HNUE JOURNAL OF SCIENCE DOI: 10.18173/2354-1059.2019-0028
Natural Science, 2019, Volume 64, Issue 6, pp. 31-38
This paper is available online at http://stdb.hnue.edu.vn

PHASE TRANSITION IN THE LINEAR SIGMA MODEL OF THE TWO-COMPONENT MIXING SYSTEM

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Abstract. The thermal and quantum phase transitions are studied basing on the Cornwall-Jackiw-Tomboulis (CJT) effective action approach for the relativistic linear sigma model of the two-component mixing system. After obtaining the expression of the thermodynamic potential in Hartree-Fock (HF) approximation, which preserves the Goldstone theorem, the numerical results show that there may be two phase transition scenarios in the system. The first scenario is the thermal phase transition which can only occur in one component. The second scenario is quantum phase transitions which can occur in both two components. In addition, both these two types of phase transitions belong the second order phase transition.
Keywords: Two-component system, CJT effective action, Hartree-Fock (HF) approximation, Goldstone theorem, thermal phase transition, quantum phase transition.

1. Introduction

The phenomenon of kaon condensation has been extensively investigated since Kaplan and Nelson [1] showed that kaon condensation could occur at a density around $3\rho_0$, where ρ_0 is the normal nuclear density. No long time after, it was proved [2] that kaons are condensed in quark matter at sufficiently high densities and low temperature is in the color-flavor-locked (CFL) phase, and their dynamics [3] is essentially described by the linear sigma model at finite density, which is not invariant under the Lorentz transformations.

In recent years many works (for example, [4-11], and others) related to phase transition, symmetry breaking and restoration, Bose-Einstein condensation, etc. have

Received April 19, 2019. Revised June 21, 2019. Accepted June 28, 2019.
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been implemented within the linear sigma model because this model is considered to be best suited for the theory of low energy phenomena of quantum chromodynamic (QCD). In the present work the linear sigma model at finite density and temperature is reconsidered by means of the Cornwall-Jackiw-Tomboulis (CJT) effective action. However, there is a serious difficulty related to re-standardizing the effective action to satisfy Goldstone theorem in the Hartree-Fock (HF) approximation. In order that the Goldstone theorem is respected and the renormalization is achieved in this approximation we adopt the gapless resummation and the renormalization prescription developed in [6, 7], respectively.

In addition, the previous models are mainly limited to one field of multiple components or two fields in the non-relativistic case. Therefore, expanding the model for describing the two-component mixing system in the relativistic case is essential because it allows to clarify many effects related to the internal structure of the stars such as neutron stars [12, 13], or the existence of quark matter in the color-flavor-locked phase at high density and low temperature [14]. This paper presents the research results initially in that direction.

2. Content

2.1. Lagrangian of the model and the CJT effective potential

Let us start from the linear sigma model of the two-component mixing system described by the lagrangian [11]:

$$L = (\partial^0\phi^*)(\partial_0\phi) - (\partial^a\phi^*)(\partial_a\phi) - i\mu_1[(\partial^0\phi^*)\phi - \phi^*(\partial_0\phi)] + (\mu_1^2 - m_1^2)(\phi^*\phi)$$
$$+(\partial^0\psi^*)(\partial_0\psi) - (\partial^a\psi^*)(\partial_a\psi) - i\mu_2[(\partial^0\psi^*)\psi - \psi^*(\partial_0\psi)] + (\mu_2^2 - m_2^2)(\psi^*\psi)$$
$$-\lambda_1(\phi^*\phi)^2 - \lambda_2(\psi^*\psi^2 - \lambda(\phi^*\phi)(\psi^*\psi). \tag{2.1}$$

Here m_1, μ_1 (m_2, μ_2) are, respectively, mass and chemical potential of the complex doublet field ϕ (ψ); $\lambda_1, \lambda_2, \lambda$ are coupling constants and $\partial_a = \frac{\partial}{\partial x^a}$, $\partial_0 = \frac{\partial}{\partial x^0}$.

This model gives the CJT effective potential $V_\beta^{CJT}(\phi_0, \psi_0, D, G)$ at finite temperature in the HF approximation, which preserves the Goldstone theorem

$$\overline{V}_\beta^{CJT}(\phi_0, \psi_0, D, G) = \ (-\mu_1^2 + m_1^2)\phi_0^2 + (-\mu_2^2 + m_2^2)\psi_0^2 + \lambda_1\phi_0^4 + \lambda_2\psi_0^4 + \lambda\phi_0^2\psi_0^2$$
$$+\frac{T}{2}\sum_n\int\frac{d^3k}{(2\pi)^3}\ \mathrm{tr}\big[\ln D^{-1}(k) + \ln G^{-1}(k) + D_0^{-1}(k;\phi_0,\psi_0)D + G_0^{-1}(k;\phi_0,\psi_0)G - 2\mathbb{1}\big]$$
$$+\frac{\lambda_1}{4}P_{11}^2 + \frac{\lambda_1}{4}P_{22}^2 + \frac{3\lambda_1}{2}P_{11}P_{22} + \frac{\lambda_2}{4}Q_{11}^2 + \frac{\lambda_2}{4}Q_{22}^2 + \frac{3\lambda_2}{2}Q_{11}Q_{22}$$
$$+\frac{\lambda}{4}P_{11}Q_{11} + \frac{\lambda}{4}P_{11}Q_{22} + \frac{\lambda}{4}P_{22}Q_{11} + \frac{\lambda}{4}P_{22}Q_{22}, \tag{2.2}$$

including the gap and SD equations:

** The gap equations*

$$
\begin{aligned}
&-\mu_1^2 + m_1^2 + 2\lambda_1\phi_0^2 + \lambda\psi_0^2 + 3\lambda_1 P_{11} + \lambda_1 P_{22} + \frac{\lambda}{2}Q_{11} + \frac{\lambda}{2}Q_{22} = 0,\\
&-\mu_2^2 + m_2^2 + \lambda\phi_0^2 + 2\lambda_2\psi_0^2 + \frac{\lambda}{2}P_{11} + \frac{\lambda}{2}P_{22} + 3\lambda_2 Q_{11} + \lambda_2 Q_{22} = 0, \qquad (2.3)
\end{aligned}
$$

**The SD equations*

$$
\begin{aligned}
M_1^2 &= -\mu_1^2 + m_1^2 + 6\lambda_1\phi_0^2 + \lambda\psi_0^2 + \lambda_1 P_{11} + 3\lambda_1 P_{22} + \frac{\lambda}{2}Q_{11} + \frac{\lambda}{2}Q_{22},\\
M_2^2 &= -\mu_2^2 + m_2^2 + \lambda\phi_0^2 + 6\lambda_2\psi_0^2 + \frac{\lambda}{2}P_{11} + \frac{\lambda}{2}P_{22} + \lambda_2 Q_{11} + 3\lambda_2 Q_{22}. \qquad (2.4)
\end{aligned}
$$

Here D, G are the complete propagators, $T = 1/\beta$ is the temperature and P and Q are the notations

$$
\begin{aligned}
P_{aa} &= T\sum_n \int \frac{d^3k}{(2\pi)^3}\, D_{aa}(\omega_n, \vec{k})\ ; \quad a = 1, 2,\\
Q_{aa} &= T\sum_n \int \frac{d^3k}{(2\pi)^3}\, G_{aa}(\omega_n, \vec{k})\ ; \quad a = 1, 2. \qquad (2.5)
\end{aligned}
$$

2.2. Numerical calculation

In this section we perform numerical calculations to study the phase transition in the linear sigma model of the two-component mixing system in accordance with the two processes when the temperature and / or the chemical potential change. These are two typical physical processes corresponding to thermal phase transition and quantum phase transition. To do this, first need to select the parameters for the model. Basing on [16] we choose masses and chemical potentials correspond to kaons, namely $m_1 = 5$ MeV, $m_2 = 4$ MeV, $\mu_1 = 4.5$ MeV, and the coupling constants selected are $\lambda_1 = 0.0048, \lambda_2 = 0.005, \lambda = 0.004$. Next, we need to determine the phase structure of the system by drawing phase balance curves. By numerical solving equations (2.4) and (2.3) we draw lines $\phi_0 = 0, \psi_0 = 0$ in the phase plane $T - \mu_2$ and obtain result as shown in Figure 1.

As can be seen in Figure 1, with $\mu_{2c1} \simeq 2$ MeV $< \mu_2 < \mu_{2c2} \simeq 3.8$ MeV there is a desert corresponding to both $\phi_0 = 0$ and $\psi_0 = 0$. Therefore with a fixed value of μ_2, only $\phi_0 \neq 0$ or $\psi_0 \neq 0$ can exist and as a result only the thermal phase transition in the sector ϕ or ψ occurs. In contrast, with one a definite value of temperature T, there may exist both $\phi_0 \neq 0$ and $\psi_0 \neq 0$ at different values of μ_2 and therefore the quantum phase transition not only in the sector ϕ but also in ψ occurs. In order to get some insight the above statements, let us examine each type of phase transition in detail.

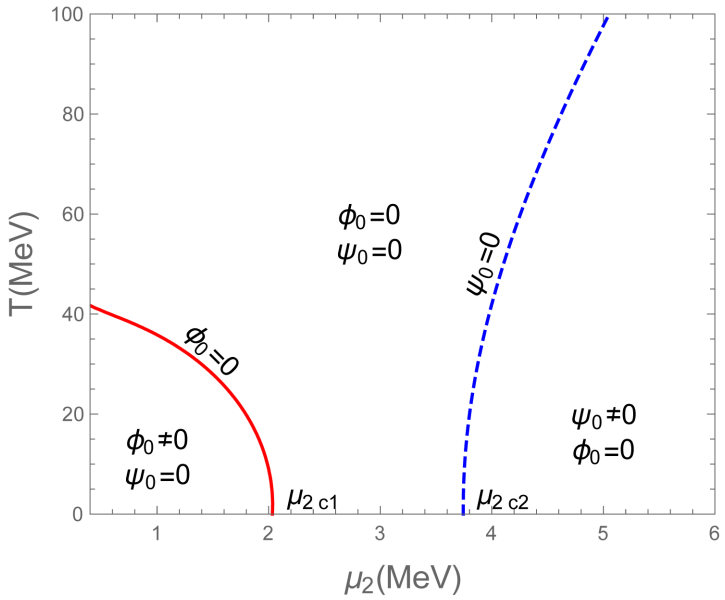

Figure 1. Phase diagram in the plane $T - \mu_2$ ***at*** $\lambda = 0.004$

2.2.1. Thermal phase transition

In order to investigate thermal phase transition in the ψ sector, we choose chemical potential $\mu_2 = 5.5$ MeV basing on Figure 1. By numerical solving the equations (2.4) and (2.3) with the selected parameters we obtain the temperature T dependence of the ψ_0 and ϕ_0 as shown in Figure 2. Obviously, ψ_0 increases steadily to zero when temperature increases to $T_{c\psi}$. That is a sign of second order phase transition.

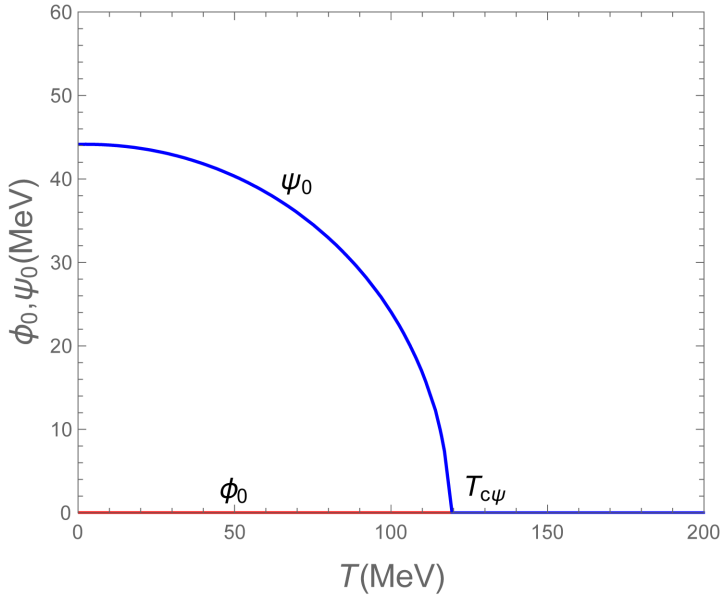

Figure 2. The T ***dependence of*** ϕ_0 ***and*** ψ_0 ***at*** $\mu_2 = 5.5MeV$

Figure 3 represents the ψ_0 dependence of the effective potential $\overline{V}_{\beta}^{CJT}(\phi_0, \psi_0, D, G)$. Local minimum (at $\psi_0 \neq 0$) of the effective potential gradually shifts to the origin and completely disappears at $T_{c\psi} \simeq 120$ MeV. That confirms a second order phase transition in the ψ sector occurs at $T_{c\psi} \simeq 120$ MeV.

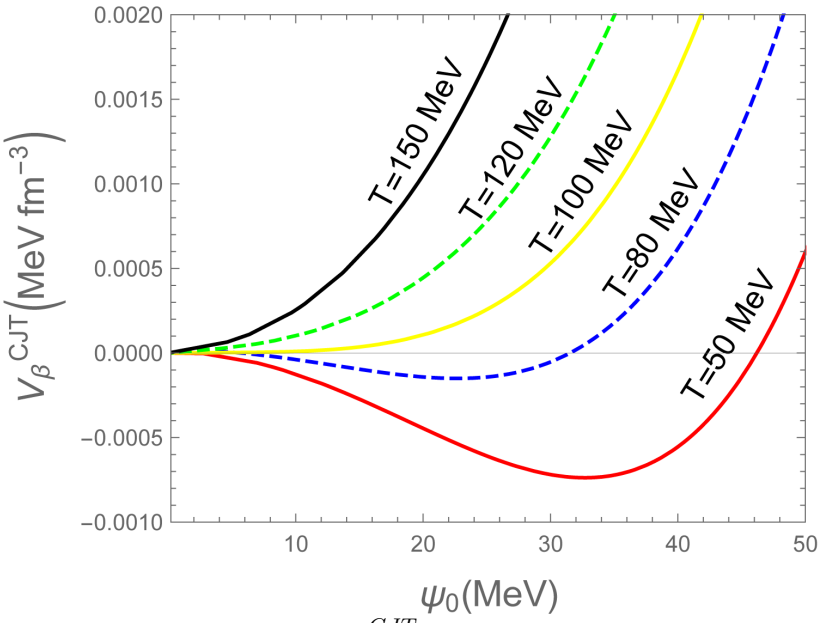

Figure 3. The ψ_0 dependence of $\overline{V}_{\beta}^{CJT}(\phi_0, \psi_0, D, G.)$ at several temperature

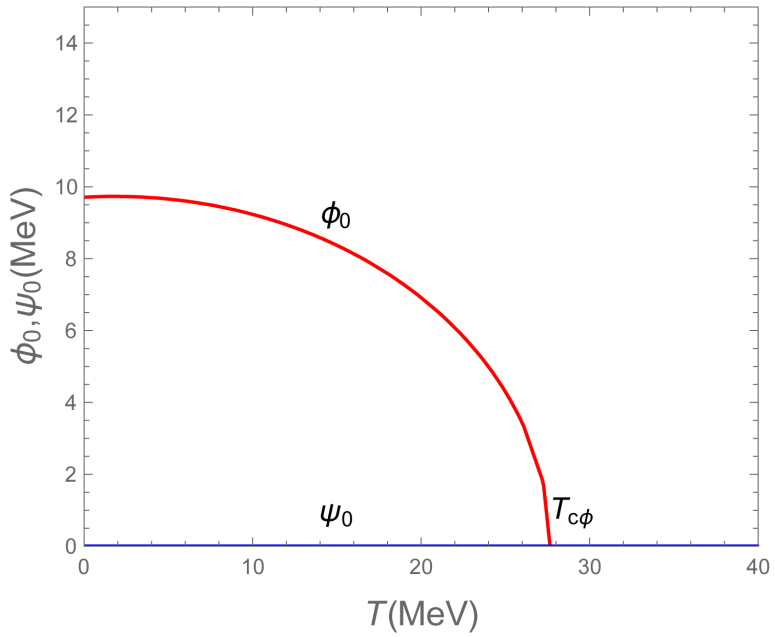

Figure 4. The T dependence of ϕ_0 and ψ_0

Analogously, in order to investigate thermal phase transition in the ϕ sector, we choose chemical potential $\mu_2 = 1.5$ MeV. Figure 4 represents the temperature dependence of the vacuum expectation values ψ_0, ϕ_0. As is seen from this figure the phase transition in the sector ϕ also is second order and takes place at temperature $T_{c\phi} \simeq 27.5$ MeV.

2.2.2. Quantum phase transition

Quantum phase transition is a phase transition that occurs at a fixed temperature when the chemical potential changes. Figure 5 shows the μ_2 dependence of ϕ_0 and ψ_0 at $T = 10$ MeV. As can be seen on this figure, when the chemical potential μ_2 increases, the ϕ_0 decreases to zero and then is replaced by the ψ_0. With $\mu_{2c1} \simeq 2$ MeV $< \mu_2 < \mu_{2c2} \simeq 3.8$ MeV both ϕ_0 and ψ_0 cannot coexist. This result is completely consistent with the comment that was taken from Figure 1. Moreover, the monotonous variation of ϕ_0 and ψ_0 also shows signs of second order phase transition. This is also clearly shown in Figure 6 drawing the dependence ψ_0 of the effective potential $\overline{V}_\beta^{CJT}(\phi_0, \psi_0, D, G)$ at several values of μ_2: When μ_2 increases over the value μ_{2c2}, the minimum of $\overline{V}_\beta^{CJT}(\phi_0, \psi_0, D, G)$ gradually moves out of the origin (corresponding to $\psi_0 = 0$). This means that the system from symmetrical phase to other symmetrical phase is broken at μ_{2c2}.

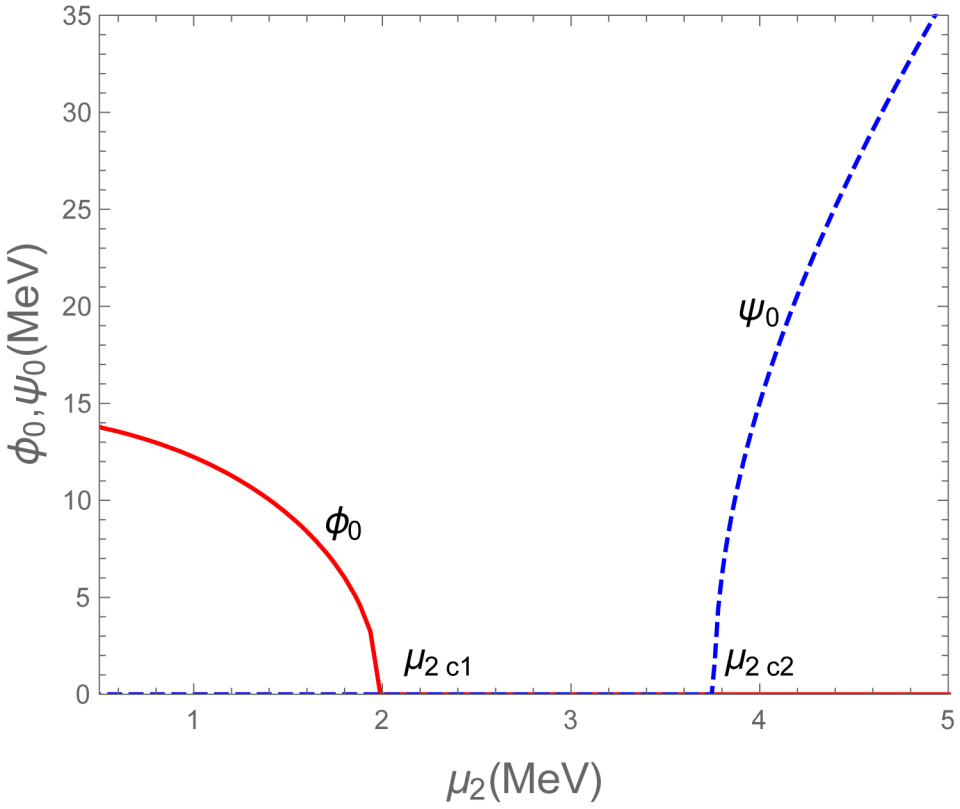

Figure 5. The μ_2 dependence of ϕ_0 and ψ_0 at $T = 10MeV$

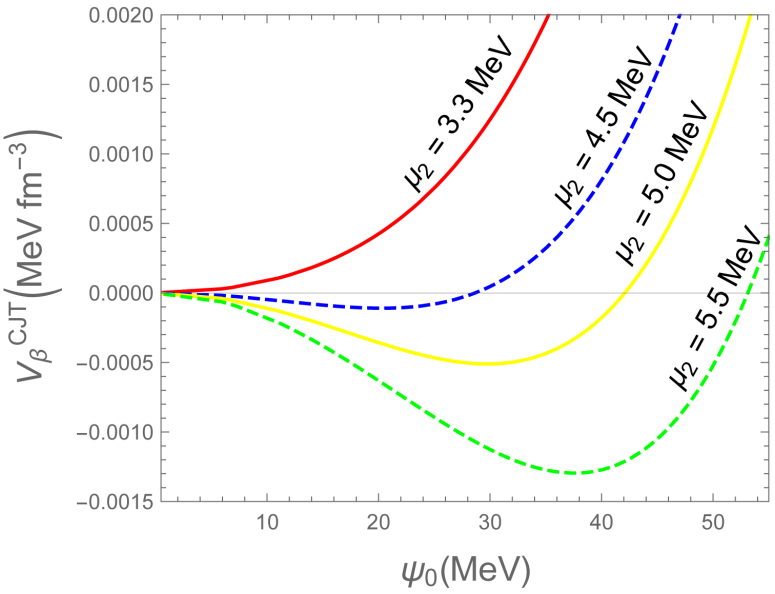

Figure 6. The ψ_0 dependence of ϕ_0, ψ_0 at several values of the chemical potential μ_2

3. Conclusion

In this paper the phase transitions in the linear sigma model were considered by means of the finite temperature CJT effective action. The main results are following:

1-The thermodynamic potential of system in the HF approximation, which is renormalized and respects Goldstone theorem.

2-There may be two phase transition scenarios in the system. The first scenario is the thermal phase transition which can only occur in one component. The second scenario is quantum phase transitions which can occur in both two components depending on the effect of temperature or chemical potential. These results are confirmed by EoS.

3-Both the thermal and quantum phase transitions belong to the second order.

Actually, in order to highlight physical properties of kaon matter we proceed to the numerical computation of phase transition patterns taking a set of kaon masses and chemical potential specified in [7], while the coupling constants are chosen from $T - \mu$ phase diagram Figure 1 to get desired scenarios. To conclude, we would like to emphasize that the formalism developed in this paper is also useful for all studies of nuclear and particle dynamics at finite density and temperature starting from the linear sigma model at different energy scales. At scale of order 10 MeV we deal with nuclear structure if the degrees of freedom are sigma, pions, and nucleons. For higher energy of order 100 MeV we are led to the chiral dynamics of hadrons with nucleons replaced by quarks and at scale of order 100 GeV the Higgs physics of the electroweak theory emerges. This is evidently the promising task for the next research.

Acknowledgment. This work is funded by the Vietnam Foundation of Education and Training Ministry.

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