

Fusion-fission in the reactions of the $^{58}\text{Ni} + ^{251}\text{Cf}$ and $^{64}\text{Zn} + ^{248}\text{Cm}$ combinations

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ABSTRACT

Introduction: In the present study, we evaluate the nucleon evaporation, alpha decay, and fission widths in the fusion-fission of the $^{58}\text{Ni} + ^{251}\text{Cf}$ and the $^{64}\text{Zn} + ^{248}\text{Cm}$ reactions for the synthesis of the super-heavy $^{309,312}126$ nuclei. **Methods:** The feasibility of the synthesis of the $^{309,312}126$ isotopes via the mentioned systems is investigated based on the widths. The widths in the excitation energy range of $E^* = 10 - 100$ MeV are calculated in the scope of the statistical model, in which the level density is calculated by using the Fermi-gas model. By employing the LISE++ code, the level densities the compound nuclei, $^{309,312}126$ nuclei, are calculated to be about $10^5 - 10^{50}$ (MeV^{-1}) in the energy range of interest. **Results:** The lifetime of the compound nuclei, $^{309,312}126$ nuclei, which are estimated based on the total width, is about $10^{-22} - 10^{-20}$ s. The fission has the largest width compared to those of the alpha decay and nucleon evaporations. Hence, the $^{58}\text{Ni} + ^{251}\text{Cf}$ and the $^{64}\text{Zn} + ^{248}\text{Cm}$ combinations are appropriate to the study of the mass distribution. In addition, the large alpha decay widths suggest the $^{309,312}126$ isotopes be the alpha-decay nuclei. **Conclusion:** The results are expected to be useful for considering measurements at facilities in the near future.

Key words: fusion, cross-section, compound nucleus, fission, super-heavy nuclei

INTRODUCTION

Recently, super-heavy elements with atomic numbers up to $Z = 118$ have been experimentally discovered so far¹⁻⁶. However, the number of isotopes is not diversified, and the production mechanism of super-heavy nuclei has not been revealed up to date. It is thought that, for heavy nuclei, the fusion mechanism can be proceeded through three main stages: (i) Coulomb barrier penetration of the projectile for the capture of target, (ii) competition of compound nucleus formation and quasi-fission processes, and (iii) survival probability of excited compound nucleus by light particle evaporation against fission as shown in **Figure 1**. There is a competition between fusion and quasi-fission processes in the interaction of heavy nuclei⁷⁻⁹. If the fusion is dominant over the quasi-fission, super-heavy nuclei can be produced. Once a hot compound nucleus is formed, it can de-excite via evaporation or fusion-fission to exist in more stable states. Therefore, it is necessary to study the probability of each stage to understand the interaction mechanism of heavy nuclei. Notice that it is possible for the appearance of the new doubling-magic numbers during the fission of super-heavy nuclei. The fission is also one of the routes reaching to the neutron-rich heavy region.

It should be noted that the cross section for the synthesis of new super-heavy elements with Z greater than 118, which is important for understanding the fusion mechanism, has large uncertainty. Since the fusions of the $^{58}\text{Ni} + ^{251}\text{Cf}$ and the $^{64}\text{Zn} + ^{248}\text{Cm}$ combinations, respectively, lead to the unknown $^{309,312}126$ nuclei, they can be candidates for discovering new super-heavy elements with the atomic numbers up to $Z = 126$. The cross section relevant to the penetration of the Coulomb barrier and leading to a contact between two colliding nuclei (process (i)) can be precisely determined in a coupled-channel calculation^{10,11}. The probability of neutron emission from excited compound nuclei to form super-heavy nuclei can be calculated within the statistical model approach (process (iii))^{12,13}. It is believed that the fusion-fission and quasi-fission give different fission properties in these reactions. Hence, the fusion probability can be determined by evaluating the fusion-fission properties in the fusions of the $^{58}\text{Ni} + ^{251}\text{Cf}$ and the $^{64}\text{Zn} + ^{248}\text{Cm}$ systems.

In order to investigate the mentioned problems, the measurements of the concerned fusions are proposed to obtain cross sections of the synthesis of elements with $Z > 118$ and to reveal the mass distribution in the fission process. In the previous studies^{7,8}, the $^{58}\text{Ni} +$

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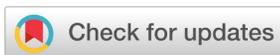
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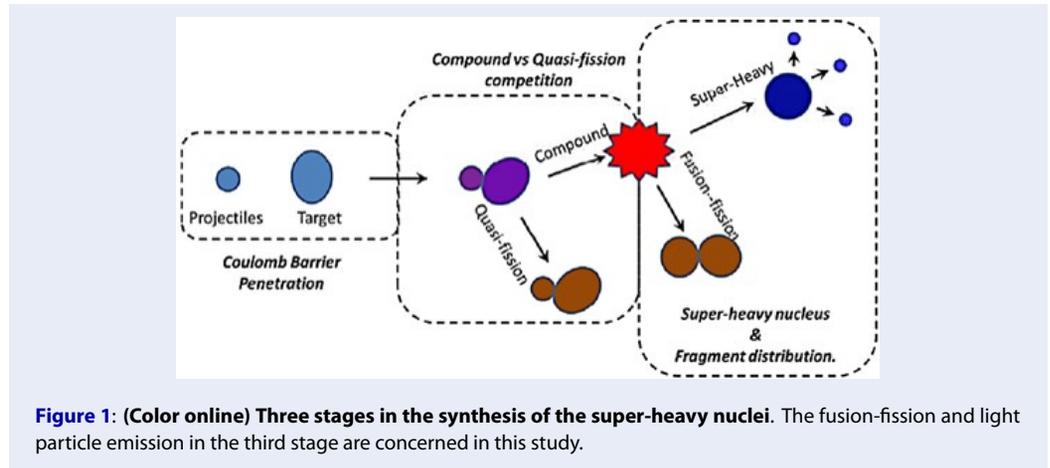


Figure 1: (Color online) Three stages in the synthesis of the super-heavy nuclei. The fusion-fission and light particle emission in the third stage are concerned in this study.

^{251}Cf and the $^{64}\text{Zn} + ^{248}\text{Cm}$ combinations have been suggested for evaluating the fission properties due to their small fusion cross sections. Because the synthesis cross section strongly depends on the probability of related processes, it is necessary to evaluate the compound formation and survival probabilities. Notice that the probabilities of the light-particle evaporation and fission are characterized by the decay- and fission-widths. Therefore, in this study, the widths of the neutron/proton evaporation, alpha emission, and fission in the de-excitation of the compound nuclei, $^{309,312}_{126}$, which are formed by the $^{58}\text{Ni} + ^{251}\text{Cf}$ and the $^{64}\text{Zn} + ^{248}\text{Cm}$ combinations, were evaluated. Besides, the level densities and lifetimes of the super-heavy $^{309,312}_{126}$ nuclei were also estimated.

THEORETICAL FRAMEWORK

As shown in **Figure 1**, the compound nucleus may de-excite via light-particle evaporation or fusion-fission processes. There is a competition between these processes. The emission of the light particles such as neutron, proton, or alpha is the main path of the evaporation. The fusion-fission proceeds with fragmentation to produce lighter isotopes. The destruction of the compound nucleus strongly depends on the probability of the decay via a certain decay mode. The decay probability, P_i , in an interval time, Δt , can be described in terms of the partial decay width, Γ_i , as

$$P_i = \frac{\Gamma_i}{\hbar} \Delta t \quad (1)$$

where $\hbar = 6.5821 \times 10^{-22}$ MeV.s is the reduced Planck's constant. The partial width can be evaluated by the Weisskopf formula ¹⁴:

$$\Gamma_i = \frac{m_i}{\pi^2 \hbar^2} (2s_i + 1) \int_0^{E^* - E_{Bi}} E_i \sigma_i \frac{\rho_i(E_D^*)}{\rho(E^*)} dE_i \quad (2)$$

in which m_i , s_i , and E_i are the mass, spin, and energy of the emitted particle, respectively; E^* and E_{Bi} denote the excitation energy of the compound nucleus and the threshold of the particle emission; σ_i is the cross section for the compound-nuclide formation via the channel of the emitted particle and daughter nucleus; $r_i(E_D^*)$ and $r_i(E^*)$ are the level densities of the daughter and compound nuclei at excitation energies E_D^* (after emission) and E^* (before emission), respectively.

The fission width, which reflects the fission probability of the compound nucleus, estimated based on Bohr-Wheeler method, is given by ^{13,15}:

$$\Gamma_f = \frac{1}{2\pi} \int_0^{E^* - B_f} \frac{\rho_f(E^* - B_f - E)}{\rho(E^*)} dE \quad (3)$$

where B_f is the fission barrier, which can be obtained from Ref. ^{16,17}; E and ρ_f are the kinetic energy of the fissioning system and the level density of the fissioning nucleus ¹⁸ in the saddle configuration at given excitation energy, respectively. Subsequently, the total width of the de-excitation is defined as:

$$\Gamma_{total} = \sum_i \Gamma_i + \Gamma_f \quad (4)$$

The level density, $\rho(E^*)$, can be described in terms of rotational ($K_{rot.}$) and vibrational ($K_{vib.}$) parameters, and the non-collective internal nuclear excitation, $\rho_{int.}(E^*)$, as ¹⁸⁻²¹

$$\rho(E^*) = K_{rot.} + K_{vib.} + \rho_{int.}(E^*) \quad (5)$$

The coefficients of the rotational and vibrational effects are given by

$$K_{rot.} = \begin{cases} I \left(\frac{E^* - \Delta}{a} \right) f(\beta_2, \beta_4) & \text{for deformed nuclei} \\ 1 & \text{for spherical nuclei} \end{cases} \quad (6)$$

and

$$K_{vib.} \approx \exp \left(0.0555 \left(A \frac{E^* - \Delta}{a} \right)^{2/3} \right) \quad (7)$$

where I and a denote the rigid-body inertia moment and nuclear level-density parameter in the Fermi-gas model^{20,21}, respectively. Notice that the level density is considered under point of view of the equidistant model²². The pairing energy is simply calculated by

$$\Delta = \begin{cases} 0 & (odd - odd) \\ 12A^{-1/2} & (odd - A) \\ 24A^{-1/2} & (even - even) \end{cases} \text{ in MeV.} \quad (8)$$

The deformation-dependent function, $f(\beta_2, \beta_4)$, is described in terms of the coefficients of quadrupole (β_2) and octupole (β_4) deformations as

$$f(\beta_2, \beta_4) = 1 + \sqrt{\frac{5\pi}{16}} \beta_2 + \sqrt{\frac{45\pi}{28}} \beta_2^2 + \frac{15\sqrt{5\pi}}{7} \beta_2 \beta_4 \quad (9)$$

The non-collective internal nuclear excitation is determined by

$$\rho_{int.}(E^*) = \frac{1}{12} \sqrt{\pi} a^{-1/4} (E^* - \Delta)^{-5/4} \exp \left(2\sqrt{a(E^* - \Delta)} \right) \quad (10)$$

Since the lifetime reflects the existence of the compound and/or residual nuclei in the fusion-fission stage, this factor plays an important role in investigations of the fission. The mean lifetime, τ , of excited nuclei can be determined based on the total width as

$$\tau = \frac{\hbar}{\Gamma_{total}}. \quad (11)$$

RESULTS

The decay widths of neutron, proton, alpha, and fission in the excitation energy range of $E^* = 10 - 100$ MeV were calculated by using Equations (2) and (3). Since the rotation energy, $E_{rot.}$, is much smaller than the value of $E_{cm.} + Q$, the $E^* = E_{cm.} + Q - E_{rot.}$ is approximately equal to $E^* = E_{cm.} + Q$ where $E_{cm.}$ and Q are the reaction energy in the center-of-mass frame and the Q-value of the fusion, respectively. The Q-values of the $^{58}\text{Ni} + ^{251}\text{Cf}$ and the $^{64}\text{Zn} + ^{248}\text{Cm}$ reactions are -249.6 and -260.2 MeV, respectively. Obviously, the fusions of these combinations are endothermic reactions because of high Coulomb barriers of the high-Z heavy-nuclide interactions. The nuclear level density was computed based on the Fermi-gas model with a consideration of the equidistant space model, as mentioned above. Notice that the LISE++

code^{23,24} was employed for the level density calculation. In this calculation, the shell and pairing corrections¹⁸ were included. The level density parameters of a were found to be about 39.5 and 40.1 for the $^{309}\text{126}$ and the $^{312}\text{126}$ isotopes, respectively. The estimated nuclear level densities of these nuclei are shown in Figure 2. By taking the calculated level density, the particle decay and fission widths were determined. The quantitative results of these quantities are presented in Tables 1 and 2. A comparison of the widths is shown in Figure 3.

Notice that the branching ratios of the partial widths to the total ones, Γ_i/Γ_{total} , describe the probabilities of decays or fission in the destruction process of the compound nucleus. To investigate the observation probability of the light particle emission and the fission from the $^{309,312}\text{126}$ nuclei, we evaluated the branching ratios of Γ_i/Γ_{total} for the alpha decay, 1n-, 1p-evaporations, and fission with excitation energies up to $E^* = 100$ MeV for the $^{58}\text{Ni} + ^{251}\text{Cf}$ and the $^{64}\text{Zn} + ^{248}\text{Cm}$ combinations. A comparison of the ratios is shown in Figure 4. The total width is the sum of the evaporation and fission widths, as described in Equation (4). We found that the total widths are approximately equal to the fission ones. Taking the total widths into Equation (1), the probabilities for destroying the compound nuclei, $^{309,312}\text{126}$, via all channels in an interval of one second, were estimated. These values are presented in the last columns of Tables 1 and 2. The probabilities for 1n-, 1p-evaporations, alpha decay, and fission in a unit of time can also be calculated based on the decay and fission widths, Γ_i , by using Equation (1).

The survival time scale of the compound nuclei can be evaluated by using the mean lifetime, which is calculated by Equation (11). The results are presented in Figure 5, Tables 1 and 2. It was found that the lifetimes of the concerned compound nuclei are in the range of $\tau = 10^{-22} - 10^{-20}$ in the excitation energy range of $E^* = 10 - 100$ MeV.

DISCUSSIONS

The level densities of the excited $^{309,312}\text{126}$ isotopes were estimated to be about $10^5 - 10^{50}$ (MeV^{-1}) in the excitation energy range of $E^* = 10 - 100$ MeV, as can be seen in Figure 2. It is found that the densities are reduced by the pairing and shell corrections. The reduction of a few factors is observed for the $^{309}\text{126}$ isotope while it is about 1 - 2 orders of magnitude for the other. This discrepancy can be understood by the different energies Δ , in the pairing correction. As described in the previous section, $\Delta = 12A^{-1/2}$ for the

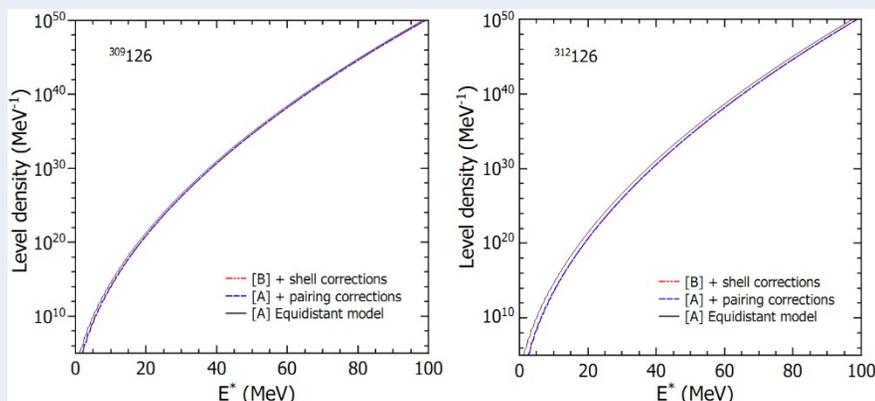


Figure 2: (Color online) Nuclear level densities of the $^{309}_{126}$ (left panel) and the $^{312}_{126}$ (right panel) nuclei were calculated based on the Fermi-gas model with the equidistant space model. The pairing and shell corrections were considered in the calculations.

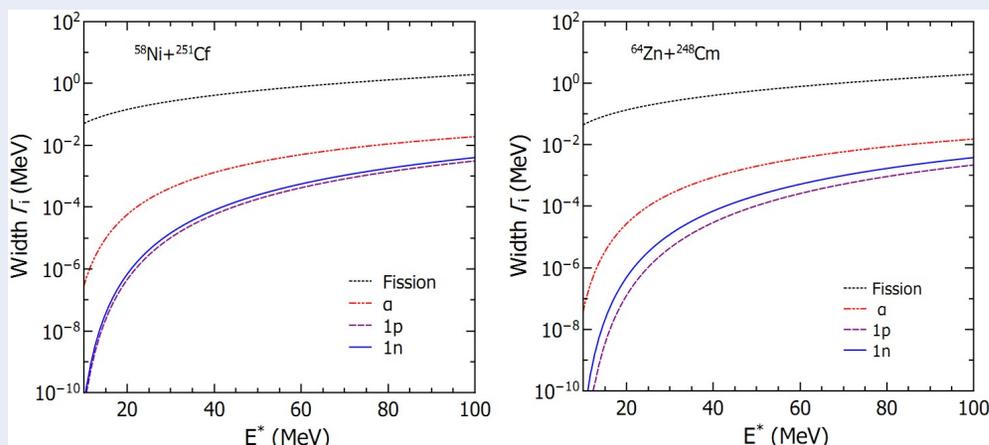


Figure 3: Color online) Comparisons of the partial decay widths of the light particle emissions and the fission width of the fission channel in the synthesis of the $^{309}_{126}$ (left panel) and the $^{312}_{126}$ (right panel) nuclei.

even-odd $^{309}_{126}$ isotope while it is $24A^{-1/2}$ for the even-even $^{312}_{126}$ nucleus.

As can be seen in Figures 3 and 4, the partial widths are rapidly (slightly) increased by excitation energies in the range of $E^* < 40$ ($E^* > 40$) MeV. This result is explained by the weak survival of the compound nuclei at high excited states. It is also observed that the fission emerges as a dominant over the other de-excitation processes. The fission widths are about 2 – 6 and 4 – 8 orders of magnitudes higher than those of the alpha decay and neutron (or proton) evaporations, respectively. The neutron widths are also larger than those of the proton emission. These results indicate that the de-excitation of the compound nuclei easily proceeds via fission and alpha decay rather than nucleon evaporations in the competition of de-

excitation channels in the third stage described in **Figure 1**. For measurement techniques, however, fission is not appropriate to identify new elements in the super-heavy nuclide production. Subsequently, alpha decay and neutron emission can be preferred to observations in laboratories. On the other hand, the results show the fact that the fragmentation strongly occurs, and it overlaps the light particle emission in the synthesis of the super-heavy nuclei. Hence, the fragmentation can be considered as the main source for the production of the medium nuclei, i.e., Fe-U isotopes. The dominance of the fission and alpha decay can be understood by the Coulomb repulsion of the high-Z elements. However, this reason is not relevant to the proton evaporation because the 1n-emission width is much larger than that of the 1p-evaporation

Table 1: Partial decay widths of neutron (Γ_n), proton (Γ_p), alpha (Γ_α), and fission (Γ_f) for the $^{309}_{126}$ isotope. The lifetime (τ) and decay probability (P) in an interval of $\Delta t = 1$ second were calculated based on the total width

E* (MeV)	Γ_n (MeV)	Γ_p (MeV)	Γ_α (MeV)	Γ_f (MeV)	τ (s)	P
8.1	5.4E-14	2.4E-14	1.7E-08	3.6E-02	1.8E-20	5.5E+19
12.1	1.7E-09	1.1E-09	1.8E-06	6.9E-02	9.6E-21	1.0E+20
16.2	8.3E-08	5.4E-08	1.6E-05	1.1E-01	6.2E-21	1.6E+20
20.2	7.5E-07	5.0E-07	6.1E-05	1.5E-01	4.5E-21	2.2E+20
24.2	3.3E-06	2.3E-06	1.6E-04	1.9E-01	3.4E-21	2.9E+20
28.3	9.9E-06	6.9E-06	3.3E-04	2.4E-01	2.7E-21	3.7E+20
32.3	2.3E-05	1.7E-05	5.8E-04	3.0E-01	2.2E-21	4.5E+20
36.4	4.7E-05	3.4E-05	9.2E-04	3.6E-01	1.8E-21	5.4E+20
40.4	8.4E-05	6.1E-05	1.4E-03	4.2E-01	1.6E-21	6.4E+20
44.4	1.4E-04	1.0E-04	1.9E-03	4.9E-01	1.3E-21	7.5E+20
48.5	2.1E-04	1.6E-04	2.6E-03	5.6E-01	1.2E-21	8.6E+20
52.0	3.0E-04	2.2E-04	3.2E-03	6.3E-01	1.0E-21	9.7E+20
56.1	4.2E-04	3.1E-04	4.1E-03	7.2E-01	9.2E-22	1.1E+21
60.1	5.7E-04	4.3E-04	5.0E-03	8.0E-01	8.2E-22	1.2E+21
64.1	7.5E-04	5.7E-04	6.1E-03	9.0E-01	7.3E-22	1.4E+21
68.2	9.6E-04	7.3E-04	7.2E-03	9.9E-01	6.6E-22	1.5E+21
72.2	1.2E-03	9.3E-04	8.5E-03	1.1E+00	6.0E-22	1.7E+21
76.3	1.5E-03	1.2E-03	9.8E-03	1.2E+00	5.4E-22	1.8E+21
80.3	1.8E-03	1.4E-03	1.1E-02	1.3E+00	4.9E-22	2.0E+21
84.3	2.2E-03	1.7E-03	1.3E-02	1.4E+00	4.5E-22	2.2E+21
88.4	2.6E-03	2.0E-03	1.4E-02	1.6E+00	4.2E-22	2.4E+21
92.4	3.1E-03	2.4E-03	1.6E-02	1.7E+00	3.8E-22	2.6E+21
96.5	3.5E-03	2.8E-03	1.8E-02	1.8E+00	3.6E-22	2.8E+21
100.5	4.1E-03	3.2E-03	1.9E-02	2.0E+00	3.3E-22	3.0E+21

even though neutron is a neutral particle. This exception suggests more investigations for the fusion-fission mechanism.

As mentioned, the partial width of the alpha decay is much larger than those of 1n- and 1p-evaporations. This result indicates that it is possible for the compound nuclei to become the alpha-decay super-heavy nuclei. This conclusion is also suggested by a previous study of the alpha-decay half-lives of the Z = 126 isotopes²⁵. Hence, the observation of the $^{309,312}_{126}$ nuclei in experiments strongly depends on the alpha-decay half-lives. By considering the increasing orientation of the widths, the neutron emission process is predicted to be comparable to the alpha decay in much higher energy range, i.e., $E^* > 400$ MeV. In

other words, for highly excited states of the compound nuclei, there is a strong competition between the alpha decay and neutron evaporation.

Since the fission width is much larger than the widths of the neutron/proton emissions, the evaporation-residue cross section should be much smaller than that of the fission. This result is totally consistent with that observed in our previous study for the synthesis cross section of the $^{309,312}_{126}$ nuclei via $^{58}\text{Ni} + ^{251}\text{Cf}$ and the $^{64}\text{Zn} + ^{248}\text{Cm}$ combinations^{7,8}. Notice that the evaporation cross sections of $^{309,312}_{126}$ were found to be extremely small, which is in the order of zb (10^{-21} barn)^{7,8}.

For the lifetimes of $^{309,312}_{126}$, it is found that the survival of the $^{312}_{126}$ isotope is longer than that of the

Table 2: Partial decay widths of neutron (Γ_n), proton (Γ_p), alpha (Γ_α), and fission (Γ_f) for the $^{312}_{126}$ isotope. The lifetime (τ) and decay probability (P) in an interval of $\Delta t = 1$ second were calculated based on the total width.

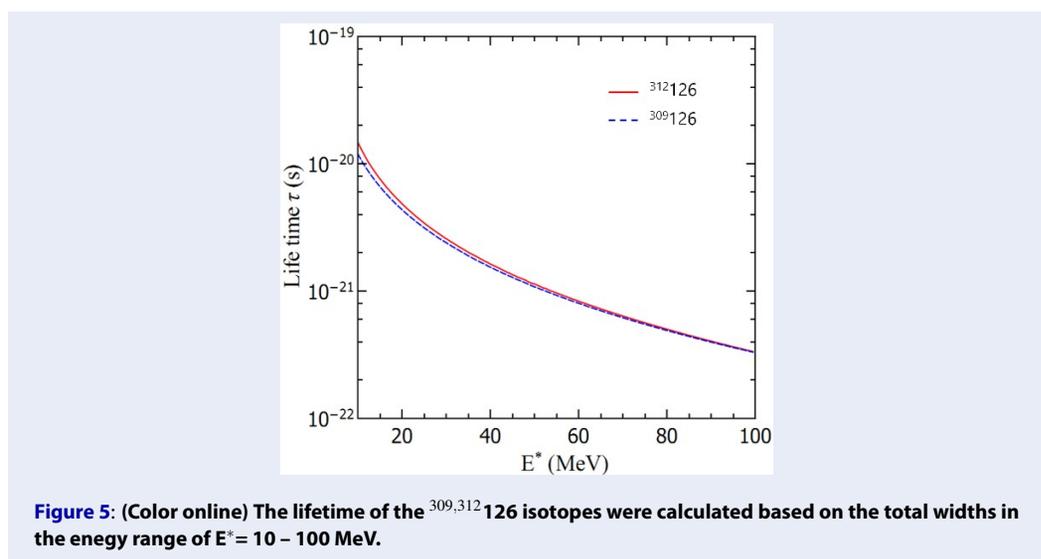
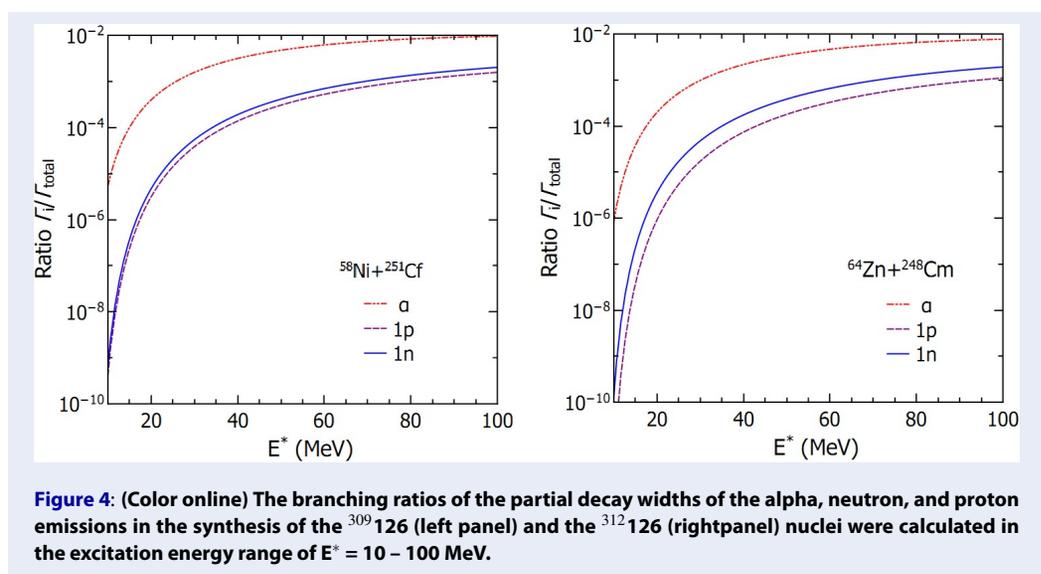
E* (MeV)	Γ_n (MeV)	Γ_p (MeV)	Γ_α (MeV)	Γ_f (MeV)	τ (s)	P
8.1	1.8E-15	1.6E-18	7.1E-10	3.1E-02	2.2E-20	4.6E+19
12.1	6.7E-10	5.9E-11	4.4E-07	6.1E-02	1.1E-20	9.3E+19
16.2	5.1E-08	9.7E-09	6.1E-06	9.7E-02	6.8E-21	1.5E+20
20.2	5.4E-07	1.4E-07	2.9E-05	1.4E-01	4.8E-21	2.1E+20
24.2	2.6E-06	8.0E-07	8.5E-05	1.8E-01	3.6E-21	2.8E+20
28.3	8.2E-06	2.8E-06	1.9E-04	2.3E-01	2.8E-21	3.5E+20
32.3	2.0E-05	7.5E-06	3.6E-04	2.9E-01	2.3E-21	4.3E+20
36.4	4.1E-05	1.7E-05	5.9E-04	3.4E-01	1.9E-21	5.2E+20
40.4	7.4E-05	3.2E-05	9.1E-04	4.1E-01	1.6E-21	6.2E+20
44.4	1.2E-04	5.5E-05	1.3E-03	4.8E-01	1.4E-21	7.2E+20
48.5	1.9E-04	8.9E-05	1.8E-03	5.5E-01	1.2E-21	8.3E+20
52.0	2.7E-04	1.3E-04	2.3E-03	6.2E-01	1.1E-21	9.4E+20
56.1	3.8E-04	1.9E-04	3.0E-03	7.0E-01	9.4E-22	1.1E+21
60.1	5.2E-04	2.6E-04	3.7E-03	7.9E-01	8.3E-22	1.2E+21
64.1	7.0E-04	3.5E-04	4.5E-03	8.8E-01	7.5E-22	1.3E+21
68.2	9.0E-04	4.7E-04	5.5E-03	9.8E-01	6.7E-22	1.5E+21
72.2	1.1E-03	6.0E-04	6.5E-03	1.1E+00	6.1E-22	1.7E+21
76.3	1.4E-03	7.6E-04	7.5E-03	1.2E+00	5.5E-22	1.8E+21
80.3	1.7E-03	9.4E-04	8.7E-03	1.3E+00	5.0E-22	2.0E+21
84.3	2.1E-03	1.1E-03	9.9E-03	1.4E+00	4.6E-22	2.2E+21
88.4	2.5E-03	1.4E-03	1.1E-02	1.6E+00	4.2E-22	2.4E+21
92.4	2.9E-03	1.6E-03	1.3E-02	1.7E+00	3.9E-22	2.6E+21
96.5	3.4E-03	1.9E-03	1.4E-02	1.8E+00	3.6E-22	2.8E+21
100.5	3.9E-03	2.2E-03	1.6E-02	2.0E+00	3.3E-22	3.0E+21

other. Besides, the survival probability of these compound nuclei is decreased by high excitation energies. This can be explained by the stronger deformation of the nuclei at highly excited states, which de-excite to more stable states by fission or emissions of light particles.

CONCLUSION

In this study, the fusion-fission of the $^{58}\text{Ni} + ^{251}\text{Cf}$ and the $^{64}\text{Zn} + ^{248}\text{Cm}$ reactions were considered in the scope of the production mechanism of unknown super-heavy isotopes, $^{309,312}_{126}$. The level densities of these isotopes were calculated based on the Fermi-gas model. It was found that energy levels of the con-

cerned nuclei are rapidly increased by excitation energies. The pairing and shell corrections slightly reduce the predicted level densities. For the competition between evaporations and fission in the de-excitation of the compound nuclei, it was observed that the fission is strongly dominant over the other processes. This result leads to a high probability of fragmentation for the medium-mass isotopes. Hence, the $^{58}\text{Ni} + ^{251}\text{Cf}$ and the $^{64}\text{Zn} + ^{248}\text{Cm}$ combinations can be preferred to the study of mass distribution in fission. On the other hand, since the alpha decay width is much larger than that of the evaporations, the $^{309,312}_{126}$ isotopes are also considered to be the alpha-decay super-heavy nuclei. In addition, it was found that the nucleon



evaporation mechanism is also a mystery in the super-heavy nuclide synthesis. Therefore, theoretical and experimental studies of the synthesis of super-heavy nuclei are strongly suggested.

COMPETING INTERESTS

The author declares that there is no conflict of interest regarding the publication of this article.

AUTHORS' CONTRIBUTIONS

The ideas, calculations, data analysis, discussion of results, and writing manuscript were performed by Dr. Nguyen Ngoc Duy.

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