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PHASE TRANSITION OF THE REISSNER-NORDSTRÖM BLACK HOLE

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Abstract. The phase transition of matter outside the four-dimensional Reissner-Nordström charged black hole have been investigated. Based on the metric we have found analytic expressions for thermodynamic quantities as temperature, pressure and isobaric specific heat. The numerical results have shown that for temperatures T less than the critical value T_c there exits a ”liquid-gas” phase transition similar to the Van der Waals fluid. In addition, also pointed out that both temperature and spatial curvature affect phase transitions, but phase transitions are always the first oder.
Keywords: phase transition, black hole, first order, critical value.

1. Introduction

Black hole is a special object, there are Hawking radiation, entropy and phase transition, etc. Although black holes microscopic mechanism is still not clear, its thermodynamic properties can be systematically studied as it is a thermodynamic system which is described by only a few physical quantities, such as mass, charge, pressure, temperature, entropy, etc. Generally, these thermodynamic quantities are described on the horizon and they are related by the first law. Then, thermodynamics of the black hole has become an interesting and challenging topic. This has gotten new attention with the development of Anti de Sitter/Conformal Field Theory (AdS/CFT) duality, since the black hole thermodynamics on holographic screen has acquired a new and interesting interpretation as a duality of the corresponding field theory [1]. Thermodynamic properties of black holes have been studied for many years [2-4]. One has been found that black holes can not only be described by conventional thermodynamic variables such as temperature, pressure, entropy, etc. but also have a rich phase structure and critical phenomena [5, 6].

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In our recent paper [7], we considered the electrically charged AdS black hole and viewed the cosmological constant as a dynamical pressure and the black hole volume as its conjugate quantity. The consequence, its thermodynamic properties become richer, the phase transition behavior of electrically charged AdS black hole is reminiscent of the liquid-gas phase transition in a Van der Waals system.

In this paper, continuing the research direction done in [6-9] we consider further on phase structure of Reissner-Nordström charged black hole. The results show that there is a transition between small and large black holes (with horizonal radius change), the system share the same oscillatory behavior in pressure-volume graph, in temperature-entropy graph. In particular, the behavior of isobaric specific heat and free energy show that the phase transitions are still the first oder when temperature or spatial curvature change.

2. Content

2.1. Basic conculations

Let us start from the RN charged BH in four dimensions Anti de Sitter (AdS_4) spacetime whose metric is given by

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2, \tag{2.1}$$

in which

$$f(r) = k - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{L^2}, \tag{2.2}$$

outside of the BH. Here M and Q are mass and charge of the BH, correspondingly; L is the AdS_4 radius (related to the cosmological constant Λ: $\Lambda = -3/L^2$) and k stands for the spatial curvature of BH. Specifically, $k > 0, k = 0$ and $k < 0$ give a spherical, planar and hyperbolic geometry, respectively. In (2.2) $d\Omega_2^2$ is the metric of the two-sphere S^2 of radius $1/\sqrt{k}$.

By definition, the radius of event horizon r_h is the root of $f(r_h) = 0$. So

$$f(r_h) = k - \frac{2M}{r_h} + \frac{Q^2}{r_h^2} + \frac{r_h^2}{L^2} = 0, \tag{2.3}$$

from which we derive

$$M = \frac{r_h}{2}\left[k + \frac{Q^2}{r_h^2} + \frac{r_h^2}{L^2}\right]. \tag{2.4}$$

Inserting (2.4) into (2.2) we obtain

$$f(r) = k\left(1 - \frac{r_h}{r}\right) + \frac{Q^2}{r^2}\left(1 - \frac{r_h}{r}\right) + \frac{r^2}{L^2}\left(1 - \frac{r_h^3}{r^3}\right). \tag{2.5}$$

The Hawking temperature T of the BH is determined by expression (formula 3.26 of [1])

$$T = \frac{f'(r_h)}{4\pi}. \tag{2.6}$$

Combining (2.4), (2.5) and (2.6) we obtain

$$T = \frac{1}{2\pi}\left(-\frac{Q^2}{r_h^3} + \frac{M}{r_h^2} + \frac{r_h}{L^2}\right) = \frac{1}{4\pi r_h}\left(k - \frac{Q^2}{r_h^2} + \frac{3r_h^2}{L^2}\right). \tag{2.7}$$

In the case of the RN charged BH the pressure P is defined as [5]:

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi}\frac{1}{L^2}, \tag{2.8}$$

and the volume is determined by:

$$V = \frac{4}{3}\pi r_h^3, \tag{2.9}$$

The entropy ζ of BH is given by (formula 3.1 and 3.2 of [1])

$$\zeta = \pi r_h^2. \tag{2.10}$$

Basing on (2.7), (2.8) and (2.10) we arrive at the isobaric specific heat

$$C_P \equiv T\left(\frac{\partial \zeta}{\partial T}\right)_P = \frac{2\pi r_h^2\left(8\pi P r_h^4 + k r_h^2 - Q^2\right)}{8\pi P r_h^4 - k r_h^2 + 3Q^2}. \tag{2.11}$$

2.2. Phase transition

In this section, the numerical calculation is investigated order to get insight into the phase transition. For convenience, below dimensionless quantities are used.

First of all, let us study the state equation $P(V,T)$. By introducing the critical quantities

$$P_c = \frac{k^2}{96\pi Q^2}; \quad V_c = \frac{8\sqrt{6}\pi Q^3}{k^{3/2}}; \quad T_c = \frac{k^{3/2}}{3\sqrt{6}\pi Q}, \tag{2.12}$$

and combining formulas (2.7), (2.8) and (2.9) we obtain

$$P/P_c = \frac{1 - 6(V/V_c)^{2/3} + 8(T/T_c)(V/V_c)}{3(V/V_c)^{4/3}}. \tag{2.13}$$

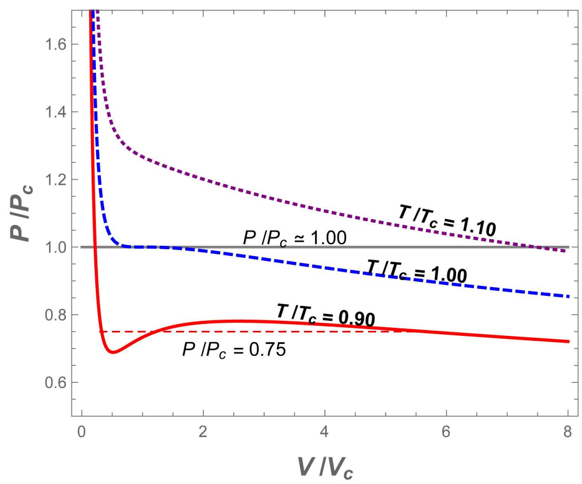

Figure 1. The volume V dependence of the pressure P at $T/T_c = 0.9; 1.0; 1.1$

Now we draw the volume V dependence of the pressure P at several values of the temperature T. Figure 1 represents the shape of pressure curves in the $P - V$ plane. As seen clearly from this figure, for $T < T_c$ there exists a minimum of pressure. It means that there is a ”liquidgas” phase transition similar to the Van der Waals fluid. In contrast, with $T > T_c$ there will be no phase transition-the system is always ”gaseous”. At $T = T_c$ the pressure curve have only inflection points, so $T = T_c$ is the critical temperature.

Next the radius horizon dependence of the Hawking temperature is concerned. Basing on the (2.7) and (2.8) we are able to write

$$T/T_c = \frac{-1 + 6(r_h/r_c)^2 + 3(P/P_c)(r_h/r_c)^4}{8(r_h/r_c)^3}, \tag{2.14}$$

where

$$r_c = Q\sqrt{\frac{6}{k}}. \tag{2.15}$$

Then we draw the radius horizon r_h dependence of the temperature T at several values of the pressure P, which given in Figure 2.

Using (2.7), (2.8) and (2.10) we get

$$T/T_c = \frac{-1 + 6(\zeta/\zeta_c) + 3(P/P_c)(\zeta/\zeta_c)^2}{8(\zeta/\zeta_c)^{3/2}}, \tag{2.16}$$

where

$$\zeta_c = \frac{6\pi Q^2}{k}. \tag{2.17}$$

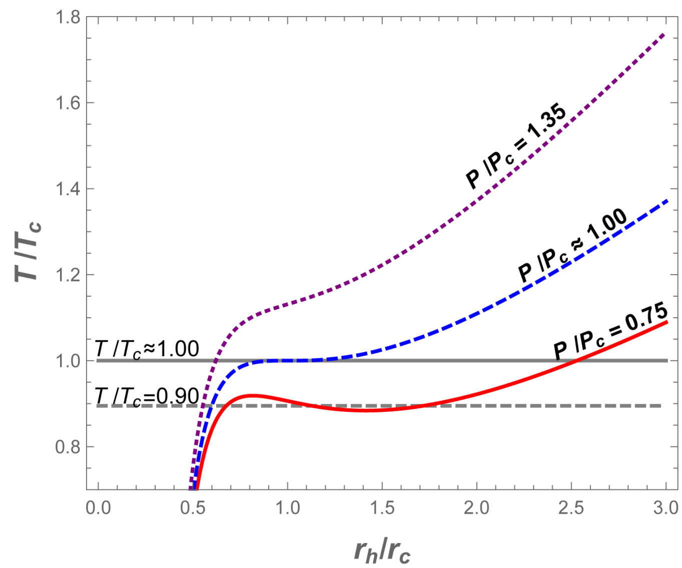

Figure 2. The r_h ***dependence of the temperature*** T ***at*** $P/P_c = 0.75; 1.00; 1.35$

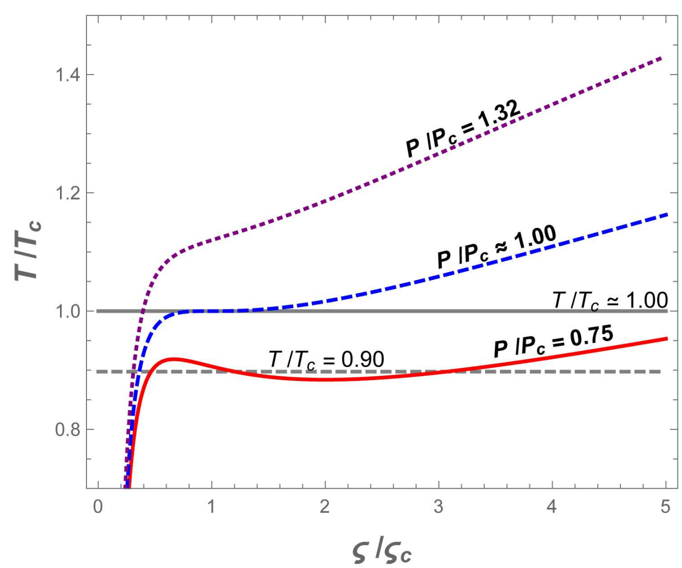

Figure 3. The ζ ***dependence of the temperature*** T ***at*** $P/P_c = 0.75; 1.00; 1.32$

Basing on (2.16) we draw the entropy dependence of the temperature. Figure 3 represents the curves of T vs ζ at several values of pressure P.

From Figures 3 and 4 it is clear that there exists a gas-liquid phase transition when $P < P_c$ and P_c is the critical pressure corresponding to the above mentions.

Here, a question appears: The phase transition in the system is first or second order? To solve this problem, it is necessary to survey isobaric specific heat. By defining the critical isobaric specific heat C_{Pc}:

$$C_{Pc} = \frac{4\pi Q^2}{k} \tag{2.18}$$

we rewrite (2.11) in the form

$$C_P/C_{Pc} = \frac{6(r_h/r_c)^5(T/T_c)}{1 - 3(r_h/r_c)^2 + 2(r_h/r_c)^3(T/T_c)}. \tag{2.19}$$

Figure 4 represents the curves of isobaric specific heat C_P vs r_h at several values of the temperature T.

As shown in Figure 4, for temperature $T < T_c$ the curve C_P has a jump at the neighborhood of $r_h = r_c$ and changes continuously for $T > T_c$. It shows that the phase transition is the first oder and T_c is the critical temperature.

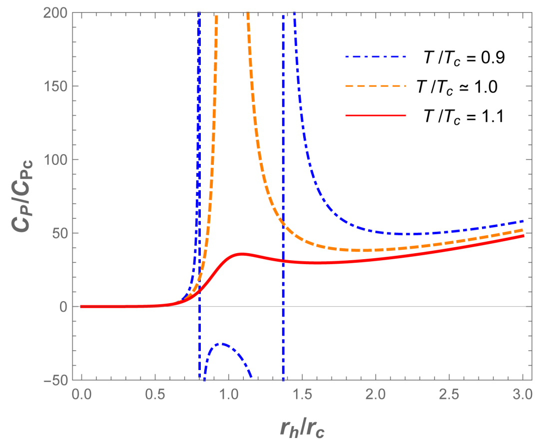

Figure 4. The r_h ***dependence of the*** C_P ***at*** $T/T_c = 0,9; 1,0; 1,1$

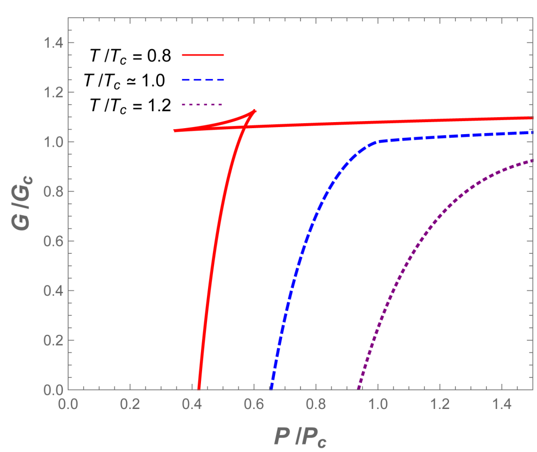

Figure 5. The T ***dependence of the*** $G(T,P)$ ***at*** $P/P_c = 0,6; 1,0; 1,4$

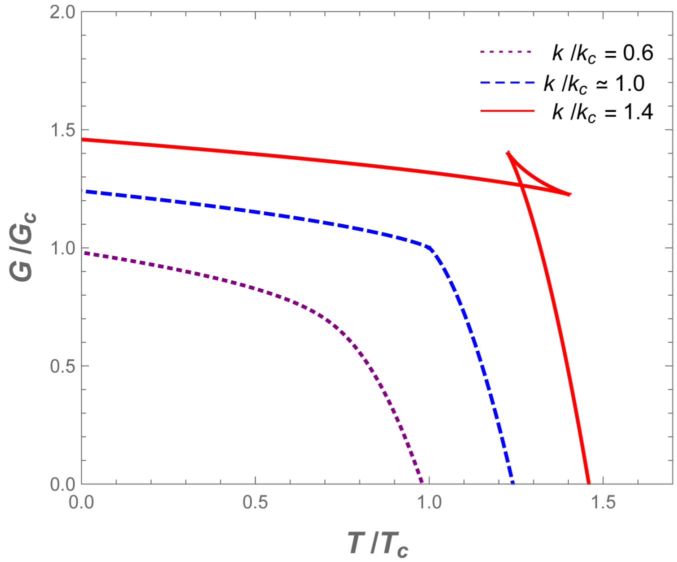

Figure 6. The T dependence of the $G(T,k)$ at $k/k_c = 0.8; 1.0; 1.2$

To confirm once again the above statement let us consider the free energy given by $G = M - T\zeta$. Based on (2.4), (2.7), (2.8), and (2.10) we can calculate

$$G/G_c = \frac{1 + 3(r_h/r_c)^2 - (T/T_c)(r_h/r_c)^3}{3(r_h/r_c)}. \tag{2.20}$$

and

$$P/P_c = \frac{1 - 6(r_h/r_c)^2 + 8(T/T_c)(r_h/r_c)^3}{3(r_h/r_c)^4}. \tag{2.21}$$

Here

$$G_c = Q\sqrt{\frac{2k}{3}}. \tag{2.22}$$

Using (2.20) and (2.21) we draw P dependence of free energy $G(P,T)$ as in Figure 5. As clearly seen from this picture, for $T < T_c$ the curve $G(P,T)$ have a singular segment. That is the sign of first-order phase transition. This conclusion is also confirmed in Figure 6 where the T dependence of free energy $G(T,k)$ is presented. It is obvious that with $k > k_c$ the curve $G(T,k)$ have a singular segment, that is a sign of first-oder phase transition. Moreover, the graph of $G(T,k)$ shows that the spatial curvature k also affects phase transition. However, the phase transition is still the first oder.

3. Conclusions

Let us now summarize the main results presented in the previous sections:

Based on the metric of RN charge BH we have found expressions for Hawking temperature, pressure and isobaric specific heat.

Using the above expressions we performed numerical calculations to study phase transition and obtained the following results:

1-With temperature T less than the critical value T_c, there exits a ”liquidgas” phase transition of the matter outside of BH similar to the Van der Waals fluid. In contrast, with $T > T_c$ there will be no phase transition: matter is always ”gaseous”.

2-Both temperature and spatial curvature affect phase transitions, but phase transitions are always first order.

To conclude, we would like to emphasize that the above results are obtained only with $k > 0$ and outside of the BH. For comprehensive conclusions, it is necessary to investigate the overall effect of spatial curvature k on the thermodynamic properties of matter.

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