HNUE JOURNAL OF SCIENCE DOI: 10.18173/2354-1075.2021-0249
Educational Sciences 2021, Volume 66, Issue 5, pp. 209-216
This paper is available online at http://stdb.hnue.edu.vn

POTENTIAL OF APPLYING REALISTIC MATHEMATICS EDUCATION TO THE TEACHING OF PROBABILITY - STATISTICS AT THE UNIVERSITIES OF MEDICINE AND PHARMACY

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Abstract. RME is a theory of mathematics education with a history of more than 50 years, widely deployed in many developed education systems around the world. Many research papers have confirmed the compatibility of RME with the principles and orientation of mathematics education, as well as with the education practices in Vietnam. Based on an overview of RME, this paper presents the potential of applying RME for teaching Probability - Statistics at the Universities of Medicine and Pharmacy.

Keywords: Probability and statistics, realistic Mathematics education, University of Medicine and Pharmacy.

1. Introduction

Realistic Mathematics Education (RME) is a theory of mathematics education that offers a pedagogical and didactical philosophy on mathematics learning and teaching, as well as on designing instructional materials for mathematics education (Bakker, [1], 2004). RME was developed in the Netherlands, within the Wiskobas project (Mathematics in the Elementary schools) in 1968. RME has succeeded in protecting high school education in the Netherlands from the influence of the educational trend of "New Math - Academic Math, separated from reality" in Europe during the 1980s.

RME is based on the initial main idea of Freudenthal that mathematics should be taught to be useful (Freudenthal, [2], 1968). Afterward, many studies were conducted at Freudenthal Institute and other organizations in the Netherlands for the sake of RME. Freudenthal presented three fundamental ideas of RME (Mathematics as a human activity; Guided reinvention and Didactical phenomenology) and developed RME [2-5]. Treffers suggests some principles of RME teaching which have been improved throughout the years by many researchers [6]. According to Marja Van den Heuvel - Panhuizen (2000), there are 6 principles in RME: 1 - Activity; 2 - Reality; 3 - Level; 4 - Intertwinement; 5 - Interactivity; and 6 - Guidance [7].

Gravemeijer analyzes and discusses instructional design as a learning process, an instruction-theoretical reflection on the use of manipulatives, mediating between concrete and abstract, educational development and developmental research in mathematics education, and implementation and effect of realistic curricula. He further explains the viewpoint of Freudenthal on mathematizing, saying that mathematizing mainly involves generalizing and formalizing. Formalizing embraces modeling, symbolizing, schematizing, and refining, and

Received December 2, 2021. Revised December 20, 2021. Accepted December 27, 2021.
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generalizing is to be understood in a reflective sense. Alongside Freudenthal, Gravemeijer asserts the key role of teaching through modelization [8-10].

Among the authors who followed Freudenthal's work, notably De Lange, Treffers, and Van den Heuvel, who developed the concept of mathematization. De Lange distinguishes between horizontal and vertical mathematization based on the kind of activities being conducted [11]. Van den Heuvel analyzes and assesses the efficiency of RME theory and keeps working in that direction [12].

Nguyen Thanh Thuy and Le Tuan Anh are the first Vietnamese authors to research RME in two Doctoral Thesis in Education Science. The thesis of Nguyen Thanh Thuy [13], defended in 2005 in the Netherlands, presented a system of recommendations to help pre-service teachers in Vietnam absorb and apply RME in teaching mathematics. [13]. Le Tuan Anh's thesis, completed in Germany, pointed out the following: although students and teachers may face some challenges in the adoption of RME, this theory would be a possible method for mathematics teaching and learning effectively appliable to high school education in Vietnam [14]. In the New 2018 Vietnamese Mathematics Curriculum, the influence of RME is perceptible [15]. The two authors also discuss the proper understanding of the ideas and principles of RME, as well as the implementation of RME in a way that's suitable to the reality in Vietnam.

Dao Tam and Nguyen Hong Ngu study the core teaching principles of RME and conceive teaching scenarios in accordance with these principles to clarify the meaning of using real-life situations in mathematics teaching and learning [16].

Tran Cuong and Nguyen Thuy Duyen show some results of RME studies and came up with solutions to help teachers design and develop practical mathematics problems for their courses, contributing to the improvement of mathematics teaching in the high schools [17].

Nguyen Tien Trung et al. conduct research on RME. Their works deal with RME in terms of policies, course building, high school mathematics textbooks, and the application of RME in high school mathematics classes in Vietnam [18-20].

Nguyen Danh Nam presents some research results on RME and suggests mathematics education principles in compliance with reality, giving directions to the renewal of mathematics teaching and learning methods [21].

Although introduced to Vietnam in the early 2000s, researchers on the application of RME have only caught the public's interest in the last 5 years. Most of these studies confirm the compatibility of RME with education principles, the orientation of mathematics education, and the reality of education in Vietnam.

Originating from the study of mathematics teaching for elementary school, RME is nowadays spreading to secondary schools and even beyond. In this article, with an overview of research on teaching based on RME, we lay out and analyze the potential of teaching Probability and Statistics at the University of Medicine and Pharmacy with RME-based methods.

2. Content

Recent works have confirmed the compatibility of RME with educational principles, the orientation of mathematics education, and the reality of education in Vietnam. For some aspects, we will compare and clarify the similarity between teaching math to young students and teaching Probability - Statistics to medical students, thus proving the latter is appropriate and has a firm scientific basis.

2.1. Historical formation and development

Mathematics was born of practical needs, as a tool for production activities as well as other sectors, leading the way for the development of science and technology. Due to its great benefits

in shaping and developing learners' minds, mathematics is taught in high schools as a compulsory subject with a large amount of devoted time.

In our work in 2017 [22], we combined a series of events showing that Probability and Statistics were also created to solve practical problems. Probability arises from games of chance, Statistics emerges from the need to record, calculate, and analyze natural and social phenomena: bets, spending, administrative management of lands, fields, populations, etc. When developed to a certain extent, knowledge about Probability and Statistics is applied in all fields of life, science, engineering, and technology.

In the medical field, in particular, Probability and Statistical help physicians use data and research results effectively: collecting, describing, processing, analyzing, making and verifying hypotheses, making informed decisions when it comes to taking care of, diagnosing and treating the patients, protecting and improving public health; For the pharmaceutical industry, Probability and Statistics help pharmacists in using a large amount of data in the experimental process to develop new drugs, comparing the effects of drugs or in pharmaceutical research.

Because of such great impacts, Probability and Statistics are also included in medical schools as a compulsory subject (called Probability - Statistics), becoming more and more essential in the training programs and currently undergoing major reforms to fully benefit future healthcare workers.

2.2. The objectives of teaching

The teaching of Mathematics in high schools aims to equip students with basic knowledge and practical skills, to develop their intelligence and moral qualities, to prepare them for a productive working life or the pursuit of higher education. In the 2018 High School Education Program, the above goals are worded as instilling moral values and developing the students' skills. Among those skills, the ability to use mathematics, with its five different components, is one of the most important.

According to the practical principle of RME theory, an essential goal of mathematics education is that the learner must be able to apply mathematics to solve practical problems [20].

The 2019 Vietnamese Education Law [21] stipulates that the general goal of a college education is to train highly qualified human resources capable of participating in scientific and technological research to create new knowledge and products, serving the needs of socio-economic development, ensuring national defense and security and international integration. It is necessary to help each student develop their virtue, intelligence, body, and beauty comprehensively; acquire professional knowledge, skills, and sense of responsibility; catch up with scientific and technological progress related to their training level; be capable of self-study, creativity, and adaptability to the working environment; have an entrepreneurial spirit and a will to help people [23]. In the field of Medicine and Pharmacy, the teaching of Probability - Statistics aims at equipping future doctors and medical staff with knowledge, skills, statistical thinking, professional competence, demonstrated by proficiently applying Probability and Statistics to solve practical questions in public healthcare [24, 25].

2.3. Applying the basic ideas and principles of RME in teaching Probability - Statistics at the Universities of Medicine and Pharmacy

As Le and Tran described in [15] (2020), teaching in accordance with RME ideas consists of the following:

- According to the practical principle, the important goal of mathematics education is that learners must be able to apply mathematics to solve practical problems.

- Lessons must start from practical scenarios that are meaningful to the students, to give them the opportunity to add meaning to the mathematical structure formed in their minds. The practical scenarios here are not strictly real-life ones but also include scenarios with mathematical content inspired from reality in a specific teaching context.

- The role of models and teaching with modeling is particularly important [11].

- The practice of mathematics is a special activity which follows a specific process and must lead to mathematical products. Students learn mathematics based on activities they experience in their daily lives.

- Students acquire knowledge by reinventing preexisting knowledge and constructing their own.

2.3.1. The practice of Probability – Statistics in the medical field

Right at the beginning of the 20th century, after R. Fisher developed the theoretical basis of modern statistical testing, A.B. Hill wrote a series of articles in the Lancet (started in 1937) on the subject of applying statistical methods in medical research; The commentary of Hill (British Medical Journal, 1947) was perhaps the first one to assert that in order to avoid clinical error, physicians must be proficient in statistical probability, have knowledge in designing and conducting studies. Therefore, it is necessary to include statistics in medical training curricula [26].

In the early 1990s, Evidence-Based Medicine (EBM) was born in Canada. This is a medical practice method based on medical data, especially research results that have been validated for publication in specialized journals. Based on this evidence, combined with clinical experience and information gathered from patients, physicians can make proactive and informed decisions about intervention, treatment, and care [27].

In the clinical practice of Evidence-Based Medicine, physicians frequently use the concepts of statistics and epidemiology. Intervention effectiveness, influencing factors, risk factors, links, etc. are all quantified as much as possible by the significant coefficients that statisticians have proposed; Physicians are required to read and understand those statistical data, sometimes create their statistics with their insights and treatment data, draw statistically significant conclusions and make treatment decisions, contributing to databases and expanding their experience. To approach and be able to practice Evidence-Based Medicine, physicians must have sufficient basic knowledge of statistics and epidemiology.

It can be said that Evidence-Based Medicine has changed the face of medical science as well as treatment practice in hospitals around the world. Nowadays, derivative concepts such as evidence-based therapeutic practice, evidence-based medication administration, evidence-based first aid, etc. are increasingly developed and popular in the medical field [28].

Medical examination and treatment by Evidence-Based Medicine - a special feature in the works of today's medical industry - to determine Probability and Statistics is a thriving activity of medical professions.

2.3.2. Opportunities for “guided reinvention” in the field of medical

In the history of the formation and development of Medicine, Epidemiology, and Statistics, these three science branches have a very close relationship: much statistical knowledge is born from the history of Medicine.

In ref. [29] (2015), Blackstone pointed out that the connection between statistics and medicine - biology - epidemiology originates in London, the capital of England. The birth of statistics - usually dated 1662 - was when J. Graunt and W. Petty studied the pathologically classified mortality rates and estimated the city's population. In 1710, J. Arbuthnot analyzed 82 years of data on the sex ratio in infants, to giving birth to what we call p-value in the modern language; In 1747, James Lind, a surgeon of the Royal Navy, conducted the first clinical trial with the administration of six different drugs to treat sailor's scurvy, helping discover Vitamin C

to cure this disease in the Navy. In 1854, John Snow performed the first modern epidemiological study by collecting data during a cholera epidemic in London, which allowed a water pump on Broad Street to be the source of the epidemic because of its connection with water from the Thames.

Since the 1950s, randomized clinical trials have been the standard for reliable research in biomedicine.

With such examples abound, once the lecturer has enough knowledge about the history and connection between Medicine and Statistics and some related sciences, the re-enactment in the form of abridged simulation allows students to rediscover under supervision, which helps them build the knowledge needed for their careers.

2.3.3. The practice environment of medicine offers many practical contexts and is favorable to the development of models in Probability - Statistics

De Lange, a classic author of RME theory, proposed three levels of word usage context ranked from low to high: (1) simple language transformation to turn real problems into mathematical issues; (2) seeking relevant mathematical knowledge to solve real problems; (3) introducing and developing mathematical models and concepts.

At level (1), any medical record can become a basis for statistical problems and questions.

At level (2), as described in section 2.3.1., all real problems in the physician's practice of Evidence-Based Medicine require statistical knowledge and can become teaching scenarios used in Medicine schools.

Illustrating level (3) is not difficult, given the high level of penetration of Statistics in Evidence-Based Medicine. According to Blackstone [29], in the field of pediatric heart surgery alone, Evidence-Based Medicine regularly uses dozens of statistical concepts including Uncertainty Confidence Limits, P Values, Risk Factor, Continuity Versus Discontinuity in Nature, Linearity Versus Nonlinearity, Nihilism Versus Predictability, Parsimony, Nomograms, Effectiveness, Appropriateness, Timing, and Indications for Intervention, Time-Related Events, Repeatable Events, Competing Risks, Weighted Events, Clinical Trials with Randomly Assigned Treatment, Clinical Studies with Nonrandomly Assigned Treatment.

2.3.4. Learning activities and resources of students for learning probability and statistics in the context of medicine and pharmacy

Freudenthal emphasizes RME's view that "Mathematics is a human activity". He explains that “Mathematics as an activity differs from mathematics that is merely taught. The product of mathematical activity is not only propositions and theorems but can also be proofs and thoughts.

Medicine differs from other disciplines by its connection to human life and health. Each patient is a unique case, and the diagnosis and symptoms are not always the same. However, comparisons are essential and require medical staff to record, collect and analyze data, draw experience for future diagnoses, medical examination, and treatment. In today’s Medicine, the concepts and principles of statistics are also applied in all activities: from diagnosis to treatment of disease, from epidemiological investigation to community diagnosis, design, and evaluation of programs interventions, from protecting and improving public health to predicting new developments that may arise [30]. Modern medical practices cannot be separated from the use of a rapidly growing amount of information and data. Accordingly, Probability - Statistics becomes more frequently and widely used.

To teach Probability - Statistics, lecturers can make students perform many types of statistical activities such as measuring, collecting data, calculating, analyzing, evaluating, making decisions, forecasting, etc.

These statistical activities have close ties with the daily activities of operating physicians, such as examination, diagnosis, treatment planning, re-examination, progress assessment, adjustment, medical records writing, discovery, treatment experience summary, epidemiological status assessment, disease model monitoring, etc. As for pharmacists, it is the counting, dispensing, and management of medicines, finding out and evaluating drug effects, thereby giving appropriate advice to support treatments, etc.

These activities all directly create products in physical form and can be performed by students during Probability - Statistics classes. The products can be easily collected and organized in records. This process of simulating professional activities helps students build new knowledge by themselves through real-life situations, under the guidance of teachers, thereby preparing them to oversee the community's health.

2.3.5. The potential of teaching Probability - Statistics by modeling

RME theory places special emphasis on the use of models in teaching mathematics. Mathematical modeling is the process of transforming a real-life problem into a mathematical one by establishing and solving mathematical models, representing, and evaluating solutions in real-life contexts, and improving mathematical models if the solution is not acceptable [31]. Here, a mathematical model is understood as a mathematical structure that roughly describes the characteristics of a certain phenomenon. A mathematical model includes mathematical objects and the relationships between those objects [32].

In teaching Probability - Statistics, the concept of the model is extremely popular when describing and explaining a real-life situation. They are tools to represent real-life scenarios in the form of mathematical language, a means of solving mathematical problems related to real-life situations. Teaching Probability - Statistics by modeling is the process of selecting and using appropriate Probability - Statistics tools to analyze real-life situations, understand them better, make decisions, and solve practical problems. Below are some familiar *patterns* in Probability - Statistics:

- A probability model includes probability space (Ω), events (or subsets of Ω), probability measurement (P) and symbols (Ω, P) [31].

- A random variable with a real value is a function measured on a space of probability: $X:(\Omega,P)\to R$, [29].

- The probability distribution of random variables is a method of expressing the relationship between possible values of a random variable and its corresponding probability [33]. Some common probability distributions of random variables are Normal distribution, Binomial distribution, etc. [34].

- Statistical data model in the form of tables, charts: Statistical data collected is usually raw data. To analyze and process data, statistical data must be modeled in the form of tables and charts to highlight the development trend of the research phenomenon.

- Statistical hypothesis testing model to verify whether the statistical hypothesis can be accepted based on the observed data sample from the population. The objective of testing the statistical hypothesis is to find out if the Probability (usually called *p*) for the null hypothesis (called H_0) is consistent with reality. If *p* is small (people usually choose lower than 0.05) we reject hypothesis H_0, which means accept hypothesis H_1, the conclusion will then be the same as hypothesis H_1. Testing models of statistics hypothesis in Medicine are done through the following steps [30]:

Step 1. Define a null hypothesis (H0) and the main hypothesis H1).

Step 2. Determine the Probability of Type I Error (α) and the Probability of Type II Error (β) and estimate the sample size based on these two probabilities.

Step 3. Collect data to test the hypotheses. Call the data D.

Step 4. Use Fisher's method of verifying the statistical significance of the Probability of data D occurring if the null hypothesis (H0) is true. p = P(D/H0).

Step 5. Draw conclusions: reject hypothesis H_0 if $p < 0.05$.

- Regression models: univariate linear regression, multivariable, logistic regression, etc.

3. Conclusions

Though originating from elementary mathematics education, RME theory is becoming increasingly useful at higher levels of education. Our analysis of formation and development history, place in the training programs, and the purpose of teaching shows the high similarity between teaching mathematics to young students and teaching Probability - Statistics to medical students. Using the main ideas and principles of teaching according to RME theory to review the teaching process of Probability - Statistics in medical schools also shows that: teaching Probability - Statistics in Universities of Medicine and Pharmacy using RME theory is very appropriate.

Research should be followed by the proposal, implementation, and testing of methods of teaching Probability - Statistics in suitable Universities of Medicine and Pharmacy that allow students at these schools to study and calculate statistics effectively and substantively in order to use them proficiently in their professional practices.

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