A MOVING ELEMENT METHOD USING TIMOSHENKO'S BEAM THEORY FOR DYNAMIC ANALYSIS OF TRAIN-TRACK SYSTEMS

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ABSTRACT

The paper presents a dynamic analysis of train-track systems supported by viscoelastic foundations by combining Timoshenko's beam theory and moving element method (MEM). In the proposed method, a three-node beam element is utilized to get a high order approximation for the deflection of Timoshenko beam. The reduced integral method is applied in order to avoid the shear-locking phenomenon when computing the shear strain energy of the rail beam. In addition, the behavior of train-track system with respect to time is deduced by using Newmark's constant acceleration method. Numerical results show that the proposed method is free of shear locking and gives a good agreement with Koh et al.'s method using Euler-Bernoulli beam theory.

Keywords: train-track system, moving element method (MEM), Timoshenko beam theory, three node beam element, shear-locking phenomenon.

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1. Introduction

Moving-load dymanic problems which are very common in engineering have received a lot of interests from researchers all around the world from quite early. It was believed that in 1847, the collapse of Stephanson's Bridge in England motivated researchers to find out accurately the effects of moving loads to structures [1]. Among a large number of structures subjected to moving load, transport engineering structures like railways, bridges or pavements have gained much concern from beginning stage [2].

Up to now many models have been developed to study the behavior of the transport structures Kirchhoff's plate theory. Moving loads have also been split into many cases such as moving constant or time depended forces, moving masses and moving vehicle systems. Besides that, to reflect the reality, the effect of foundation on which the structures lie has been counted and divided into many cases like elastic and viscoelastic foundations.

Initially, analytic methods have been used widely to study the effect of moving load to transport structures. Some of these methods which can be named here are the method of using Green's function and integral equations [3], method of expansion of the eigen-functions [4], Galerkin's method [5] and the Fourier transform method (FTM) [6, 7]. For example, in 1958 and 1959, P. M. Mathews [6, 7] made an analysis of vibration of an infinite uniform beam on elastic foundation under an alternation force moving at a constant velocity. In this work, a new coordinate system moving with the force was defined and the deflection of the beam modeled by Euler-Bernoulli's beam theory was then found by using Fourier transform technique.

In recent decades with the quick development of digital computers, many numerical methods such as finite element method (FEM), mesh-free method, finite difference method have been proposed. These new methods have made the researching of mechanics in general and of moving load problems in particular become more convenient and expandable. Along with this, all branches of transport have obtained many great advances with huge increase in speed and weight of vehicles. This fact has made the researching of dynamic responses of structures under moving vehicles become more and more important. Although proving the strength in analyzing various structures in engineering, finite element method (FEM) [8] has encountered many obstacles when facing with moving load problems. For example, a very large domain of structure subjected to high speed moving load requires a mesh with a lot of degrees of freedom (DOFs). Besides that, keeping track of load positions related to mesh nodes is unavoidable. These restrictions have increased much computational cost. Therefore, recently, to solve this kind of problems, some new approaches which commonly use moving coordinates have appeared and improved their efficiency.

In 2000 and 2001, Chen and Huang [9, 10] studied infinite Timoshenko beam subjected to a moving load on viscoelastic foundation. By using a moving coordinate as Matthew [6, 7], the dynamic stiffness matrix of the beam with the velocity component inside is established. In 2003, C. G. Koh et al. [11] proposed a new approach called moving element method (MEM), which was a combination of FEM and moving coordinate, for analyzing the dynamic of train-track systems. In their work, the Euler-Bernoulli rail beam on viscoelastic foundation was discretized into elements which 'flow' with the moving vehicles/forces. The train was modeled by a spring-mass-damper system which is then coupled with the beam equations to form governing equations. In 2013, this problem is then expanded by Ang and Dai to deal with the abrupt change of foundation stiffness [12].

After being released, MEM has being extended to research other types of structure under moving load. In 2007, Koh et al. [13] used their new method to analyze the half-space continuum under different moving load types such as strip loads and concentrated loads. In 2009, MEM for random vibration analysis of vehicles on Kirchhoff plate supported by Kelvin foundation was presented by Xu et al. [14].

In this paper, the application of MEM in analyzing train-track system is extended by using Timoshenko beam theory to model rail beam. With this extension, shear strain of rail beam is counted and the model becomes more appropriate to reality. Because of the specific property of MEM, a three-node element is used to gain a high order approximation for both deflection and rotation of Timoshenko beam. The reduced integral method is then applied when computing the shear strain energy of the rail beam to avoid the shear-locking phenomenon. In numerical example section, both moving constant force and moving vehicle system are taken to study. The behavior of train-track system with respect to time is deduced by using Newmark's constant acceleration method. The reliability and accuracy of the proposed method which is called MEM-T for brief are verified by comparing its numerical results with those of Koh [11] by using Euler-Bernoulli beam theory (briefly, MEM-E).

2. Formulation of the moving element method (MEM-T)

In this paper, the train-track system is modeled simply as in Figure 1. The train is modeled as a spring-mass-damper system which has only 3 DOFs (u_1, u_2, u_3) corresponding to three vertical displacements of three masses (m_1, m_2, m_3). These massess represent the wheel-axle system, the bogie, and the train body, respectively. Between the lowest mass m_1 and the rail beam there are a spring k_1 and a damper c_1 which implement the Hertzian contact effects. Between the three masses there are two suspension systems which are modeled by two couples of spring and damper, (k_2, c_2) and (k_3, c_3). The train is assumed to move at a constant speed V in the positive x-direction. The rail is modeled as an infinite Timoshenko beam with Young's modulus E, shear modulus G, second moment of area I and mass per unit length m . This beam is supported on a viscoelastic foundation whose stiffness and damping per unit length are k and c , respectively.

Instead of starting from the governing differential equations of Timoshenko beam on viscoelastic foundation, we use energy approach to find out the weak form of the dynamic problem. By combining principle of virtual work with a moving coordinate, the governing equations of Timoshenko beam on viscoelastic foundation is formulated in this coordinate. The motion equations of vehicle system are then assembled with rail beam equations to form the overall equations.

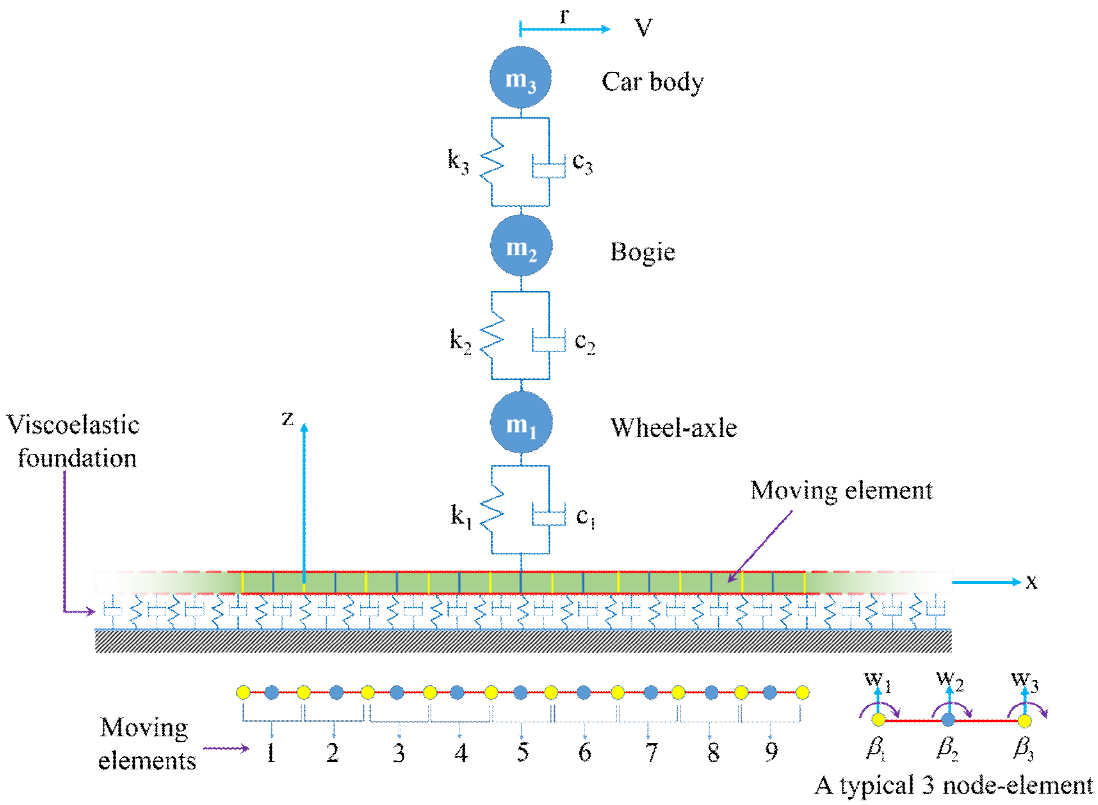

Figure 1. Train-track model

In detail, we only consider a segment of the rail beam such that both its upstream and downstream ends are sufficiently far from the wheel contact point. As a result, the forces and moments at these ends are possible to be negligible. We have the displacements of the segment of the infinite Timoshenko beam are given by

$$\begin{cases} u(x,z) = z\beta(x) \\ w(x,z) = w(x) \end{cases}. \tag{1}$$

Then the strain and stress fields are easily deduced as

$$\begin{cases} \varepsilon_x = z\dfrac{\partial \beta}{\partial x} \\ \gamma_{xz} = \beta + \dfrac{\partial w}{\partial x} \end{cases} \Rightarrow \begin{cases} \sigma_x = E\varepsilon_x = Ez\dfrac{\partial \beta}{\partial x} \\ \tau_{xz} = \mu G\gamma_{xz} = \mu G\left(\beta + \dfrac{\partial w}{\partial x}\right) \end{cases}, \tag{2}$$

where μ is the shear coefficient which is usually given by $5/6$.

If we set two operators of bending and shear strain and the displacement field of Timoshenko beam which contains only the deflection w and rotation β as

$$\mathbf{L}_1 = \begin{bmatrix} 0 & \dfrac{\partial}{\partial x} \end{bmatrix}, \mathbf{L}_2 = \begin{bmatrix} \dfrac{\partial}{\partial x} & 1 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} w \\ \beta \end{bmatrix}, \tag{3}$$

then the energy strain of Timoshenko beam which is a sum of bending and shear component is written briefly as

$$U = \frac{1}{2} EI \int_L (\mathbf{L}_1 \mathbf{u})^2 \, dx + \frac{1}{2} \mu GA \int_L (\mathbf{L}_2 \mathbf{u})^2 \, dx, \tag{4}$$

where L is the length of the beam segment and A is the area of beam cross section.

2.1. Energy method for dynamic analysis of train-track system

In this section, we use the principle of virtual work to formulate the weak form of Timoshenko beam on viscoelastic foundation. This principle states that for any compatible and small virtual displacements imposed on the body, the total internal virtual work done W_I must be equal to the total external virtual work done W_E.

The internal virtual work of rail beam can be expressed as

$$W_I = \delta U = EI \int (\mathbf{L}_1 \delta \mathbf{u})^T (\mathbf{L}_1 \mathbf{u}) dx + kGA \int_L (\mathbf{L}_2 \delta \mathbf{u})^T (\mathbf{L}_2 \mathbf{u}) dx, \tag{5}$$

whereas the external virtual work done W_E consists of the works done by the dynamic force at the wheel contact point, the inertial force of beam, the elastic and damping forces of the foundation.

$$W_E = W_E^1 + W_E^2 + W_E^3 + W_E^4. \tag{6}$$

Particularly,

$$\begin{cases} W_E^1 = \int_L F(t) \delta^{Dirac} (x - Vt) \delta w dx \\ W_E^2 = -\int_L (\delta w) m \ddot{w} dx \end{cases} \qquad \begin{cases} W_E^3 = -\int_L (\delta w) k w dx \\ W_E^4 = -\int_L (\delta w) c \dot{w} dx \end{cases}, \tag{7}$$

in which $F(t)$ given in Eq. (21) is the dynamic force at the wheel contact point and δ^{Dirac} denotes the Dirac-delta function.

Note that x-coordinate is a fix coordinate in the longitudinal direction of the beam with the origin is chosen such that the contact point is at $x = 0$ when time $t = 0$. To avoid keeping track of the load position, a moving coordinate system is defined as follow

$$r = x - Vt. \tag{8}$$

Applying the chain rule we can easily transform the derivatives with respect to the fix coordinate x and the derivatives with respect to time t as follows

$$\begin{cases} \dfrac{\partial}{\partial x} \to \dfrac{\partial}{\partial r} \\ \dfrac{\partial^2}{\partial x^2} \to \dfrac{\partial^2}{\partial r^2} \end{cases} \qquad \begin{cases} \dfrac{\partial}{\partial t} \to \dfrac{\partial}{\partial t} - V \dfrac{\partial}{\partial r} \\ \dfrac{\partial^2}{\partial t^2} \to \dfrac{\partial^2}{\partial t^2} - 2V \dfrac{\partial^2}{\partial r \partial t} + V^2 \dfrac{\partial^2}{\partial r^2} \end{cases}. \tag{9}$$

At the point of time $t = 0$, in the fixed co-ordinate, the position of the beam segment is $[-L/2, L/2]$ and in general, at arbitrary point of time t this position is $[vt - L/2, vt + L/2]$. By using the moving coordinate system above the interesting domain of beam is always $\left[-L/2, L/2\right]$ and the position at which the moving force affect the beam is always $r=0$. So there's no need to keep track of moving load.

In the moving coordinate we have the internal and external virtual works are rewritten as follows.

$$W_I = EI\int_L \left(\bar{\mathbf{L}}_1\delta\bar{\mathbf{u}}\right)^T \left(\bar{\mathbf{L}}_1\bar{\mathbf{u}}\right)dr + kGA\int_L \left(\bar{\mathbf{L}}_2\delta\bar{\mathbf{u}}\right)^T \left(\bar{\mathbf{L}}_2\bar{\mathbf{u}}\right)dr \tag{10}$$

where

$$\bar{\mathbf{L}}_1 = \begin{bmatrix} 0 & \dfrac{\partial}{\partial r}\end{bmatrix}, \bar{\mathbf{L}}_2 = \begin{bmatrix} \dfrac{\partial}{\partial r} & 1\end{bmatrix}, \bar{\mathbf{u}} = \begin{bmatrix} \bar{w} \\ \bar{\beta}\end{bmatrix} \tag{11}$$

are the two operators of bending and shear strain and the displacement field of Timoshenko beam in moving coordinate.

Similarly, the total external virtual work in this moving coordinate is given by

$$\begin{aligned} W_E &= \int_L F(t)\delta^{Dirac}(r)\delta\bar{w}dr + \\ &\int_L (\delta\bar{w})\left[\left(-m\ddot{\bar{w}}\right) + \left(2mV\frac{\partial\dot{\bar{w}}}{\partial r} - c\dot{\bar{w}}\right) + \left(-mV^2\frac{\partial^2\bar{w}}{\partial r^2} + cV\frac{\partial\bar{w}}{\partial r} - k\bar{w}\right)\right]dr \end{aligned}. \tag{12}$$

2.2. Foundation of MEM

As in equation (12) we can see that there is a second order derivative with respect to r so we use three node elements to discretize the displacement field (Figure 1).

Consider a typical three node element whose length is l_e. The shape functions for this element is given by

$$N_1 = \frac{l_e^2 - 3l_e r + 2r^2}{l_e^2}, \quad N_2 = \frac{4r\left(l_e - r\right)}{l_e^2}, \quad N_3 = -\frac{r\left(l_e - 2r\right)}{l_e^2}. \tag{13}$$

Then the displacement field is approximated as follows

$$\begin{cases} \bar{\mathbf{u}} = \left[\mathbf{N}\right]\left\{\bar{\mathbf{q}}_e\right\} \\ \bar{w} = \left\lfloor \mathbf{N}_{\bar{w}'} \right\rfloor \left\{\bar{\mathbf{q}}_e\right\} \end{cases}, \tag{14}$$

in which $\left\{\bar{\mathbf{q}}_e\right\}^T = \left[\bar{w}_1 \quad \bar{\beta}_1 \quad \bar{w}_2 \quad \bar{\beta}_2 \quad \bar{w}_3 \quad \bar{\beta}_3\right]$ is a vector of six DOFs of a beam element and the matrices of shape functions are

$$\left[\mathbf{N}\right] = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \quad \left\lfloor \mathbf{N}_{\bar{w}} \right\rfloor = \left[N_1 \quad 0 \quad N_2 \quad 0 \quad N_3 \quad 0\right]. \tag{15}$$

Substitute Eq. (14) into Eq. (10) and (12), we have the approximated internal and external virtual works are given by

$$W_I = \sum_{e=1}^{Ne}\left[\left\{\delta\bar{\mathbf{q}}_e\right\}^T \left[EI\int_0^{l_e}\left(\bar{\mathbf{L}}_1\mathbf{N}\right)^T\left(\bar{\mathbf{L}}_1\mathbf{N}\right)dr + \mu GA\int_0^{l_e}\left(\bar{\mathbf{L}}_2\mathbf{N}\right)^T\left(\bar{\mathbf{L}}_2\mathbf{N}\right)dr\right]\left\{\bar{\mathbf{q}}_e\right\}\right], \tag{16}$$

$$W_E = \sum_{e=1}^{Ne} \begin{bmatrix} \{\delta\bar{\mathbf{q}}_e\}^T \int_0^{l_e} F(t)\delta^{Dirac}(r)\mathbf{N}_{\bar{w}}^T dr + \\ \{\delta\bar{\mathbf{q}}_e\}^T \int_0^{l_e} \left[\left(-m\mathbf{N}_{\bar{w}}^T\mathbf{N}_{\bar{w}}\right)\{\ddot{\bar{\mathbf{q}}}_e\} + \left(2mV\mathbf{N}_{\bar{w}}^T\mathbf{N}_{\bar{w},r} - c\mathbf{N}_{\bar{w}}^T\mathbf{N}_{\bar{w}}\right)\{\dot{\bar{\mathbf{q}}}_e\}\right]dr + \\ \{\delta\bar{\mathbf{q}}_e\}^T \int_0^{l_e} \left[\left(-mV^2\mathbf{N}_{\bar{w}}^T\mathbf{N}_{\bar{w},rr} + cV\mathbf{N}_{\bar{w}}^T\mathbf{N}_{\bar{w},r} - k\mathbf{N}_{\bar{w}}^T\mathbf{N}_{\bar{w}}\right)\{\bar{\mathbf{q}}_e\}\right]dr \end{bmatrix}. \quad (17)$$

Finally, the weak form of the problem is deduced from the principle of virtual work, $W_I = W_E$. Therefore

$$\sum_{e=1}^{Ne}\{\delta\bar{\mathbf{q}}_e\}^T \mathbf{F}_e = \sum_{e=1}^{Ne}\{\delta\bar{\mathbf{q}}_e\}^T \int_0^{l_e}\left[\mathbf{M}_e\{\ddot{\bar{\mathbf{q}}}_e\} + \mathbf{C}_e\{\dot{\bar{\mathbf{q}}}_e\} + \mathbf{K}_e\{\bar{\mathbf{q}}_e\}\right]dr, \quad (18)$$

where

$$\begin{gathered} \mathbf{F}_e = \int_0^{l_e} F(t)\delta^{Dirac}(r)\mathbf{N}_w^T dr \\ \mathbf{M}_e = \int_0^{l_e}\left[\left(m\mathbf{N}_{\bar{w}}^T\mathbf{N}_{\bar{w}}\right)\right]dr \\ \mathbf{C}_e = \int_0^{l_e}\left(-2mV\mathbf{N}_{\bar{w}}^T\mathbf{N}_{\bar{w},r} + c\mathbf{N}_{\bar{w}}^T\mathbf{N}_{\bar{w}}\right) \\ \mathbf{K}_e = EI\int_0^{l_e}\left(\bar{\mathbf{L}}_1\mathbf{N}\right)^T\left(\bar{\mathbf{L}}_1\mathbf{N}\right)dr + \mu GA\int_0^{le}\left(\bar{\mathbf{L}}_2\mathbf{N}\right)^T\left(\bar{\mathbf{L}}_2\mathbf{N}\right)dr + \\ \int_0^{l_e}\left[\left(mv^2\mathbf{N}_{\bar{w}}^T\mathbf{N}_{\bar{w},rr} - cV\mathbf{N}_{\bar{w}}^T\mathbf{N}_{\bar{w},r} + k\mathbf{N}_{\bar{w}}^T\mathbf{N}_{\bar{w}}\right)\right]dr \end{gathered} . \quad (19).$$

For simplicity, the Gaussian quadrature rule is used to compute these matrices above. In addition, reduced integration technique is applied when computing the shear component of the stiffness matrix $\mathbf{K}_e$ in order to overcome the shear-locking phenomenon. Particularly, instead of using three Gaussian points as normal (because second order shape functions are used), we only use two points here. Because the virtual displacement vector $\delta\bar{\mathbf{q}}_e$ is arbitrary, it can be eliminated to give the governing equations of the rail beam.

2.3. The coupled equations of motion for the train-track model

Besides the effect of moving gravity load, train-track vibration is also caused by the roughness of the rail. Therefore it is necessary to include the rail corrugation in the formulation. By using Newton's second law, we can easy find the force equilibrium equations for each mass in train-track model as

$$\begin{aligned} m_3\ddot{u}_3 + k_3(u_2 - u_3) + c_3(\dot{u}_2 - \dot{u}_3) &= -m_3 g \\ m_2\ddot{u}_2 + k_2(u_2 - u_1) + c_2(\dot{u}_2 - \dot{u}_1) + k_3(u_2 - u_3) + c_3(\dot{u}_2 - \dot{u}_3) &= -m_2 g \\ m_1\ddot{u}_1 + k_1(u_1 - w_0) + c_1(\dot{u}_1 - \dot{w}_0) + k_2(u_1 - u_2) + c_2(\dot{u}_1 - \dot{u}_2) &= -m_1 g + \underbrace{(k_1 y_c + c_1\dot{y}_c)}_{F_c} \end{aligned} \quad (20)$$

in which y_c is the rail corrugation expressed as a function of time, $w_0 = \bar{w}|_{r=0}$ is the rail deflection, both at wheel contact point, and g is the acceleration due to gravity. From the third equation in Eq. (20) we have the wheel contact force is given by

$$F(t) = c_1(\dot{u}_1 - \dot{w}_0 - \dot{y}_c) + k_1(u_1 - w_0 - y_c). \quad (21)$$

Eq. (20) then is coupled with the rail beam to create equations of motion for train-track model

$$\mathbf{M}\ddot{\mathbf{z}} + \mathbf{C}\dot{\mathbf{z}} + \mathbf{K}\mathbf{z} = \mathbf{P}. \quad (22)$$

Here $\mathbf{z}$ is the displacement vector which consists all DOFs of beam and three DOFs of train system. $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are structural matrices which obtained by assembling the corresponding matrices of the beam model and the train model. Finally, Newmark's constant acceleration method is used to solve the above dynamic equation.

3. Numerical examples

In this section, to verify the efficiency of the proposed method, we will use some problems proposed by Koh et al. in their first paper of MEM [11]. Particularly, parameters for track and vehicle are given as in

with an addition is that the Poisson's ratio of rail beam is $\nu = 0.3$. Here we will illustrate two cases of moving load, constant force at constant velocity for the first case and 3-DOF vehicle at constant velocity for another. Both MEM-T and MEM-E methods are implemented. To put all of these methods in a same condition, we only use a regular mesh of 100 beam elements as being used by Koh et al. [11].

Table 1. Parameters of train-track system

Track parameters					
L	50 m		m	60.0 kg/m	
E	2.00×10^{11} Pa		k	1.00×10^{7} N/m^2	
I	3.06×10^{-5} m^4		c	4900 Ns/m^2	
Vehicle parameters					
m_1	350 kg	m_2	250 kg	m_3	3500 kg
k_1	8.00×10^{9} N/m	k_2	1.26×10^{6} N/m	k_3	1.41×10^{5} N/m
c_1	6.70×10^{5} Ns/m	c_2	7.10×10^{3} Ns/m	c_3	8.87×10^{3} Ns/m

Besides that, in the case of moving force, the shear-locking phenomenon is illustrated by comparing results from different methods when the thickness of rail beam changed. It's assumed that h is the thickness of rail beam corresponding to the original data of Koh et al. and the area of beam cross section and the second moment of area is given by $A = h$ and $I = Ah^2/12$, respectively.

3.1. Constant force at constant velocity

In this example, the rail beam is assumed to be smooth and the damping of foundation is removed. We only consider the effect of the total gravity load of three mass moving at a constant velocity of 20 m/s. Because the excitation is a constant force, the solution is time invariant at the steady state. All time derivatives in Eq. (22) are vanished. As a results, there's no need to using Newmark's method. Besides solution from MEM-E method proposed by Koh at al. [11], another one called quasi-static solution from Ref [15] is used as a comparator. This solution is given explicitly by

$$\overline{w}(r) = Ae^{-|\alpha r|}\left[\cos(|\alpha r|) + \sin(|\alpha r|)\right] \tag{23}$$

in which $A = \alpha F / 2k$ with F is the constant moving load and $\alpha = (k / 4EI)^{1/4}$.

Table 2. Deflection of Beam at Contact Point (mm)

Thickness	Deflection of Beam at Contact Point (mm)			
	Quasi-static [**Error! Reference source not found.**]	(MEM-E) [**Error! Reference source not found.**]	MEM-T (Reduced Int)	MEM-T (Full Int)
8h	-0.3380	-0.3381	-0.3393	-0.3408
4h	-0.5684	-0.5685	-0.5699	-0.5731
2h	-0.9560	-0.9562	-0.9580	-0.9679
h	-1.6078	-1.6088	-1.6110	-1.6455
h/2	-2.7039	-2.7075	-2.7111	-2.7845
h/4	-4.5474	-4.5458	-4.5663	-4.4142
h/8	-7.6478	-7.4411	-7.6797	-6.0722

Table 3. Error Percent to Quasi-static solution

Thickness	Error Percent to Quasi-static solution (%)		
	Koh et al. [**Error! Reference source not found.**] (MEM-E)	MEM-T (Reduced Int)	MEM-T (Full Int)
8h	0.0314	0.3923	0.8401
4h	0.0096	0.2650	0.8250
2h	0.0258	0.2069	1.2472
h	0.0662	0.1988	2.3461
h/2	0.1335	0.2672	2.9792
h/4	-0.0353	0.4155	-2.9302
h/8	-2.7035	0.4170	-20.6023

The deflections of beam at contact point corresponding to different cases of beam thickness are computed and presented in Table 2. Table 3 shows the error percent of other methods compared to the quasi-static solution. It’s obvious that MEM-T using reduced integration gives good agreement with MEM-E and quasi-static solution. In addition, MEM-T using reduced integration is also free of shear-locking when the rail beam is thinner whereas MEM-T using full integration is not. Figure 2, Figure 3 represents the rail displacement profiles computed by the mentioned methods. Clearly, no matter if the beam is thinner or thicker, the proposed method gives excellent agreements with both MEM-E method and quasi-static solution.

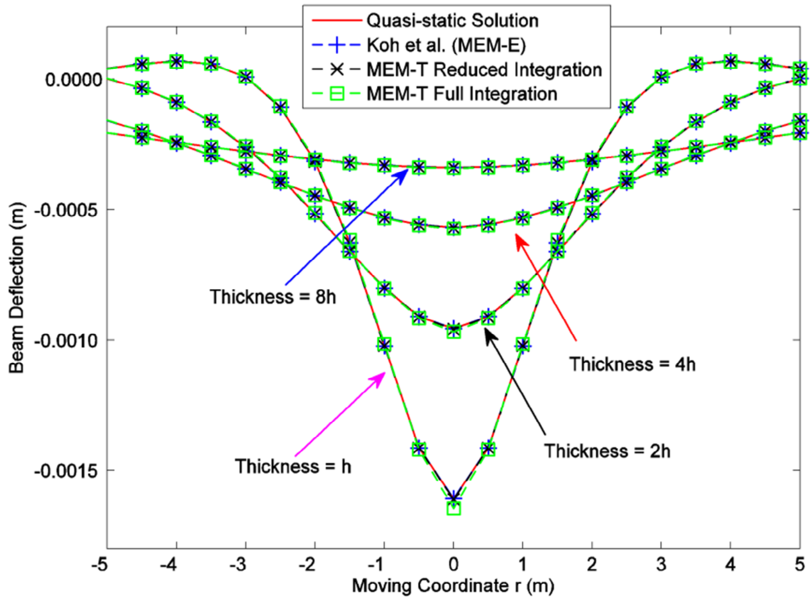

Figure 2. Rail displacement profile corresponding to some cases of beam thickness

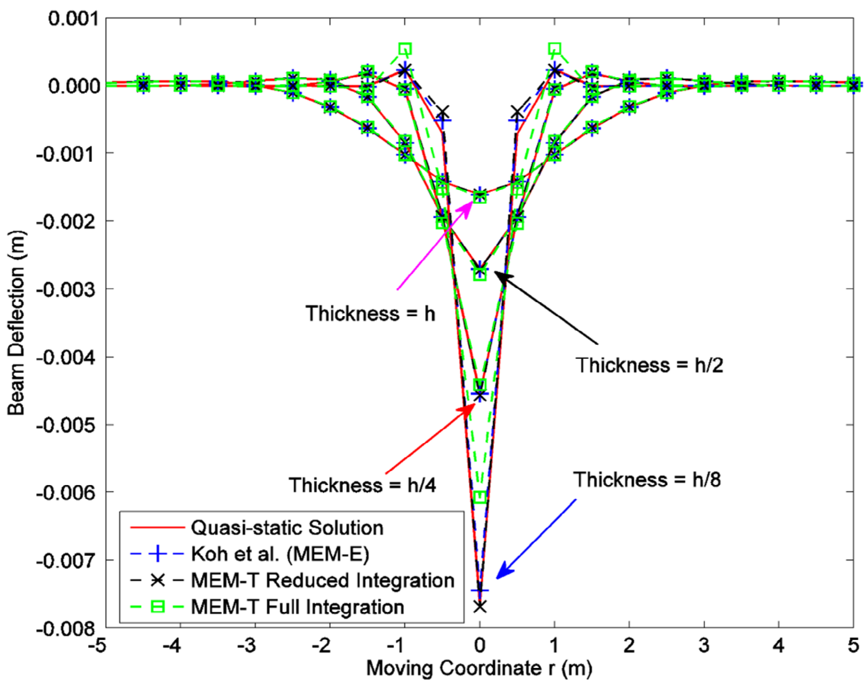

Figure 3. Rail displacement profile corresponding to some cases of beam thickness

3.2. 3-DOF vehicle at constant velocity

We now consider a moving vehicle load

instead of a pure force. In this case, the rail corrugation is counted as an excitation and is given by a periodic function,

$$y_c(x) = y_{c0}\sin(2\pi x / \lambda_c) = y_{c0}\sin(2\pi Vt / \lambda_c) \quad (24)$$

where the amplitude and wavelength are given by $y_{c0} = 0.5$ mm, $\lambda_c = 0.5$ m, respectively. The vehicle is modeled as a spring-mass-damper moving on the rail beam at a constant velocity, $V = 20$ m/s. The displacement of beam at wheel contact point and displacements of three masses which represent three components of the train is taken to study.

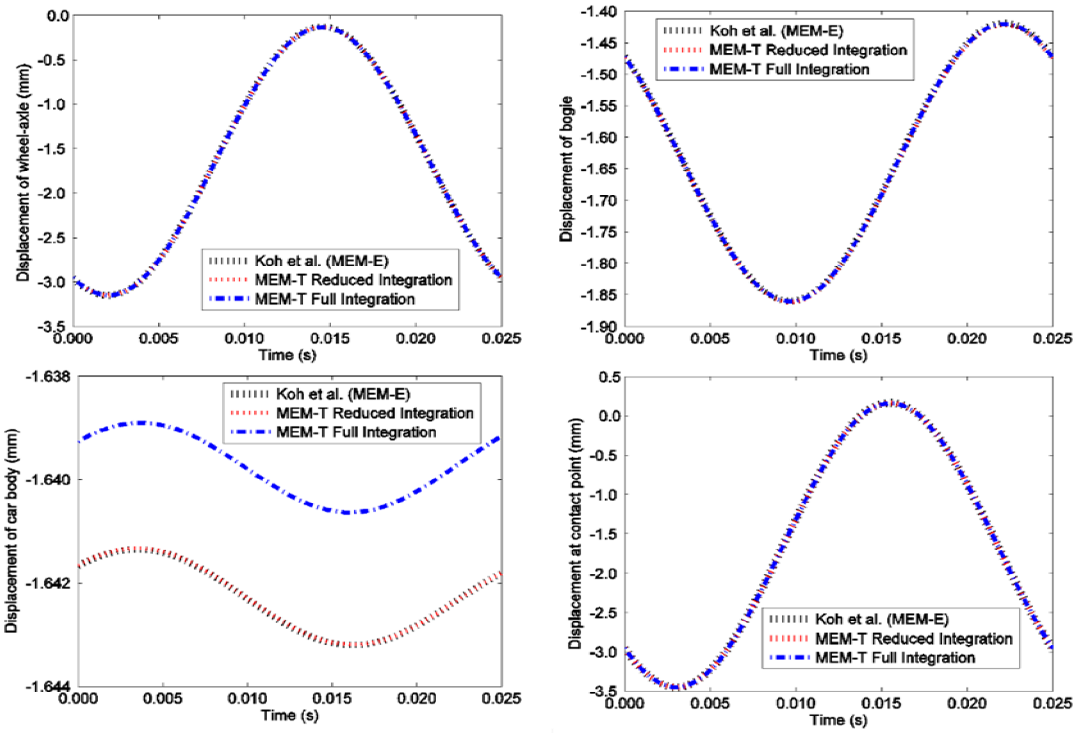

Figure 4. Displacements of train masses and displacement of beam at the contact point

The dynamic equations of MEM is solved by using Newmark's constant acceleration with a time step of 0.0001s and at-rest initial conditions. As in previous example, MEM-E and MEM-T with reduced or full integration used are implemented. Figure 4 shows the dynamic responses of the rail displacement at the contact point and the displacements of three masses in a typical corrugation cycle, T= λ_c / V = 0.025s. In these results, the static responses due to self-weight of the three masses are excluded. We can see an excellent agreement between MEM-T using reduced integration and MEM-E proposed by Koh et al.

4. Conclusion

In this paper, the moving element method (MEM) is extended to analyze the dynamic behaviors of train-track system which is modeled by a Timoshenko beam on viscoelastic foundation subjected to a moving spring-mass-damper system. The proposed method is hence called as the MEM-T. The coupled train-track governing equations are then established and solved by Newmark's constant acceleration. In the MEM-T, the reduced integration is used to avoid the shear locking. The numerical examples show that the results by the MEM-T agree well with those by Koh et al. [11] using Euler beam theory. The obtained results are very promising to extend to analyze the dynamic behavior of beam structures made by composite and FGM.

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