# OPTIMAL PROVISIONING OF OPTICAL NETWORKS WITH ASYMMETRIC NODES 

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#### Abstract

Wavelength Switched Optical Networks (WSONs) have been designed to take advantage of all optical switching fabrics with a high level of automation and efficiency. Therein, the Wavelength Selective Switches (WSS) represent the core switching elements with a technology enabling multi-degree Reconfigurable Optical Add/Drop Multiplexers (ROADM) architectures with colorless and directionless switching. In this paper, we propose an optimization model to establish the best ROADM switching connectivity to maximize the grade of service, for a given number of ports. We show that the grade of service can vary significantly, up to $30 \%$, depending on the switching connectivity. Besides, the larger the network is, the more the variance increases: from $20 \%$ to $30 \%$, when the number of nodes varies from 14 to 24 .


Keywords: asymmetric networks, ROADM, routing, wavelength assignment.

## 1. Introduction

Recent developments in the Wavelength Selective Switch (WSS) technology enable multi-degree Reconfigurable Optical Add/Drop Multiplexers (ROADM) architectures with colorless, directionless, and contention-less switching. WSS is regarded as a very promising enabler for future reconfigurable wavelength division multiplexing (WDM) mesh networks, see, e.g., [1]. WSS selects individual wavelengths from multiple ingress ports and switches them to a common egress port, a key property of the WSS based ROADM referred to as Asymmetric Switching: in an optical switching element, the optical signal from one direction can only reach a subset of other directions. Such restrictions have been hardly considered in the studies on the RWA (Routing and Wavelength Assignment) problem.

Currently, most of the proposed RWA algorithms either assume a network with an ideal physical layer [2-4] or a network with physical layer impairments [5], with fully flexible node architectures. Very few studies consider RWA algorithms assuming nodes with architectural constraints such as the ones associated with asymmetric switching.

[^0]In this study, we propose a new ILP (Integer Linear Programming) model, called RWA_AN (RWA with asymmetric nodes), derived from the one of Jaumard, Meyer, and Thiongane [6] for the classical RWA problem. The resulting model is a large-scale optimization ILP model, which allows the exact solution of quite large RWA instances, i.e., up to 670 wavelengths, assuming all nodes are asymmetric and that the switching connectivity matrix is given. We next modify the RWA_AN Model and design the RWA_OAS Model (RWA with an optimized asymmetric switch matrix) in order to find the best switching connectivity matrix for a given number of ports and a given number of switching connections, concerning the grade of service (GoS), and compare the resulting GoS with the one of the first models.

## 2. Content

### 2.1. RWA_AN/RWA_OAS problems

### 2.1.1. Related works

In WDM networks, many papers have already appeared on the RWA problem. As it is a highly combinatorial problem, various heuristic scheme solutions have been proposed under different traffic assumptions with static or dynamic patterns, with single or multi hops, and for various objectives. Several compact ILP formulations have been also proposed for this problem: see [7] for surveys in the asymmetrical and symmetrical traffic cases respectively. Several improvements as well as comparisons of all these formulations can be found in [6]. However, none of the above studies consider the internal switching structures of optical nodes.

Chen et al. in [8] proposed two solution schemes, link-state (LS) and distance vector (DV) schemes, for dynamic light-path provisioning in optical WDM mesh networks with asymmetric nodes. In LS schemes, two proposed algorithms are the asymmetric switching-aware (ASA) Dijkstra's algorithm (the K -shortest path-based algorithm) and the entire path searching (EPS) algorithm. Results show that the ASA Dijkstra's algorithm has a high blocking probability while the computational complexity of the EPS algorithm is factorial, therefore non-polynomial. Hence, those algorithms cannot scale well when the network size increases. For the DV scheme, the authors proposed a routing solution based on information diffusion. Results show that the resulting algorithm can achieve a low blocking probability with low computational complexity.

In [9], the authors study how to provide resilience against node failures in WDM networks with asymmetric nodes. It implies the search for pairs of node disjoint paths, one for a working path and another for a backup path. While Bhandari's method [10] (indeed, Suurballe and Tarjan's algorithm [11]) can quickly compute optimal disjoint paths in WDM networks with symmetrical nodes, the same algorithm may fail in networks that have asymmetric nodes. The authors proposed an approach for adapting Bhandari's method to avoid the trap issues due to asymmetric nodes. However, the time complexity of the resulting algorithm is exponential and the proof of the optimality is not provided.

### 2.1.2. Statement of the RWA_AN and RWA_OAS problems

We consider a WDM optical network represented by a directed multigraph $G=(V, L)$ with node set $V=\left\{v_{1}, v_{2}, \ldots, v_{n}\right\}$ where each node is associated with a node of the physical network, and with arc set $L=\left\{l_{1}, l_{2}, \ldots, l_{n}\right\}$ where each arc is associated with a fiber link of the physical network: the number of arcs from $v$ to $v^{\prime}$ is equal to the number of fibers supporting traffic from $v$ to $v^{\prime}$. See Figure 1(a) for an example of a multigraph representing a multifiber optical network.

(a) Multigraph

(b) Expanded Multigraph

Figure 1. Directed multigraphs and expanded multigraphs

We will also use a so-called expanded directed graph $G^{E}=\left(V^{E}, L^{E}\right)$ where $V^{E}=$ $\cup_{v \in V} P O R T^{v}$ where $P O R T^{v}$ is the set of ports of node $v$, and $L^{E}=\left(\cup_{v \in V} L^{v}\right) \cup L$ where $L^{v}$ is set of links connecting ports of node $v$. An example of an expanded directed graph is shown in Figure 1(b).

The set of available wavelengths is denoted by $\Lambda=\left\{\lambda_{1}, \lambda_{2}, \ldots, \lambda_{w}\right\}$ with $W=|\Lambda|$. Traffic is described by set $T$ where $T_{s d}$ defines the number of connection requests from $v_{s}$ to $v_{d}$. Let $S D=\left\{\left(v_{s}, v_{d}\right) \in V \times V: T_{s d}>0\right\}$. We only consider single-hop routing, i.e., the same wavelength is used from source to destination for all connection requests.

We next study the two following RWA problems with asymmetric nodes:

- Problem RWA_AN, i.e., RWA with asymmetric nodes. Given an expanded multigraph $G^{E}$ corresponding to a WDM optical network with asymmetric nodes (for a given set of asymmetric switch connections), and a set of requested connections, and a suitable light-path $(p, \lambda)$ for each granted connection, where a lightpath is defined by the combination of a routing path $p$ and a wavelength so that no two paths sharing a link are assigned the same wavelength. We study the objective of maximizing the number of accepted connections (or the Grade of Service (GoS)), that is equivalently minimizing the blocking rate. This objective is most relevant when there is not enough transport capacity, i.e., enough available wavelengths, to accommodate all connection requests.
- Problem RWA_OAS, i.e., RWA with an optimized asymmetric switch matrix. Given an expanded multigraph $G^{E}$ corresponding to a WDM optical network with limited switching capabilities (i.e., number of switch con-nections between the ports of a node $v$, denoted by $S_{v}$ ), and the (asymmetric) switching node configuration that maximizes the GoS.


### 2.2. Solution of RWA with asymmetric nodes

### 2.2.1. RWA_AN model

The proposed optimization model relies on the concept of configurations. Let $C$ define the set of all wavelength configurations where a wavelength configuration is associated with a maximal set of link disjoint paths, all routed on the same wavelength, that can be used for satisfying a given fraction of the connections. A wavelength configuration $c$ is represented by a vector $a^{c}$ such that: $a_{s d}^{c}=$ number of connection requests from $v_{s}$ to $v_{d}$ that are supported by configuration $c$. A wavelength configuration $c$ is maximal if there does not exist another wavelength configuration $c^{\prime}$ such that $a^{c^{\prime}} \geq a^{c}$.

There are two sets of variables in the model. Let $z_{c}$ represent the number of selected occurrences of configuration $c$, each with a different wavelength. Variables $y_{s d}$ define the number of accepted connections from $v_{s}$ to $v_{d}$ for all $\left(v_{s}, v_{d}\right)$ in $S D$.

The objective function can be formulated as follows:

$$
\begin{equation*}
\max \sum_{\left(v_{s, v_{d}}\right) \in S D} y_{s d} \tag{1}
\end{equation*}
$$

subject to:

$$
\begin{gather*}
\sum_{c \in C} z_{c} \leq W  \tag{2}\\
\sum_{c \in C} a_{s d}^{c} z_{c} \geq y_{s d} \quad\left(v_{s}, v_{d}\right) \in S D  \tag{3}\\
y_{s d} \leq T_{s d} \quad\left(v_{s}, v_{d}\right) \in S D  \tag{4}\\
z_{c} \in \mathbb{N} \quad c \in C \tag{5}
\end{gather*}
$$

Constraints (2) ensure that we assign no more than the number of available wavelengths. Constraints (3) guarantee full support for each requested connection. Constraints (4) ensure that the number of accepted connections for a given pair sourcedestination does not exceed the demand.

### 2.2.2. Solution of the RWA_AN Model

## * Generalities

A straightforward way to solve the ILP model of the previous would be to enumerate all potential wavelength configurations. Although easy, it will not be scalable. Indeed, the ILP model has a natural decomposition scheme which allows its linear relaxation to be solved by column generation techniques.

The Column Generation method is nowadays a well-known technique for solving efficiently large-scale optimization problems [2]. The challenge lies in the modeling for identifying a proper decomposition of the original problem into a so-called master problem and one or several so-called pricing problems. The master problem corresponds to a linear program subject to the first set of explicit constraints and the second set of implicit constraints expressed throughout properties of the coefficients of the constraint
matrix. The pricing problems consist of the optimization of the so-called reduced cost subject to the set of implicit constraints: It either identifies favorable columns to be added to the master problem or indicates that no such column exists. The solution scheme is a two-step process where we first solve the linear relaxation of the master problem using column generation techniques, and then design an algorithm (e.g., rounding off algorithm or the ILP solution of the restricted master problem) in order to derive an integer solution such that the so-called optimality gap $\left(z_{I L P}-z_{L P}^{*}\right) / z_{L P}^{*}$, (where $z_{L P}^{*}$ is the optimal value of the linear relaxation, and $z_{I L P}$ is the incumbent integer solution) is as small as possible. The optimization model corresponds to a master problem with a pricing problem, which is described as follows:

## * Pricing problem

We introduce one set of decision variables $\alpha=\left(\alpha_{l}^{s d}\right)$ such that $\alpha_{l}^{s d}=1$ if there exists a light-path from $v_{s}$ to $v_{d}$, which goes through link $l, 0$ otherwise.
The objective of the pricing problem, $\operatorname{RCOST}(\alpha)$, is weighted with the dual variables. $u^{(2)} \geq 0$ be the value of the dual variable associated with constraint (2) and $u_{s d}^{(3)} \geq 0$ the values of the dual variables associated with constraint (3) in the optimal linear relaxation solution of the restricted master problem, i.e., the problem (1)-(5).
The pricing problem can be written as follows:

$$
\begin{equation*}
R \operatorname{cost}(\alpha)=-u^{(2)}+\sum_{\left(v_{s}, v_{d}\right) \in S D} \sum_{l \in \omega^{+}\left(v_{s}\right)} \alpha_{l}^{s d} u_{s d}^{(3)} \tag{6}
\end{equation*}
$$

subject to:

$$
\begin{array}{cl}
\sum_{\left(v_{s}, v_{d}\right) \in S D} \alpha_{l}^{s d} l \in L^{E} \\
\sum_{l \in \omega^{+}(v)} \alpha_{l}^{s d}=\sum_{l \in \omega^{-}(v)} \alpha_{l}^{s d} & \left(v_{s}, v_{d}\right) \in S D, v \in V^{E} \backslash\left(v_{s}, v_{d}\right) \\
\sum_{l \in \omega^{+}(v)} \alpha_{l}^{s d} \leq T_{s d} & \left(v_{s}, v_{d}\right) \in S D \\
\sum_{l \in \omega^{-}(v)} \alpha_{l}^{s d}=0 & \left(v_{s}, v_{d}\right) \in S D \\
\sum_{l \in L^{v}} \alpha_{l}^{s d}=\sum_{l \in \omega^{-}(v)} \alpha_{l}^{s d} & \left(v_{s}, v_{d}\right) \in S D, v \in V^{E} \backslash\left(v_{s}, v_{d}\right) \\
\alpha_{l}^{s d} \in\{0,1\} \tag{12}
\end{array} \quad\left(v_{s}, v_{d}\right) \in S D, l \in L^{E} .
$$

Constraints (7) and (8) define a set of disjoint paths, i.e., a configuration. Constraints (9) and (10) ensure that we grant no more than the number of requested connections. Constraints (11) ensure that each path only goes through at most one internal connection of an asymmetric node.

The restricted master problem, i.e., the master problem with a very limited number of configurations and the pricing problem are solved alternately until the optimality
condition is met, i.e., the pricing problem cannot generate any new configuration with a negative reduced, see again [10] for more details on a CG-ILP solution scheme. Consequently, if $\operatorname{RCOST}(\alpha) \leq 0$, then problem (1)-(5) has been solved to optimality. Otherwise, the routing configuration c defined by the vector $\left(a_{s d}^{c}\right)$ with $a_{s d}^{c}=$ $\sum_{l \in \omega^{+}(v)} \alpha_{l}^{s d}$ for $\left(v_{s}, v_{d}\right) \in S D$ is added to the current restricted master problem, which is solved again. Once the linear relaxation of the restricted master is optimally solved, we solve the ILP model resulting from the set of columns of the last solved restricted master problem in order to output an ILP solution for RWA AN Problem (1)-(5).

### 2.3. RWA with optimal asymmetric switch node configurations

### 2.3.1. RWA_OAS model

We modify the definition of the wavelength configurations we used in the previous section as follows: each configuration $c$ is now represented by two binary vectors $a^{c}$ (same definition as before) and $b^{c}$ where $b_{l}^{c}=1$ if configuration $c$ uses link $l=L^{v}$ (i.e., internal port connection) and 0 otherwise.

We also need to introduce one more set of variables: $x_{l}=1$ if link $l$ is chosen for an internal port connection of an asymmetric node, and 0 otherwise.
RWA_OAS Model has the same objective as RWA_AN, and includes the same set of constraints, as well as the following set of additional constraints:

$$
\begin{gather*}
\sum_{c \in C} b_{l}^{c} z_{c} \leq W x_{l} \quad v \in V, l \in L^{E}  \tag{13}\\
\sum_{l \in L^{v}} x_{l} \leq S_{v} \quad v \in V  \tag{14}\\
\sum_{l \in \omega^{+}(v)} x_{l} \geq 1 ; \sum_{l \in \omega^{-}(v)} x_{l} \geq 1 \quad v \in V^{E} \tag{15}
\end{gather*}
$$

Constraints (13) ensure that link 1 is used in a configuration only if it is selected for an internal port connection in a switching matrix (i.e., $x_{l}=1$ ). Constraints (14) ensure that the number of internal port connections of an asymmetric node does not exceed the limit on the number of internal port connections for that node. Constraints (15) ensure that there is at least one internal port connection per node in the expanded graph $\left(G^{E}\right)$.

### 2.3.2. Solution of RWA_OAS model

The solution scheme of RWA_OAS Model follows the one for the RWA_AN model, i.e., a CG-ILP solution scheme, which requires the definition and the solution of a pricing problem in order to generate the configurations. Let $u^{(13)}$ be the dual value associated with constraint (13). The objective function of the pricing problem can be written as follows:

$$
\begin{equation*}
R \operatorname{CoST}(\alpha)=u^{(2)}+\sum_{\left(v_{s}, v_{d}\right) \in S D} \sum_{l \in \omega^{+}\left(v_{s}\right)} \alpha_{l}^{s d} u_{s d}^{(3)}-\sum_{v \in S D} \sum_{l \in L^{v}} x_{l} u^{(13)} \tag{16}
\end{equation*}
$$

We use the same set of constraints as for the pricing problem of RWA_AN, together with some modified constraints that are next described. Replace the set of constraints (7) by the following constraint set:

$$
\begin{equation*}
\sum_{\left(v_{s}, v_{d}\right) \in S D} \alpha_{l}^{s d} \leq x_{l} \quad l \in L^{E} \tag{17}
\end{equation*}
$$

Constraints (17) ensure that link 1 is only chosen for the configuration under construction if $x_{l}=1$, i.e., it is chosen for connection of a switching matrix.

Add the following new set of constraints:

$$
\begin{equation*}
\sum_{l \in L^{v}} x_{l} \leq S_{v} \quad v \in V \tag{18}
\end{equation*}
$$

In order to ensure that the number of internal port connections (IPC) of an asymmetric node does not exceed the IPC number for that node.

The initial step of the solution process, i.e., the solution of the linear relaxation of the RWA_OAS model is the same as for the RWA_AN model, using a column generation algorithm. Next, we aim at finding an integer solution to the RWA_OAS problem. We found that this integer solution consists of the integer solution of the RWA_AN model and its respective set of asymmetric switch connections which is defined by a combination of values $x_{l}$ for $l \in L^{v}, v \in V$. Hence, we propose a two-step process. In the first step, we identify the binary values of the $x$ variables using a sequential rounding-off mechanism (see Algorithm 1 below). Once all the $x$ variables have been set to either 0 or 1 (i.e., internal port connections have been selected), we solve the remaining problem with an ILP solver.

> Algorithm 1 Rounding-based algorithm for setting the integer values of the $x$ variables $x^{I P} \leftarrow x^{L P}$
> while $\exists x_{l}^{I P} \notin Z^{+}$do
> Select the variable $x_{l}^{I P}$ with the largest fractional value

$x_{l}^{I P} \leftarrow \operatorname{ROUND}\left(x_{l}^{L P}\right)$
Solve the CG_ILP model where the restricted master problem is (1)-(5), (13)-(15) and the pricing problem (6), (8)-(12), (17)-(18) with the additional constraint $x_{l}=x_{l}^{I P}$ $x^{I P} \leftarrow x^{L P}$
end while

[^1]
### 2.4. Numerical results

The two RWA_AN and RWA_OAS models were solved using the solution process described in Sections 3 and 4. Algorithms were implemented using the OPL programming language and solved using CPLEX 12.5. Programs were run on a 2.2 GHz AMD Opteron 64-bit processor with 4GB RAM.

We next describe the data instances and then discuss the quality of the solutions provided by both models. We then look at the grade of service vs. switching connectivity for a given number of ports, under two switching scenarios.

### 2.4.1. Network and data instances

We run experiments on the NSFNET topology: 14 nodes, 42 directed links [11]. The internal node edges describe the optical switching capabilities. Note that each switching edge corresponds to two switching directed capabilities in opposite directions. The numbers beside nodes define the number of switching capabilities of nodes for the RWA_OAS model. $(x)$ indicates that they are $x / 2$ bidirectional switching capabilities between the ports, i.e., whenever one can switch from port $\pi$ to port $\pi$, we assume it is also possible to switch from $\pi^{\prime}$ to port $\pi$. Those switching edges are randomly generated for the RWA_AN model. The number of wavelengths is set to 30 .

We run experiments on two different topologies: the 14 -node, 42 -(directed) link NSFNET and the 24-node, 86-(directed) link USANET [12]. In Figure 2, the blue lines describe the internal port connections for each node. They are randomly generated for the RWA_AN model while red numbers beside nodes define a limit on the number of the switching capabilities of nodes for the RWA_OAS model: $(x)$ indicates that they can be $x / 2$ bidirectional switching capabilities between the ports, i.e., whenever one can transfer from port $\pi$ to port $\pi^{\prime}$, we assume it is also possible from $\pi^{\prime}$ to port $\pi$. The number of wavelengths is set to 30 for both topologies.


Figure 2. Network topology
For each network topology, we consider 10 traffic instances. For the first traffic instance (i.e., SD_0), the directed traffic demand matrix $T=\left[T_{s d}\right]$ is generated by drawing the (integer) traffic demands (in units of light-paths) uniformly at random in
$\{0,1,2,3,4,5\}$. The following traffic instances correspond to incremental traffic: $S D_{-} \in$ $S D_{-}^{(i+1)}$ where $S D_{-}(i+1)$ is built upon $S D_{-}$by randomly adding from 1 up to 5 more requests for each pair of nodes. Table 1 gives the detailed characteristics of the request sets. For each traffic in- stance, we provide the number of node pairs with requests ( $|S D|$ ) and the overall number of traffic requests $\left(\sum_{\left\{v_{s}, v_{d}\right\} \in S D} d_{s d}\right)$. Note that 182 is the maximum number of node pairs.

### 2.4.2. Quality of the RWA_AN and RWA_OAS solutions

We here discuss the accuracy of the solutions obtained by both models using the solution processes. The accuracy is indicated in the columns entitled $\varepsilon$ in Table 4. We can see that $\varepsilon$ varies from $0.0 \%$ to $8.8 \%$, meaning that the output solutions are always within a $9 \%$ accuracy. While these accuracies are satisfactory, they could be easily reduced to about $1 \%$ with the help of a heuristic for generating an initial solution. Moreover, note that the results correspond to the largest traffic instances solved $\varepsilon$-optimally so far with 30 wavelengths.

We next compare the grades of service resulting from the solutions of models RWA_AN and RWA_OAS, see the last column in Table 4. We can observe a $16.2 \%$ average increase of the grades of service, $G o S_{A N}^{1}$ and $G o S_{O A S}^{1}$ for all traffic instances assuming a first port scenario as described by the number $(x)$ in Figure 3(a). Therefore, optimizing the switching configurations for a given number of ports makes a significant difference. Let us see the resulting topologies in Figure 2. Indeed, Figure 2(b) provides the solution of the $S D \_2$ instance where all switching connectivity are bidirectional, while Figure 2(c) describes the solution of the $S D_{-} 9$ instance where several switching connectivities are unidirectional (see node $v_{5}$ ).

Table 1. Performance of the RWA_AN/RWA_OAS model solutions - NSFNET network with 30 wavelengths

| Traffic | RWA_AN Model |  |  | RWA_OAS Model |  |  |  | RWA_AN FULL |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $z_{A N}^{L P}$ | $z_{A N}^{I L P}$ | $\begin{gathered} \varepsilon \\ (\%) \end{gathered}$ | $z_{O A S}^{I L P}$ | $z_{O A S}^{I L P}$ | $\begin{gathered} \varepsilon \\ (\%) \end{gathered}$ | $\frac{z_{O A S}^{I L P}-z_{A N}^{I L P}}{z_{A N}^{I L P}}$ <br> (\%) | $z_{F U L L}^{I L P}$ | $z_{F U L L}^{\text {ILP }}$ | $\begin{gathered} \varepsilon \\ (\%) \end{gathered}$ | $\frac{z_{F U L L}^{I L P}-z_{A N}^{I L P}}{z_{A N}^{I L P}}$ <br> (\%) |
| SD_0 | 346.0 | 331 | 4.3 | 346.0 | 335 | 3.2 | 1.2 | 346.0 | 339 | 2.0 | 2.4 |
| SD_1 | 494.3 | 477 | 3.5 | 548.0 | 527 | 3.8 | 10.5 | 548.0 | 527 | 3.8 | 10.5 |
| SD_2 | 597.4 | 569 | 4.8 | 681.3 | 657 | 3.6 | 15.5 | 681.5 | 658 | 3.4 | 15.6 |
| SD_3 | 676.8 | 661 | 2.3 | 766.8 | 748 | 2.5 | 13.2 | 767.8 | 751 | 2.2 | 13.6 |
| SD_4 | 739.6 | 731 | 1.2 | 833.7 | 803 | 3.7 | 9.8 | 835.7 | 805 | 3.7 | 10.1 |
| SD_5 | 794.2 | 781 | 1.7 | 884.3 | 865 | 2.2 | 10.8 | 889.4 | 867 | 2.5 | 11.0 |
| SD_6 | 839.9 | 821 | 2.2 | 924.0 | 913 | 1.2 | 11.2 | 932.1 | 917 | 1.6 | 11.7 |
| SD_7 | 888.2 | 872 | 1.8 | 967.0 | 923 | 4.5 | 5.8 | 965.7 | 923 | 4.4 | 5.8 |
| SD_8 | 903.0 | 902 | 0.1 | 1,008.0 | 998 | 1.0 | 10.6 | 1,008.0 | 998 | 1.0 | 10.6 |
| SD_9 | 918.9 | 917 | 0.2 | 1,048.6 | 1,034 | 1.4 | 12.8 | 1,048.9 | 1,033 | 1.5 | 12.6 |
| SD_10 | 933.0 | 933 | 0.0 | 1,084.9 | 1,063 | 2.0 | 13.9 | 1,085.5 | 1,075 | 1.0 | 15.2 |

Table 2. Performance of the RWA_AN/RWA_OAS model solutions

- NSFNET network when the number of wavelengths increases from 30 to 600

| Traffic instances | RWA_AN Model |  |  |  |  | RWA_OAS Model |  |  |  |  | Comparison $\frac{z_{O A S}^{I L P}-z_{A N}^{I L P}}{z_{A N}^{I L P}}$ <br> (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $z_{A N}^{L P}$ | $z_{A N}^{I L P}$ | $\begin{gathered} \varepsilon \\ (\%) \end{gathered}$ | Configurations |  | $z_{O A S}^{I L P}$ | $z_{O A S}^{I L P}$ | $\begin{gathered} \varepsilon \\ (\%) \end{gathered}$ | Configurati ons |  |  |
|  |  |  |  | \#Gen | \#Select |  |  |  | $\begin{gathered} \text { \#Ge } \\ \text { n. } \end{gathered}$ | \#Se <br> lect. |  |
| SD_0 | 346.0 | 330 | 4.6 | 417 | 29 | 346.0 | 338 | 2.3 | 61 | 23 | 2.4 |
| SD_1 | 695.6 | 676 | 2.8 | 513 | 56 | 696.0 | 692 | 0.6 | 64 | 28 | 2.4 |
| SD_2 | 1043.0 | 994 | 4.7 | 470 | 65 | 1043.0 | 1038 | 0.5 | 60 | 36 | 4.4 |
| SD_3 | 1413.0 | 1363 | 3.5 | 538 | 76 | 1413.0 | 1413 | 0.0 | 42 | 21 | 3.7 |
| SD_4 | 1793.0 | 1714 | 4.4 | 604 | 82 | 1797.0 | 1783 | 0.8 | 61 | 39 | 4.0 |
| SD_5 | 2165.0 | 2076 | 4.1 | 625 | 92 | 2191.0 | 2191 | 0.0 | 63 | 34 | 5.5 |
| SD_6 | 2516.2 | 2443 | 2.9 | 601 | 97 | 2541.0 | 2538 | 0.1 | 70 | 40 | 3.9 |
| SD_7 | 2866.8 | 2762 | 3.7 | 577 | 87 | 2880.0 | 2880 | 0.0 | 62 | 37 | 4.3 |
| SD_8 | 3185.7 | 3124 | 1.9 | 519 | 96 | 3229.0 | 3225 | 0.1 | 66 | 42 | 3.2 |
| SD_9 | 3586.8 | 3516 | 2.0 | 600 | 101 | 3611.0 | 3607 | 0.1 | 61 | 41 | 2.6 |
| SD_10 | 3949.3 | 3878 | 1.8 | 567 | 100 | 3973.0 | 3965 | 0.2 | 66 | 41 | 2.2 |

### 2.4.3. Characteristic of the RWA_AN and RWA_OAS solutions

Table 3 also shows the number of generated configurations. We observe that only a very small number of configurations are generated while there are millions of possible configurations, thanks to the column generation technique which allows reaching an optimal solution of the linear relaxation without the requirement of an explicit enumeration of all the configurations. The number of selected configurations, which are part of the near optimal ILP solutions, is even smaller as can be observed in Table 4.

Table 3. Performance of the RWA_AN/RWA_OAS model solutions - USANET network with 100 wavelengths

|  | RWA_AN Model |  |  | RWA_OAS Model |  |  |  | RWA_AN FULL |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $z_{A N}^{L P}$ | $z_{A N}^{I L P}$ | $\begin{gathered} \varepsilon \\ (\%) \end{gathered}$ | $z_{O A S}^{I L P}$ | $z_{O A S}^{I L P}$ | $\begin{gathered} \varepsilon \\ (\%) \end{gathered}$ | $\begin{gathered} \frac{z_{O A S}^{I L P}-z_{A N}^{I L P}}{z_{A N}^{I L P}} \\ (\%) \end{gathered}$ | $z_{F U L L}^{I L P}$ | $z_{F U L L}^{I L P}$ | $\begin{gathered} \varepsilon \\ (\%) \end{gathered}$ | $\begin{gathered} \frac{z_{F U L L}^{I L P}-z_{A N}^{I L P}}{z_{A N}^{I L P}} \\ (\%) \end{gathered}$ |
| SD_0 | 856.9 | 816 | 4.8 | 1,049.0 | 1,036 | 1.2 | 27.0 | 1,049.0 | 1,049 | 0.0 | 28.6 |
| SD_1 | 1,468.0 | 1,412 | 3.8 | 2,007.8 | 1,911 | 4.8 | 35.3 | 2,018.9 | 1,923 | 4.8 | 36.2 |
| SD_2 | 1,863.4 | 1,758 | 5.6 | 2,541.6 | 2,423 | 4.7 | 37.8 | 2,569.0 | 2,455 | 4.4 | 39.6 |
| SD_3 | 2,260.3 | 2,209 | 2.3 | 3,023.1 | 2,913 | 3.6 | 31.9 | 3,056.0 | 2,943 | 3.7 | 33.2 |
| SD_4 | 2,507.4 | 2,393 | 4.6 | 3,412.2 | 3,257 | 4.5 | 36.1 | 3,424.3 | 3,271 | 4.5 | 36.7 |
| SD_5 | 2,740.3 | 2,642 | 3.6 | 3,688.0 | 3,517 | 4.6 | 33.1 | 3,708.0 | 3,537 | 4.6 | 33.9 |
| SD_6 | 2,930.5 | 2,793 | 4.7 | 3,918.8 | 3,746 | 4.4 | 34.1 | 3,947.0 | 3,757 | 4.8 | 34.5 |
| SD_7 | 3,124.3 | 3,058 | 2.1 | 4,116.3 | 3,964 | 3.7 | 29.6 | 4,139.0 | 4,052 | 2.1 | 32.5 |
| SD_8 | 3,271.1 | 3,131 | 4.3 | 4,280.5 | 4,077 | 4.8 | 30.2 | 4,332.0 | 4,186 | 3.4 | 33.7 |
| SD_9 | 3,456.6 | 3,384 | 2.1 | 4,498.6 | 4,286 | 4.7 | 26.7 | 4,557.0 | 4,367 | 4.2 | 29.0 |
| SD_10 | 3,598.3 | 3,497 | 2.8 | 4,638.2 | 4,491 | 3.2 | 28.4 | 4,701.0 | 4,628 | 1.6 | 32.3 |

Table 4. Performance of the RWA_AN/RWA_OAS model solutions

- NSFNET network when the number of wavelengths increases from 100 to 1600

|  | RWA_AN Model |  |  |  |  | RWA_OAS Model |  |  |  |  | Comparison $\frac{z_{O A S}^{I L P}-z_{A N}^{I L P}}{z_{A N}^{I L P}}$ <br> (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $z_{A N}^{L P}$ | $z_{A N}^{I L P}$ | $\begin{gathered} \varepsilon \\ (\%) \end{gathered}$ | Configurations |  | $z_{\text {OAS }}^{I L P}$ | $z_{O A S}^{I L P}$ | $\begin{gathered} \varepsilon \\ (\%) \end{gathered}$ | Configurations |  |  |
|  |  |  |  | \#Gen. | \#Select. |  |  |  | \#Gen. | \#Select. |  |
| SD_0 | 995.6 | 983 | 1.3 | 347 | 147 | 1,049.0 | 1,049.0 | 0.0 | 84 | 44 | 6.7 |
| SD_1 | 1,972.0 | 1,939 | 1.7 | 446 | 150 | 2,168.0 | 2,108.0 | 2.8 | 184 | 97 | 8.7 |
| SD_2 | 2,831.0 | 2,804 | 1.0 | 448 | 152 | 3,054.0 | 2,948.0 | 3.5 | 410 | 111 | 5.1 |
| SD_3 | 3,609.0 | 3,583 | 0.7 | 442 | 147 | 3,879.0 | 3,702.0 | 4.6 | 387 | 105 | 3.3 |
| SD_4 | 4,407.0 | 4,351 | 1.3 | 503 | 139 | 4,727.0 | 4,623.0 | 2.2 | 356 | 117 | 6.3 |
| SD_5 | 5,281.0 | 5,252 | 0.5 | 436 | 138 | 5,560.0 | 5,413.0 | 2.6 | 360 | 114 | 3.1 |
| SD_6 | 6,088.0 | 6,056 | 0.5 | 513 | 161 | 6,388.0 | 6,188.0 | 3.1 | 387 | 126 | 2.2 |
| SD_7 | 6,873.0 | 6,831 | 0.6 | 567 | 160 | 7,206.0 | 7,013.0 | 2.7 | 390 | 129 | 2.7 |
| SD_8 | 7,642.0 | 7,604 | 0.5 | 575 | 144 | 8,025.2 | 7,849.0 | 2.2 | 376 | 137 | 3.2 |
| SD_9 | 8,478.0 | 8,417 | 0.7 | 632 | 155 | 8,875.0 | 8,726.0 | 1.7 | 376 | 142 | 3.7 |
| SD_10 | 9,263.0 | 9,216 | 0.5 | 618 | 146 | 9,677.6 | 9,529.0 | 1.5 | 380 | 142 | 3.4 |

### 2.4.4. Grade of service vs. switching connectivity

In Figure 3, we investigate the impact of increasing the number of ports on the grade of service, and then the optimization of the switching connectivity on the grade of service. In order to do so, we run experiments on different sets of traffic requests, and for a different number of ports, for both models.


Figure 3. Impact of the number of internal connections

Indeed, we increase the number of switching capabilities by one bidirectional IPC (two directional IPCs in opposite directions) for the following set of nodes: $\left\{v_{3}, v_{4}, v_{7}, v_{8}\right\}$. While this increased switching capability does not affect much the results of the RWA_OAS Model (see lines entitled NSF_OAS and NSF_OAS_ADD), it is different for the RWA_AN Model (see lines entitled NSF_AN and NSF_AN_ADD): we can see that it allows a $9.6 \%$ average increase of the grades of service for the traffic
instances. However, if the switching connectivity matrix is optimized according to the solution of the RWA_OAS model, no additional IPC is required for improving the grades of service. Note that even the improved GoS of model RWA_AN, i.e., $\operatorname{GoS}_{A N}^{2}$, does not reach the values of the $G o S_{O A S}^{2}$. Consequently, it is really worth optimizing the switching capabilities of a ROADM in order to save on the number of ports, or the switching connectivity requirements.

### 2.4.5. Analysis of the number of hops

In Figure 4, we provide the percentage of wavelength paths based on the number of hops (i.e., the number of links) in the optimal integer solutions for both models. Indeed, lines entitled "1-hop", "2-hops", "3-hops" and " $\geq$ 4-hops" describe the proportion of wavelength paths with $1,2,3$, or more than 4 links of the length respectively. When the number of traffic demands increases, going from SD_0 to SD_19, we can see that the number of "1-hop" paths goes up while the number of " $\geq 4$-hops" paths goes down, even to zero for almost traffic instances. The results show that: with respect to the objective of maximizing the number of requests in a network with a fixed number of wavelengths, when traffic demands are increased, the solution consists essentially of single hop optical paths.


Figure 4. Analysis of the number of hops

## 3. Conclusions

We have proposed a scalable and efficient optimization model for determining the best switching matrices for each ROADM, for a given number of ports, and have shown how critical is such a choice to maximize the grade of service. Future work will include the adaptation of this model to dynamic traffic to take advantage of the flexibility of ROADMs.

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[^1]:    Algorithm 1 is started with the optimal LP (Linear Programming) relaxation solution, $x^{I P}$, as output by the column generation algorithm. If all $x^{I P}$ for $l \in L^{v}, v \in V$ have integer values, an optimum asymmetric switch matrix has been found for all asymmetric nodes and there is no need to proceed with Algorithm 1, use the same solution approach as for finding an integer solution for Model RWA_AN. On the other hand, if at least one variable $x_{l}$ has a fractional value in $x^{I P}$, one of them with maximum fractional value is selected and rounded to its closest integer value. Then, the resulting restricted master problem with one more integer $x_{l}$ variable is reoptimized, meaning the pricing problem is solved again until the LP optimality condition is met again. This process continues until there is no remaining variable $x_{l}$ with a fractional value.

