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STATE-FEEDBACK CONTROL OF DISCRETE-TIME STOCHASTIC LINEAR SYSTEMS WITH MARKOVIAN SWITCHING

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Abstract. This paper is concerned with the stabilization problem via state-feedback control of discrete-time jumping systems with stochastic multiplicative noises. The jumping process of the system is driven by a discrete-time Markov chain with finite states and partially known transition probabilities. Sufficient conditions are established in terms of tractable linear matrix inequalities to design a mode-dependent stabilizing state-feedback controller. A numerical example is provided to validate the effectiveness of the obtained result.

Keywords: multiplicative noises, Markov jump systems, stochastic stability, linear matrix inequalities.

1. Introduction

Stochastic bilinear systems, or systems with stochastic multiplicative noises, play an important role in modeling real-world phenomena in biology, economic, engineering and many other areas [1-2]. Due to various practical applications, the study on analysis and control of stochastic bilinear systems has attracted considerable research attention in the past few decades (see, [3-6] and the references therein).

Markov jump systems (MJSs) governed by a finite set of subsystems together with a transition signal determined by a Markov chain to specify the active mode form an important class of hybrid stochastic systems. They are typically used to describe dynamics of practical and physical processes subject to random abrupt changes in system state variables, external inputs and structure parameters caused by sudden component failures, environmental noises or random loss package in interconnections [7-10]. Many results on stability analysis, H_∞ control, dynamic output feedback control, and state bounding

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for various types of Markov jump linear systems (MJLSs) have been reported recently (see, e.g., [11-19]). Besides that stochastic bilinear systems with Markovian switching have been investigated [20, 21]. In [21], necessary and sufficient conditions in the form of linear matrix inequalities (LMIs) were derived ensuring stochastic stability of a class of discrete-time MJLSs with multiplicative noises. The problem robust H_∞ control of this type of systems was also studied in [22]. However, in the existing results so far the transition probabilities of the jumping process are assumed to be fully accessible and completely known. This restriction is not reasonable in practice and will narrow the applicability of the proposed control method. To the authors' knowledge, the problem of robust stabilization of uncertain discrete-time stochastic bilinear systems with Markovian switching and partially unknown transition probabilities have not been fully investigated in the literature.

In this paper, we address the problem of state-feedback control of discrete-time stochastic bilinear systems with Markovian switching. The transition probability matrix of the jumping process can be partially deficient. Based on a stochastic version of the Lyapunov matrix inequality, sufficient conditions are established in terms of tractable LMIs to design a desired state-feedback controller (SCF) that stabilizes the system. A numerical example is provided to verify the effectiveness of the obtained results.

2. Preliminaries

2.1. Notation

$\mathbb{Z}$ and $\mathbb{Z}^+$ are the set of integers and positive integers, respectively, and $\mathbb{Z}^a = \{k \in \mathbb{Z} : k \geq a\}$ for an integer $a \in \mathbb{Z}$. $\mathbb{E}[.]$ denotes the expectation operator in some probability space $(\Omega, \mathcal{F}, \mathbb{P})$. $\mathbb{R}^n$ is the n-dimensional Euclidean space with the vector norm $\|.\|$ and $\mathbb{R}^{n\times p}$ is the set of $n\times p$ matrices. $\mathbb{S}_n^+$ defines the set of symmetric positive definite matrices. $\mathrm{diag}\{A, B\}$ denotes the diagonal matrix formulated by stacking blocks A and B.

2.2. Problem formulation

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space. Consider the following discrete-time linear system with multiplicative stochastic noise and Markovian switching

$$\begin{aligned} x(k+1) &= A_1(r_k)x(k) + B_1(r_k)u(k) \\ &\quad + [A_2(r_k)x(k) + B_2(r_k)u(k)]w(k),\ k \in \mathbb{Z}^0, \end{aligned} \tag{2.1}$$

where $x(k) \in \mathbb{R}^n$ is the vector state, $u(k) \in \mathbb{R}^p$ is the control input, the system matrices $A_1(r_k)$, $B_1(r_k)$, $A_2(r_k)$ and $B_2(r_k)$ belong to $\{A_{1i}, B_{1i}, A_{2i}, B_{2i}, i \in \mathcal{M}\}$, where A_{1i}, B_{1i}, A_{2i} and B_{2i}, $i \in \mathcal{M}$, are known constant matrices. For the notational simplicity, whenever $r_k = i \in \mathcal{M}$, matrices $A_1(r_k), B_1(r_k), A_2(r_k), B_2(r_k)$ will be denoted as A_{1i}, B_{1i}, A_{2i} and B_{2i}, respectively. $\{w(k), k \in \mathbb{Z}^0\}$ is a sequence of scalar-valued independent random variables with

$$\mathbb{E}[w(k)] = 0, \mathbb{E}[w(k)]^2 = 1. \tag{2.2}$$

The jumping parameters $\{r_k, k \in \mathbb{Z}^0\}$ govern a discrete-time Markov chain specifying the system mode which takes value in a finite set $\mathcal{M} = \{1, 2, \ldots, m\}$ with transition probabilities (TPs) given by

$$\mathbb{P}\left(r_{k+1} = j | r_k = i\right) = \pi_{ij}, \quad i, j \in \mathcal{M},$$

where $p_{ij} \geq 0$, $i, j \in \mathcal{M}$ and $\sum_{j=1}^{m} p_{ij} = 1$ for all $i \in \mathcal{M}$. We denote $\Pi = (\pi_{ij})$ the transition probability matrix and $p = (p_1, p_2, \ldots, p_m)$ the initial probability distribution, where $p_i = \mathbb{P}(r_0 = i)$, $i \in \mathcal{M}$. It is assumed that the jumping process $\{r_k\}$ and stochastic $\{w(k)\}$ are independent and the transition probability matrix Π is only partially accessible, that is, some entries of Π can be completely unknown. In the sequel, we denote by $\hat{\pi}_{ij}$ the unknown entry $\pi_{ij} \in \Pi$, $\mathcal{M}_a^{(i)}$ and $\mathcal{M}_{na}^{(i)}$ the sets of indices of known and unknown TPs in row $\Pi_i = \begin{bmatrix} \pi_{i1} & \pi_{i2} & \ldots & \pi_{im} \end{bmatrix}$ of Π, respectively,

$$\mathcal{M}_a^{(i)} = \left\{ j \in \mathcal{M} : \ \pi_{ij} \text{ is known} \right\}, \ \mathcal{M}_{na}^{(i)} = \left\{ j \in \mathcal{M} : \ \pi_{ij} \text{ is unknown} \right\}. \tag{2.3}$$

Moreover, if $\mathcal{M}_a^{(i)} \neq \emptyset$, we denote $\mathcal{M}_a^{(i)} = (\mu_1^i, \mu_2^i, \ldots, \mu_l^i), 1 \leq l \leq m$. That is, in the ith row of Π, entries $\pi_{i\mu_1^i}, \pi_{i\mu_2^i}, \ldots, \pi_{i\mu_l^i}$ are known.

For control system (2.1), a mode-dependent SFC is designed in the form

$$u(k) = K(r_k)x(k), \tag{2.4}$$

where $K(r_k) \in \{K_i, i \in \mathcal{M}\}$ is the controller gain which will be designed. With the controller (2.4), the closed-loop system of (2.1) is given by

$$x(k+1) = A_{1c}(r_k)x(k) + A_{2c}(r_k)x(k)w(k), \ k \in \mathbb{Z}^0, \tag{2.5}$$

where $A_{1c}(r_k) = A_1(r_k) + B_1(r_k)K(r_k)$ and $A_{2c}(r_k) = A_2(r_k) + B_2(r_k)K(r_k)$.

Definition 2.1 (see [21])**.** *The open-loop system of (2.1) (i.e. with $u(k) = 0$) is said to be stochastically stable if there exists a constant $T(r_0, x_0)$ such that*

$$\mathbb{E}\Big[\sum_{k=0}^{\infty} x^\top(k)x(k)|r_0, x_0\Big] \leq T(r_0, x_0).$$

Definition 2.2. *System (2.1) is said to be stochastically stabilizable if there exists an SFC in the form of (2.4) such that the closed-loop system (2.5) is stochastically stable for any initial condition (r_0, x_0).*

The main objective of this paper is to establish conditions to design an SFC (2.4) which makes the closed-loop system of (2.1) with partially unknown transition probabilities stochastically stable.

2.3. Auxiliary lemmas

In this section, we introduce some technical lemmas which will be useful for our later derivation.

Lemma 2.1 (Schur complement)**.** *Given matrices M, L, Q of appropriate dimensions where M and Q are symmetric and $Q > 0$. Then, $M + L^\top Q^{-1} L < 0$ if and only if*

$$\begin{bmatrix} M & L^\top \\ L & -Q \end{bmatrix} < 0 \tag{2.6}$$

or equivalently

$$\begin{bmatrix} -Q & L \\ L^\top & M \end{bmatrix} < 0. \tag{2.7}$$

The following lemma gives necessary and sufficient conditions for the stochastic stability of the open-loop system of (2.1) (see [21]).

Lemma 2.2. *The open-loop system of* (2.1) *(i.e. $u(k) = 0$) is stochastically stable if and only if there exist matrices $Q_i \in \mathbb{S}_n^+$, $i \in \mathcal{M}$, such that one of the two following conditions holds*

(i) *For all $i \in \mathcal{M}$, the following algebraic Riccati inequality (ARI) holds*

$$A_{1i}^\top G_i A_{1i} + A_{2i}^\top G_i A_{2i} - Q_i < 0, \tag{2.8}$$

where $G_i = \sum_{j=1}^m \pi_{ij} Q_j$.

(ii) *The following LMIs hold*

$$\begin{bmatrix} -Q_i & J_{1i}^\top & J_{2i}^\top \\ J_{1i} & -Q & 0 \\ J_{2i} & 0 & -Q \end{bmatrix} < 0, \quad i \in \mathcal{M}, \tag{2.9}$$

where $Q = \operatorname{diag}\{Q_1, Q_2, \dots, Q_m\}$ and

$$\begin{aligned} J_{1i}^\top &= \begin{bmatrix} \sqrt{\pi_{i1}} A_{1i}^\top Q_1 & \sqrt{\pi_{i1}} A_{1i}^\top Q_2 & \cdots & \sqrt{\pi_{im}} A_{1i}^\top Q_m \end{bmatrix}, \\ J_{2i}^\top &= \begin{bmatrix} \sqrt{\pi_{i1}} A_{2i}^\top Q_1 & \sqrt{\pi_{i1}} A_{2i}^\top Q_2 & \cdots & \sqrt{\pi_{im}} A_{2i}^\top Q_m \end{bmatrix}. \end{aligned}$$

3. Main results

In this section, we first derive conditions to ensure that system (2.1) with partially unknown transition probabilities (2.3) is stochastically stable. Then, based on the proposed stability conditions, an SFC in the form of (2.4) is designed.

Theorem 3.1. *The open-loop system of* (2.1) *with deficient TPs* (2.3) *is stochastically stable if there exist matrices $Q_i \in \mathbb{S}_n^+$, $i \in \mathcal{M}$, such that*

$$\begin{bmatrix} -\pi_a^i Q_i & \tilde{J}_{1i}^\top & \tilde{J}_{2i}^\top \\ \tilde{J}_{1i} & -\tilde{Q} & 0 \\ \tilde{J}_{2i} & 0 & -\tilde{Q} \end{bmatrix} < 0 \tag{3.1}$$

and

$$\begin{bmatrix} -Q_i & A_{1i}^\top Q_j & A_{2i}^\top Q_j \\ Q_j A_{1i} & -Q_j & 0 \\ Q_j A_{2i} & 0 & -Q_j \end{bmatrix} < 0, \quad j \in \mathcal{M}_{na}^{(i)}, \tag{3.2}$$

where

$$\begin{aligned} \tilde{J}_{1i}^\top &= \left[\sqrt{\pi_{i\mu_1^i}} A_{1i}^\top Q_{\mu_1^i} \quad \sqrt{\pi_{i\mu_2^i}} A_{1i}^\top Q_{\mu_2^i} \quad \cdots \quad \sqrt{\pi_{i\mu_l^i}} A_{1i}^\top Q_{\mu_l^i}\right], \\ \tilde{J}_{2i}^\top &= \left[\sqrt{\pi_{i\mu_1^i}} A_{2i}^\top Q_{\mu_1^i} \quad \sqrt{\pi_{i\mu_2^i}} A_{2i}^\top Q_{\mu_2^i} \quad \cdots \quad \sqrt{\pi_{i\mu_l^i}} A_{2i}^\top Q_{\mu_1^i}\right], \\ \tilde{Q} &= \mathrm{diag}\{Q_{\mu_1^i}, \cdots, Q_{\mu_l^i}\}, \\ \pi_a^i &= \sum_{j \in \mathcal{M}_a^{(i)}} \pi_{ij}. \end{aligned}$$

Proof. According to condition (2.9) of Lemma 2.2, system (2.1) with $u(k) = 0$ is stochastically stable if and only if there exist matrices $Q_i \in \mathbb{S}_n^+$, $i \in \mathcal{M}$, such that

$$A_{1i}^\top G_i A_{1i} + A_{2i}^\top G_i A_{2i} - Q_i < 0, \tag{3.3}$$

where $G_i = \sum_{j=1}^m \pi_{ij} Q_j$. It is fact that $\sum_{j=1}^m p_{ij} = 1$ for all $i \in \mathcal{M}$. Thus, condition (3.3) is equivalent to one of the following two conditions

$$\sum_{j=1}^m \pi_{ij} \left[A_{1i}^\top Q_j A_{1i} + A_{2i}^\top Q_j A_{2i}\right] - \sum_{j=1}^m \pi_{ij} Q_i < 0 \tag{3.4}$$

or

$$\begin{aligned} &\sum_{j \in \mathcal{M}_a^{(i)}} \pi_{ij} \left[A_{1i}^\top Q_j A_{1i} + A_{2i}^\top Q_j A_{2i}) - Q_i\right] \\ &\quad + \sum_{j \in \mathcal{M}_{na}^{(i)}} \pi_{ij} \left[A_{1i}^\top Q_j A_{1i} + A_{2i}^\top Q_j A_{2i}) - Q_i\right] < 0. \end{aligned} \tag{3.5}$$

Let $\tilde{G}_i = \sum_{j \in \mathcal{M}_a^{(i)}} \pi_{ij} Q_j$ and $\pi_a^i = \sum_{j \in \mathcal{M}_a^{(i)}} \pi_{ij}$. Note that $p_{ij} \geq 0$ for all $i, j \in \mathcal{M}$, condition (3.5) holds if the two following conditions hold

$$A_{1i}^\top \tilde{G}_i A_{1i} + A_{2i}^\top \tilde{G}_i A_{2i} - \pi_a^i Q_i < 0, \tag{3.6}$$

$$A_{1i}^\top Q_j A_{1i} + A_{2i}^\top Q_j A_{2i} - Q_i < 0, \ j \in \mathcal{M}_{na}^{(i)}. \tag{3.7}$$

By Schur complement Lemma 2.1, conditions (3.6) and (3.7) can be recast into the following LMIs

$$\begin{bmatrix} -\pi_a^i Q_i & \tilde{J}_{1i}^\top & \tilde{J}_{2i}^\top \\ \tilde{J}_{1i} & -\tilde{Q} & 0 \\ \tilde{J}_{2i} & 0 & -\tilde{Q} \end{bmatrix} < 0 \tag{3.8}$$

and

$$\begin{bmatrix} -Q_i & A_{1i}^\top Q_j & A_{2i}^\top Q_j \\ Q_j A_{1i} & -Q_j & 0 \\ Q_j A_{2i} & 0 & -Q_j \end{bmatrix} < 0, j \in \mathcal{M}_{na}^{(i)}.$$

This completes the proof. □

We now establish conditions by which system (2.1) is stochastically stabilizable as given in the following theorem.

Theorem 3.2. *System* (2.1) *with deficient TPs* (2.3) *is stochastically stabilizable if there exist matrices* $X_i \in \mathbb{S}_n^+$ *and* Y_i, $i \in \mathcal{M}$, *such that*

$$\begin{bmatrix} -\pi_a^i X_i & \tilde{J}_{1i}^\top & \tilde{J}_{2i}^\top \\ \tilde{J}_{1i} & -\tilde{X} & 0 \\ \tilde{J}_{2i} & 0 & -\tilde{X} \end{bmatrix} < 0 \tag{3.9}$$

and

$$\begin{bmatrix} -X_i & (A_{1i}X_i + B_{1i}Y_i)^\top & (A_{2i}X_i + B_{2i}Y_i)^\top \\ * & -X_j & 0 \\ * & * & -X_j \end{bmatrix} < 0,\ j \in \mathcal{M}_{na}^{(i)}, \tag{3.10}$$

where

$$\begin{aligned} \tilde{J}_{1i}^\top &= \left[\sqrt{\pi_{i\mu_1^i}}(A_{1i}X_i + B_{1i}Y_i)^\top \quad \cdots \quad \sqrt{\pi_{i\mu_l^i}}(A_{1i}X_i + B_{1i}Y_i)^\top \right], \\ \tilde{J}_{2i}^\top &= \left[\sqrt{\pi_{i\mu_1^i}}(A_{2i}X_i + B_{2i}Y_i)^\top \quad \cdots \quad \sqrt{\pi_{i\mu_l^i}}(A_{2i}X_i + B_{2i}Y_i)^\top \right], \\ \tilde{X} &= (X_{\mu_1^i}, \dots, X_{\mu_l^i}). \end{aligned}$$

The controller gains K_i, $i \in \mathcal{M}$, *are given by* $K_i = Y_i X_i^{-1}$.

Proof. It is only necessary to show that the closed-loop system (2.5) is stochastically stable. According to Lemma 2.2, system (2.5) is stochastically stable if and only if there exist matrices $Q_i \in \mathbb{S}_n^+$, $i \in \mathcal{M}$, such that

$$A_{1ci}^\top G_i A_{1ci} + A_{2ci}^\top G_i A_{2ci} - Q_i < 0, \tag{3.11}$$

where $G_i = \sum_{j=1}^m \pi_{ij} Q_j$, $A_{1ci} = A_{1i} + B_{1i}K_i$ and $A_{2ci} = A_{2i} + B_{2i}K_i$.

Let $X_i = Q_i^{-1}$. By pre- and post-multiplying equation (3.11) with X_i, we get

$$X_i A_{1ci}^\top G_i A_{1ci} X_i + X_i A_{2ci}^\top G_i A_{2ci} X_i - X_i < 0. \tag{3.12}$$

By similar arguments used in the proof of Theorem 3.1, we can see that condition (3.12) holds if

$$X_i A_{1ci}^\top \tilde{G}_i A_{1ci} X_i + X_i A_{2ci}^\top \tilde{G}_i A_{2ci} X_i - \pi_a^i X_i < 0 \tag{3.13}$$

and

$$X_i A_{1ci}^\top Q_j A_{1ci} X_i + X_i A_{2ci}^\top Q_j A_{2ci} X_i - X_i < 0, j \in \mathcal{M}_{na}^{(i)}. \tag{3.14}$$

We now define $Y_i = K_i X_i$ then, by Schur complement lemma, conditions (3.13) and (3.14) are equivalent to (3.10) and (3.11), respectively. The proof is completed. $\square$

Remark 3.1. *When the transition rate of the jumping process of system* (2.1) *is fully accessible (transition probabilities are completely known), the derived conditions in Theorem 3.2 are reduced to those of Theorem 2 in [21]. Thus, the result of Theorem 3.2 in this paper can be regarded as an extension of the result of [21].*

Remark 3.2. *When the transition rate of the jumping process of system* (2.1) *is completely unknown, condition (3.9) in Theorem 3.2 is omitted and condition (3.10) is now required to feasible for all* $i, j \in \mathcal{M}$.

4. An illustrative example

Consider a two-mode uncertain system in the form of (2.1) with the following data

$$A_{11} = \begin{bmatrix} 0.5 & 0.4 \\ 0.1 & 1.15 \end{bmatrix},\ B_{11} = \begin{bmatrix} 1.0 \\ 0.5 \end{bmatrix},\ A_{21} = \begin{bmatrix} 0.1 & 0.25 \\ 0 & 0 \end{bmatrix},\ B_{21} = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix},$$
$$A_{12} = \begin{bmatrix} 0.8 & 0.4 \\ 0.25 & 1.05 \end{bmatrix},\ B_{12} = \begin{bmatrix} 0.6 \\ 1.0 \end{bmatrix},\ A_{22} = \begin{bmatrix} 0.5 & 0.25 \\ 0 & 0 \end{bmatrix},\ B_{22} = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}.$$

The transition probability matrix is fully inaccessible, that is,

$$\Pi = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix},$$

where $?$ stands for unknown entries. It can be verified using the LMI toolbox in MATLAB that condition (3.2) is not feasible for all $i, j \in \{1, 2\}$. Thus, Theorem 3.1 cannot guarantee the stability of the open-loop system. A simulation result with initial state $x(0) = [1 \quad 1]^\top$ is given in Figure 1. It can be seen that the open-loop system is unstable.

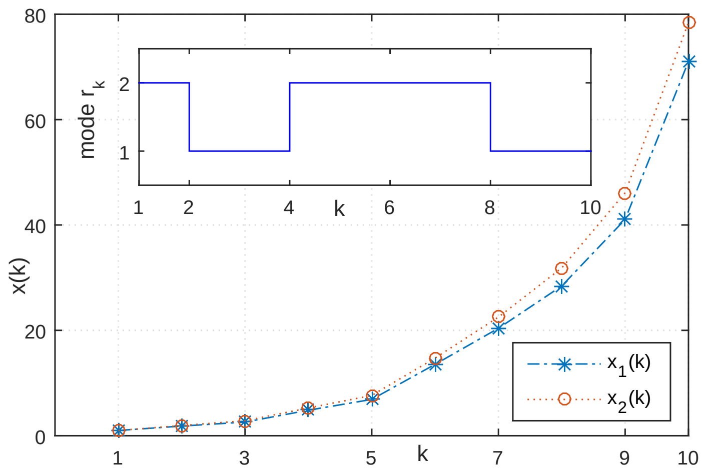

Figure 1. A state trajectory of the open-loop system with random mode

We now apply Theorem 3.2 to design a mode-dependent SFC in the form of (2.4). that makes the closed-loop system (2.5) stochastically stable. By solving condition (3.10) using MATLAB LMI toolbox we obtain the controller gains

$$K_1 = \begin{bmatrix} -0.1838 & -0.5237 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -0.3943 & -0.8955 \end{bmatrix}.$$

A state trajectory of the closed-loop system with the obtained controller is given in Figure 2. The simulation results demonstrates the effectiveness of the design method proposed in this paper.

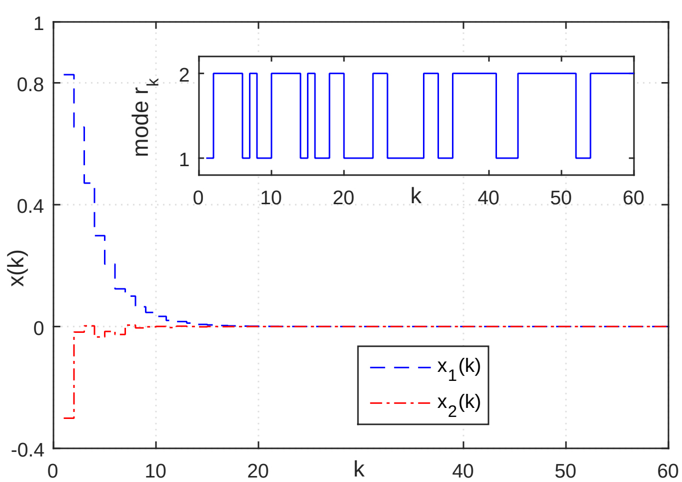

Figure 2. A state trajectory of the closed-loop system

5. Conclusions

In this paper, the stabilization problem via mode-dependent state-feedback controller has been studied for a class of discrete-time stochastic systems with Markovian switching and multiplicative noises. Sufficient conditions have been derived in the form of tractable LMIs to design a desired stabilizing state feedback controller. An example has been provided to illustrate the effectiveness of the obtained result.

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