays.*

## 1. Introduction

In the scope of functional differential equations, stability problem has been the subject of investigable research attention. Among the well-known Lyapunov stability method, the Lyapunov functional is a powerful tool for stability analysis of time-delay systems [2], [3], [7], [10]. Based on the Lyapunov function, delay-dependent stability criteria for these systems are established in terms of linear matrix inequalities (LMIs). On the other hand, the exponential stability problem for differential systems has received the attention of many mathematicians in recent times. However, to the best our knowledge, the problem of exponential stability differential systems with state delays has not been fully investigated to date, especially for nonlinear systems with time-varying. The stability criteria have mainly been given for linear systems with constant delay, linear system with time-varying delay [4], [8], [12], [11]. There are very few results about exponential stability for nonlinear systems with time-varying delay. In this research, we have considered the exponential stability problem for a class of nonlinear system with time-varying delays. Based on an improved Lyapunov-Krasovskii functional combined with Wirtinger-based integral inequality, the sufficient condition for the exponential stability for nonlinear systems has been derived in term of LMIs.

*Notations:* The following notations will be used throughout this paper. $R^+$ denotes the set of all nonnegative real numbers; $R^n$ denotes the $n-$dimensional Euclidean space with the norm $\|.\|$ and scalar product $\langle x, y\rangle = x^T y$ of two vectors $x, y$; $\lambda_{max}(A)$ ($\lambda_{min}(A)$, resp.) denotes the maximal (the minimal, resp.) number of the real part of eigenvalues of $A$; $A^T$ denotes the transpose of the matrix $A$ and $I$ denotes the identity matrix; $Q \geq 0$ ($Q > 0$, resp.) means that $Q$ is semi-positive definite (positive definite, resp.) i.e. $x^T Q x \geq 0$ for all

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$x \in R^n$ (resp. $x^T Qx, > 0$ for all $x \neq 0$); $A \geq B$ means $A - B \geq 0$; $C^1([-\tau, 0], R^n)$ denotes the set of $R^n$- valued continuous functions on $[-\tau, 0]$ with the nor $\| \varphi \|_\tau = \max\{\sup_{-\tau \leq t \leq 0} \| \varphi(t) \|, \sup_{-\tau \leq t \leq 0} \| \dot{\varphi}(t) \|\}$.

The segment of the trajectory $x(t)$ is denoted by $x_t = \{x(t+s) : s \in [-\tau, 0]\}$ with its norm $\| x_t \| = \sup_{s \in [-\tau, 0]} \| x(t+s) \|$.

## 2. Preliminaries and problem statement

Consider the following system with mixed time varying delays

$$\begin{cases} \dot{x}(t) = & Ax(t) + Bx(t-h(t)) + Hg(t, x(t), x(t-h(t))), \quad t \geq 0, \\ x(t) = & \varphi(t), \quad t \in [-\tau, 0], \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the system state; $A, B, H \in R^{n \times n}$ are real known system matrices with appropriate dimensions; The time varying delays $h(t), d(t)$ are continuous functions satisfying $0 < h_1 \leq h(t) \leq h_2$ and $\dot{h}(t) \leq \beta$ where $h_1, h_2$ are lower and upper bounds of the time varying delays $h(t)$. $\varphi(t) \in C([-h_2, 0], R^n)$ is the compatible initial function specifying the intial state system. The nonlinear functions $g : R^+ \times R^n \times R^n \to R^n$ satisfies

$$g^T(.)g(.) \leq a^2 x^T(t) E^T Ex(t) + b^2 x^T(t-h(t)) F^T Fx(t-h(t)) \tag{2}$$

where $E, F$ are symmetric positive definite matrices and $a, b$ are any real numbers.

**Definition 1.** *System (1) is said to be $\alpha-$ exponentially stable for $\alpha > 0$ if there exist $N > 0$ such that, for any compatible initial conditions $\varphi(t)$ the solution $x(t, \varphi)$ satisfies*

$$\| x(t, \varphi) \| \leq N \| \varphi \|_\tau e^{-\alpha t}, \quad \forall t \geq 0.$$

We introduce the following technical well-known propositions and lemma, which will be used in the proof of our results.

**Proposition 1.** *(Matrix Cauchy inequality [5]) For any $M, N \in R^{n \times n}$, $M = M^T > 0$ and $x, y \in R^n$ then $2x^T Ny \leq x^T Mx + y^T N^T M^{-1} Ny.$*

**Proposition 2.** *([13]) any symmetric positive definite matrix $M$, scalar $\nu > 0$ and vector function $\omega : [0, \nu] \to R^n$ such that the integrals concerned are well defined, then*

$$\left( \int_0^\nu \omega(s) ds \right)^T M \left( \int_0^\nu \omega(s) ds \right) \leq \nu \int_0^\nu \omega^T(s) M \omega(s) ds.$$

**Proposition 3.** *(Schur complement Lemma [5]) For given matrices $X, Y, Z$ with appropriate dimensions satisfying $X = X^T, Y = Y^T > 0.$*

*Then* $\begin{bmatrix} X & Z^T \\ Z & -Y \end{bmatrix} < 0$

if and only if $X + Z^T Y^{-1} Z < 0.$

**Proposition 4.** *(Wirtinger-based integral inequality [15]) For a give* $n\times n$ *matrix* $W>0$ *and a function* $w:[a,b]\to R^n$ *whose derivative* $\dot{w}\in PC([a,b],R^n)$, *the following inequality holds* $\int_a^b \dot{w}^{\mathrm{T}}(s)W\dot{w}(s)ds \ge \frac{1}{b-a}\xi^{\mathrm{T}}W\xi$ (3)

where $W=diag\{W,3W\}$ and $\xi=col\{w(b)-w(a),w(b)+w(a)-\frac{2}{b-a}\int_a^b w(s)ds\}$.

## 3. Main results

In this section, we propose new conditions ensuring the regularity, impulse free and exponential stability of system (1) as presented in the following theorem.

Firstly, given $\alpha>0$. We denote:

$$\Phi_1(t)=x(t),\quad \Phi_2(t)=x(t-h(t)),\quad \Phi_3(t)=x(t-h_1),\quad \Phi_4(t)=x(t-h_2),\ h_{12}=h_2-h_1,$$

$$\Phi_5(t)=\dot{x}(t),\quad \Phi_6(t)=(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds),\quad \Phi_7(t)=(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds),$$

$$\Phi_8(t)=g(t,x(t),x(t-h(t))),\ \Upsilon(t)=\begin{bmatrix}\Phi_1^{\mathrm{T}}(t) & \Phi_2^{\mathrm{T}}(t) & \Phi_3^{\mathrm{T}}(t) & \Phi_4^{\mathrm{T}}(t) & \Phi_5^{\mathrm{T}}(t) & \Phi_6^{\mathrm{T}}(t) & \Phi_7^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}},$$

$$\Pi(1,1)=PA+A^{\mathrm{T}}P^{\mathrm{T}}+Q+2\alpha P+\varepsilon a^2E^{\mathrm{T}}E-e^{-2\alpha h_1}W_1-e^{-2\alpha h_2}W_2,$$

$$\Pi(1,2)=PB,$$

$$\Pi(1,5)=A^{\mathrm{T}}(h_1^2W_1+h_2^2W_2+h_{12}^2W),$$

$$\Pi(1,8)=PH,$$

$$\Pi(2,2)=-(1-\beta)e^{-2\alpha h_2}Q-8e^{-2\alpha h_2}W+\varepsilon b^2F^{\mathrm{T}}F,$$

$$\Pi(2,3)=-2e^{-2\alpha h_2}W,$$

$$\Pi(2,4)=-2e^{-2\alpha h_2}W,$$

$$\Pi(2,5)=B^{\mathrm{T}}(h_1^2W_1+h_2^2W_2+h_{12}^2W),$$

$$\Pi(2,6)=6e^{-2\alpha h_2}W,$$

$$\Pi(2,7)=6e^{-2\alpha h_2}W,$$

$$\Pi(3,3)=e^{-2\alpha h_1}Z-e^{-2\alpha h_1}W_1-4e^{-2\alpha h_2}W,$$

$$\Pi(3,7)=6e^{-2\alpha h_2}W,$$

$$\Pi(4,4)=-e^{-2\alpha h_2}Z-e^{2\alpha h_2}W_2-4e^{-2\alpha h_2}W,$$

$$\Pi(4,6)=6e^{-2\alpha h_2}W,$$

$$\Pi(5,5)=-(h_1^2W_1+h_2^2W_2+h_{12}^2W),$$

$\Pi(5,8)=(h_1^2W_1+h_2^2W_2+h_{12}^2W)H,$

$\Pi(6,6)=-12e^{-2\alpha h_2}W,$

$\Pi(7,7)=-12e^{-2\alpha h_2}W,$

$\Pi(8,8)=-\varepsilon I.$

**Theorem 1.** *For given scalars* $\beta>0, 0<h_1\le h_2$ *and* $\alpha>0$.

*System (1) is* $\alpha-$*exponentially if there exist symmetric positive definite matrices* $P,Q,Z,W,W_i$ $(i=1,2)$ *and any number* $\varepsilon>0$ *satisfying the following LMI:*

$$\begin{bmatrix} \Pi(1,1) & \Pi(1,2) & 0 & 0 & \Pi(1,5) & 0 & 0 & \Pi(1,8) \\ \star & \Pi(2,2) & \Pi(2,3) & \Pi(2,4) & \Pi(2,5) & \Pi(2,6) & \Pi(2,7) & 0 \\ \star & \star & \Pi(3,3) & 0 & 0 & 0 & \Pi(3,7) & 0 \\ \star & \star & \star & \Pi(4,4) & 0 & \Pi(4,6) & 0 & 0 \\ \star & \star & \star & \star & \Pi(5,5) & 0 & 0 & \Pi(5,8) \\ \star & \star & \star & \star & \star & \Pi(6,6) & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \Pi(7,7) & 0 \\ \star & \star & \star & \star & \star & \star & \star & \Pi(8,8) \end{bmatrix}<0. \quad (4)$$

*Proof.* We construct the following Lyapunov-Krasovskii function (LKF)

$$V(t,x_t)=V_1+V_2+V_3+V_4+V_5+V_6 \quad (5)$$

where

$$V_1=x^{\mathrm{T}}(t)Px(t),$$

$$V_2=\int_{t-h(t)}^{t}e^{2\alpha(s-t)}x^{\mathrm{T}}(s)Qx(s)ds,$$

$$V_3=\int_{t-h_2}^{t-h_1}e^{2\alpha(s-t)}x^{\mathrm{T}}(s)Zx(s)ds,$$

$$V_4=h_1\int_{-h_1}^{0}\int_{t+s}^{t}e^{2\alpha(u-t)}\dot{x}^{\mathrm{T}}(u)W_1\dot{x}(u)duds,$$

$$V_5=h_2\int_{-h_2}^{0}\int_{t+s}^{t}e^{2\alpha(u-t)}\dot{x}^{\mathrm{T}}(u)W_2\dot{x}(u)duds,$$

$$V_6=h_{12}\int_{-h_2}^{-h_1}\int_{t+s}^{t}e^{2\alpha(u-t)}\dot{x}^{\mathrm{T}}(u)W\dot{x}(u)duds.$$

It is easy to see that $\lambda_1\|x(t)\|^2\le V(t,x_t)\le\lambda_2\|x_t\|^2,$ (6)

where $x_t$ denotes the segment $\{x(t+s):s\in[\tau;0]\}, \lambda_1=\lambda_{min}(P)$ and

$$\lambda_2=\lambda_{max}(P)+e^{-2\alpha h_1}\lambda_{max}(Z)+h_2\lambda_{max}(Q)+\frac{h_1^3}{2}\lambda_{max}(W_1)+\frac{h_2^3}{2}\lambda_{max}(W_2)+\frac{h_{12}^2(h_1+h_2)}{2}\lambda_{max}(W).$$

Taking derivative of $V_1$ in $t$ along the trajectory of the system, we have

$$\dot{V}_1 = 2x^T(t)P\dot{x}(t) = 2x^{\mathrm{T}}(t)P[Ax(t) + Bx(t-h(t)) + Hg(t, x(t), x(t-h(t)))] \tag{7}$$

$$= x^{\mathrm{T}}(t)[PA + A^{\mathrm{T}}P^{\mathrm{T}} + 2\alpha P]x(t) + 2x^{\mathrm{T}}(t)PBx(t-h(t)) + 2x^{\mathrm{T}}(t)PHg(t, x(t), x(t-h(t))) - 2\alpha V_1.$$

From (2), it is easy to see that

$$\varepsilon(a^2 x(t)^T E^T Ex(t) + b^2 x(t-h(t))^T F^T Fx(t-h(t)) - g^T(.)g(.) \geq 0 \tag{8}$$

where any $\varepsilon > 0$. From (7) and (8), we have

$$\dot{V} \leq x^{\mathrm{T}}(t)[PA + A^{\mathrm{T}}P^{\mathrm{T}} + 2\alpha P]x(t) + 2x^{\mathrm{T}}(t)PBx(t-h(t)) + 2x^{\mathrm{T}}(t)PHg(t, x(t), x(t-h(t))) - 2\alpha V_1 \tag{9}$$

$$+\varepsilon(a^2 x(t)^{\mathrm{T}} E^{\mathrm{T}} Ex(t) + b^2 x(t-h(t))^{\mathrm{T}} F^{\mathrm{T}} Fx(t-h(t)) - g^{\mathrm{T}}(.)g(.)).$$

Next, the time-derivative of $V_k, k = 2,3,...,8,$ are computed and estimated as follows

$$\dot{V}_2 = x^{\mathrm{T}}(t)Qx(t) - (1-\dot{h}(t))e^{-2\alpha h(t)}x^{\mathrm{T}}(t-h(t))Qx(t-h(t)) - 2\alpha V_2; \tag{10}$$

$$\leq x^{\mathrm{T}}(t)Qx(t) - (1-\beta)e^{-2\alpha h_2}x^{\mathrm{T}}(t-h(t))Qx(t-h(t)) - 2\alpha V_2;$$

$$\dot{V}_3 = e^{-2\alpha h_1}x^{\mathrm{T}}(t-h_1)Zx(t-h_1) - e^{-2\alpha h_2}x^{\mathrm{T}}(t-h_2)Zx(t-h_2) - 2\alpha V_3; \tag{11}$$

$$\dot{V}_4 = h_1^2\dot{x}^{\mathrm{T}}(t)W_1\dot{x}(t) - h_1\int_{t-h_1}^{t} e^{2\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)W_1\dot{x}(s)ds - 2\alpha V_4 \tag{12}$$

$$\leq h_1^2\dot{x}^{\mathrm{T}}(t)W_1\dot{x}(t) - h_1 e^{-2\alpha h_1}\int_{t-h_1}^{t} \dot{x}^{\mathrm{T}}(s)W_1\dot{x}(s)ds - 2\alpha V_4;$$

$$\dot{V}_5 = h_2^2\dot{x}^{\mathrm{T}}(t)W_2\dot{x}(t) - h_2\int_{t-h_2}^{t} e^{2\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)W_2\dot{x}(s)ds - 2\alpha V_5 \tag{13}$$

$$\leq h_2^2\dot{x}^{\mathrm{T}}(t)W_2\dot{x}(t) - h_2 e^{-2\alpha h_2}\int_{t-h_2}^{t} \dot{x}^{\mathrm{T}}(s)W_2\dot{x}(s)ds - 2\alpha V_5;$$

$$\dot{V}_6 = h_{12}^2\dot{x}^{\mathrm{T}}(t)W\dot{x}(t) - h_{12}\int_{t-h_2}^{t-h_1} e^{2\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)W\dot{x}(s)ds - 2\alpha V_6 \tag{14}$$

$$\leq h_{12}^2\dot{x}^{\mathrm{T}}(t)W\dot{x}(t) - h_{12} e^{-2\alpha h_2}\int_{t-h_2}^{t-h_1} \dot{x}^{\mathrm{T}}(s)W\dot{x}(s)ds - 2\alpha V_6.$$

Applying the Proposition 2, we have

$$-h_1\int_{t-h_1}^{t} \dot{x}^{\mathrm{T}}(s)W_1\dot{x}(s)ds \leq -[x(t) - x(t-h_1)]^{\mathrm{T}} W_1[x(t) - x(t-h_1)] \tag{15}$$

and $$-h_2\int_{t-h_2}^{t} \dot{x}^{\mathrm{T}}(s)W_2\dot{x}(s)ds \leq -[x(t) - x(t-h_2)]^{\mathrm{T}} W_2[x(t) - x(t-h_2)] \tag{16}$$

Besides, we have

$$-h_{12}\int_{t-h_2}^{t-h_1} \dot{x}^{\mathrm{T}}(s)W\dot{x}(s)ds = -h_{12}\int_{t-h_2}^{t-h(t)} \dot{x}^{\mathrm{T}}(s)W\dot{x}(s)ds - h_{12}\int_{t-h(t)}^{t-h_1} \dot{x}^{\mathrm{T}}(s)W\dot{x}(s)ds.$$

Applying the Proposition 2, we have

$$-h_{12}\int_{t-h_2}^{t-h(t)}\dot{x}^{\mathrm{T}}(s)W\dot{x}(s)ds \tag{17}$$
$$\le -\frac{h_{12}}{h_2-h(t)}[(x(t-h(t))-x(t-h_2))^{\mathrm{T}}W(x(t-h(t))-x(t-h_2))$$
$$+3(x(t-h(t))+x(t-h_2))^{\mathrm{T}}W(x(t-h(t))+x(t-h_2))$$
$$+12(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)^{\mathrm{T}}W(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)$$
$$-12(x(t-h(t))+x(t-h_2))^{\mathrm{T}}W_2(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)]$$
$$\le -4x^{\mathrm{T}}(t-h(t))Wx(t-h(t))$$
$$-4x^{\mathrm{T}}(t-h_2)Wx(t-h_2)-4x^{\mathrm{T}}(t-h(t))W_2x(t-h_2)$$
$$+12x^{\mathrm{T}}(t-h(t))W(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)$$
$$+12x^{\mathrm{T}}(t-h_2)W(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)$$
$$-12(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds)^{\mathrm{T}}W(\frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x(s)ds).$$

Similarly, we have

$$-h_{12}\int_{t-h(t)}^{t-h_1}\dot{x}^{\mathrm{T}}(s)W\dot{x}(s)ds \tag{18}$$
$$\le -\frac{h_{12}}{h(t)-h_1}[(x(t-h_1)-x(t-h(t)))^{\mathrm{T}}W(x(t-h_1)-x(t-h(t)))$$
$$+3(x(t-h_1)+x(t-h(t)))^{\mathrm{T}}W(x(t-h_1)+x(t-h(t)))$$
$$+12(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)^{\mathrm{T}}W(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)$$
$$-12(x(t-h_1)+x(t-h(t)))^{\mathrm{T}}W(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)]$$
$$\le -4x^{\mathrm{T}}(t-h_1)Wx(t-h_1)-4x^{\mathrm{T}}(t-h(t))Wx(t-h(t))$$
$$-4x^{\mathrm{T}}(t-h_1)Wx(t-h(t))$$
$$+12x^{\mathrm{T}}(t-h_1)W(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)$$
$$+12x^{\mathrm{T}}(t-h(t))W(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)$$
$$-12(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds)^{\mathrm{T}}W(\frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x(s)ds).$$

On the other hand, using the following identities

$-\dot{x}(t)+Ax(t)+Bx(t-h(t))+Hg(t,x(t),x(t-h(t)))=0,$ we obtain

$$2\dot{x}(t)^{\mathrm{T}}[h_1^2W_1+h_2^2W_2+h_{12}^2W][-\dot{x}(t)+Ax(t)+Bx(t-h(t))+Hg(t,x(t),x(t-h(t)))]=0. \quad (19)$$

Therefore, from (9) to (21), it implies $\dot{V}(t,x_t)+2\alpha V(t,x_t)\le \Upsilon^{\mathrm{T}}(t)\Pi\Upsilon(t), \quad \forall t\ge 0,$ (20)

$$\text{where } \Pi=\begin{bmatrix} \Pi(1,1) & \Pi(1,2) & 0 & 0 & \Pi(1,5) & 0 & 0 & \Pi(1,8)\\ \star & \Pi(2,2) & \Pi(2,3) & \Pi(2,4) & \Pi(2,5) & \Pi(2,6) & \Pi(2,7) & 0\\ \star & \star & \Pi(3,3) & 0 & 0 & 0 & \Pi(3,7) & 0\\ \star & \star & \star & \Pi(4,4) & 0 & \Pi(4,6) & 0 & 0\\ \star & \star & \star & \star & \Pi(5,5) & 0 & 0 & \Pi(5,8)\\ \star & \star & \star & \star & \star & \Pi(6,6) & 0 & 0\\ \star & \star & \star & \star & \star & \star & \Pi(7,7) & 0\\ \star & \star & \star & \star & \star & \star & \star & \Pi(8,8) \end{bmatrix}$$

from inequality (4), we have $\Pi<0$. Consequently $\dot{V}(t,x_t)+2\alpha V(t,x_t)\le 0, \forall t\ge 0,$ and then $V(t,x_t)\le V(0,\phi)e^{-2\alpha t}\le \lambda_2\|\phi\|_\tau^2 e^{-2\alpha t}, \quad t\ge 0.$ Taking (6) into account, we obtain

$$\|x(t)\|\le\sqrt{\frac{\lambda_2}{\lambda_1}}\|\phi\|_\tau e^{-\alpha t}:=N\|\phi\|_\tau e^{-\alpha t}, \quad t\ge 0. \quad (21)$$

**Remark 1.** *If $\beta$ is unknown or $h(t)$ is not differentiable, then the following result can be obtained from Theorem 1 by setting $Q=0$, which will be introduced as Corollary 1.*

**Corollary 1.** *For given scalars $0<h_1\le h_2$ and $\alpha>0$. System (1) is $\alpha-$exponentially if there exist symmetric positive definite matrices $P,Z,W,W_i$ $(i=1,2)$ and any number $\varepsilon>0$ satisfying the following LMI:*

$$\begin{bmatrix} \Pi(1,1) & \Pi(1,2) & 0 & 0 & \Pi(1,5) & 0 & 0 & \Pi(1,8)\\ \star & \Pi(2,2) & \Pi(2,3) & \Pi(2,4) & \Pi(2,5) & \Pi(2,6) & \Pi(2,7) & 0\\ \star & \star & \Pi(3,3) & 0 & 0 & 0 & \Pi(3,7) & 0\\ \star & \star & \star & \Pi(4,4) & 0 & \Pi(4,6) & 0 & 0\\ \star & \star & \star & \star & \Pi(5,5) & 0 & 0 & \Pi(5,8)\\ \star & \star & \star & \star & \star & \Pi(6,6) & 0 & 0\\ \star & \star & \star & \star & \star & \star & \Pi(7,7) & 0\\ \star & \star & \star & \star & \star & \star & \star & \Pi(8,8) \end{bmatrix}<0. \quad (22)$$

where $\Pi(1,1)=PA+A^{\mathrm{T}}P^{\mathrm{T}}+2\alpha P+\varepsilon a^2E^{\mathrm{T}}E-e^{-2\alpha h_1}W_1-e^{-2\alpha h_2}W_2,$

$$\Pi(2,2)=-8e^{-2\alpha h_2}W+\varepsilon b^2F^{\mathrm{T}}F,$$

In other cases, $\Pi(i,j)$ are defined as in Theorem 1.

## 4. Conclusion

This paper has dealt with the problem of exponential stability analysis for a class of nonlinear systems with interval time-varying delays. A constructive approach and new delay-dependent condition in terms of linear matrix inequality have been proposed based on an improved LKF. Our condition guarantees the exponential stability of the system with special exponential delay rate.

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