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G-JITTER INDUCED FREE CONVECTION OVER A VERTICAL FLAT PLATE

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ABSTRACT

The effect of the periodic oscillation of the gravitational field, known as g-jitter, on the free convection from a vertical plate is investigated in the present paper. The problem has been simplified by the laminar boundary layer and Boussinesq approximations. The fully implicit finite-difference scheme is used to solve the dimensionless system of the governing equations. The results for laminar flow of air (Pr = 0.72) and water (Pr = 7.00) are presented for different values of the amplitudes and frequencies of the g-jitter. The results presented show the steady periodic variation of Nusselt number and the friction coefficient with the amplitude and frequency of the gravitational acceleration oscillation. It is found that the Prandtl number as well as the amplitude and the frequency of the oscillating gravitational acceleration affect considerably the periodic oscillation of the Nusselt number and the skin friction on the vertical plate.

Nomenclature

$A(\Re)$	amplitude of $\Re$	
Cf	skin friction coefficient	
$f'(\eta)$	non-dimensional velocity	
g	gravitational acceleration	ms-2
Gr	Grashof number based on L	
k	thermal conductivity	Wm-1K-1
L	plate height	m
Nu	Nusselt number	
Pr	Prandtl number	
qw	wall heat flux	Wm-2
t	time	s
T	temperature	K
u, v	velocity components	ms-1
U,V	non-dimensional velocity components	
x, y	Cartesian coordinates	m
X,Y	non-dimensional Cartesian coordinates	

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Greek Symbols

α	thermal diffusivity	m2s-1
β	coefficient of volume expansion	K-1
ε	non-dimensional amplitude	
η	non-dimensional similarity variable	
θ	non-dimensional temperature	
ν	kinematic viscosity	m2s-1
ρ	density	kgm-3
τ	non-dimensional time	
τ_w	wall shear stress	Nm-2
ω	frequency	s-1
Ω	non-dimensional frequency	

1. INTRODUCTION

It is known that in many situations the presence of a temperature gradient and a gravitational field can generate buoyancy convective flows. Recent technological implications have given rise to increased interest in oscillating natural and mixed convection driven by fluctuating forces associated with microgravity. This fluctuating gravity is referred to as g-jitter. It is reported by Wadih and Roux [1] that vibrations can either substantially enhance or retard heat transfer and thus drastically affect the convection. In low-gravity or microgravity environments, it can be expected that reduction or elimination of natural convection may enhance the properties and performance of materials such as crystals. The time-dependent gravitational field is of interest in space laboratory experiments, in areas of crystal growth and other applications. It is also of importance in the large-scale convection of atmosphere. Aboard orbiting spacecrafts all objects experience low-amplitude perturbed accelerations, or g-jitter, caused by crew activities and space craft maneouvers, equipment vibrations, solar drag and other sources [2, 3]. Many theoretical and experimental studies dealing with material processing or physics of fluids under the micro-gravity conditions aboard an orbiting spacecraft have been carried out in recent years. There is a growing literature, which tries to characterize the g-jitter environment and the review articles by Alexander [4] and Nelson [5] give a good summary of earlier work on convective flows in viscous (Newtonian) fluids. There have also been a number of recent studies which investigate the effect of g-jitter on such viscous fluids and also on porous media, e.g. Amin [6], Farooq and Homsy [7], Li [8, 9], Malashetty and Padmavathi [10], Pan and Li [11], Rees and Pop [12, 13, 14], and Chamkha [15].

The aim of the present study is to investigate numerically the effect of the presence of the g-jitter, (with different amplitudes and frequencies) on the periodic free convection of a viscous fluid (air and water) over a vertical plate resulting from a step change in its surface temperature.

2. BASIC EQUATIONS

The continuity and boundary layer momentum and energy equations in two-dimensional unsteady conditions can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g(t)\beta\left(T-T_\infty\right) \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where the vertical wall is considered to be along x-axis and y-axis is normal to it. The above governing equations are subjected to the following initial and boundary conditions:

$t<0$: $u=v=0$, $T=0$ for all x and y (4a)

$t\geq 0$: $u=v=0$, $T=T_w$ on y = 0; $u\to 0$, $T\to T_\infty$ as $y\to\infty$ (4b)

Following Rees and Pop [14], we consider a simple model problem in which the gravitational field is assumed to oscillate periodically over an average value g_o in the following form:

$$g(t)=g_o\{1+\varepsilon\cos(\omega t)\} \tag{5}$$

where ε is the amplitude and ω is the frequency of the g-jitter. In order to simplify the problem and to generalize the results, the above equations are written in a non-dimensional form by employing the following dimensionless variables:

$$X=x/L;\quad Y=\frac{y}{L}Gr^{1/4};\quad \tau=t/t_c;\quad \Omega=\omega t_c$$

$$U=u/u_c;\quad V=\frac{v}{u_c}Gr^{1/4};\quad \theta=\frac{T-T_\infty}{\Delta T} \tag{6a}$$

where u_c, t_c are the characteristic velocity and time scales respectively, L is the plate height and $Gr=g_o\beta L^3\Delta T/\nu^2$ is the Grashof number based on the characteristic length L, the average value of gravity acceleration and the temperature difference $\Delta T=T_w-T_\infty$. The characteristic velocity and time scales used in the present analysis are same as those given by Saeid [16, 17], which are:

$$u_c=\sqrt{g_o\beta L\Delta T};\quad t_c=\sqrt{L/(g_o\beta\Delta T)} \tag{6b}$$

Substituting (6) into equations (1) - (3) we obtain:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{7}$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\nu\frac{\partial^2 U}{\partial Y^2}+\{1+\varepsilon\cos(\Omega\tau)\}\theta \tag{8}$$

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{9}$$

where $\Pr=\nu/\alpha$ is the Prandtl number. The dimensionless initial and boundary conditions (4) become:

$\tau<0$: $U=V=\theta=0$, for all X and Y (10a)

$\tau\geq 0$: $U=V=0$, $\theta=1$ on Y = 0; $U\to 0$, $\theta\to 0$ as $Y\to\infty$ (10b)

The skin friction coefficient and the Nusselt number are defined respectively as:

$$C_f=\frac{\tau_w}{\rho u_c^2};\quad Nu=\frac{q_w x}{k\Delta T} \tag{11}$$

Where ρ and k are the density and the thermal conductivity of the fluid and τ_w, q_w are the wall shear stress and the wall heat flux defined respectively as:

$$\tau_w = \rho\nu\left(\frac{\partial u}{\partial y}\right)_{y=0}; \qquad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{12}$$

Substituting (12) in (11) and using the non-dimensional variables (6) leads to the following expressions for the skin friction coefficient and the Nusselt number:

$$\frac{C_f}{Gr^{1/2}} = \left(\frac{\partial U}{\partial Y}\right)_{Y=0}; \qquad \frac{Nu}{Gr^{1/4}} = -X\left(\frac{\partial \theta}{\partial Y}\right)_{Y=0} \tag{13}$$

No analytical solution is known for equations (7)-(9) with the initial and boundary conditions (10). Therefore, the present periodic free convection problem is solved numerically using the Cartesian coordinates, which is used by various authors [16 - 19] to solve the free convection problems.

3. NUMERICAL SCHEME

The momentum and energy equations (8) and (9) are integrated over a control volume using the fully implicit scheme which is unconditionally stable. The power-law scheme is used for the convection-diffusion formulation [20]. Finally, the finite-difference equation corresponding to the continuity equation (11) is developed using the expansion point , $\left(i+1, j-\frac{1}{2}\right)$ where i and j are the indices along X and Y respectively [21]. The resulting finite-difference equation is:

$$V_{i+1,j} = V_{i+1,j-1} - \frac{\Delta Y_w}{2(\Delta X_n)}\left(U_{i+1,j} + U_{i+1,j-1} - U_{i,j} - U_{i,j-1}\right) \tag{14}$$

where ΔY_w and ΔX_n are the grid spaces west and north of the point (i, j) respectively. The solution domain, therefore, consists of grid points at which the discretization equations are applied. In this domain X by definition varies from 0 to 1. But the choice of the value of Y, corresponding to $Y = \infty$, has an important influence on the solution. The effect of different values to represent $Y = \infty$ on the numerical scheme has been investigated and it is concluded that the value of $Y = 10$ is sufficiently large. Further larger values of Y produced the results with indistinguishable difference. The stretched grid has been selected in both X and Y direction such that the grid points clustered near the wall and near the leading edge of the flat plate as there are steep variation of the velocities and temperatures in these regions. The algorithm needs iteration for the coupled equations (7) - (9). First iteration starts to solve the discretized energy equation from zero initial velocities and temperatures in all the grid points using line-by-line tridiagonal-matrix algorithm. This means that equation (9) is reduced to the transient heat conduction equation. The resultant temperature field is then used to solve the discretized momentum equation to find U profiles from the zero V values. Finally the V profiles are found from the solution of the discretized continuity equation (14) explicitly. The iteration then continues to solve for θ, U , and V using the pervious iteration values until iterative converged solution is obtained. The convergence condition used for the three dependent variables θ, U , and V is:

$$Max\left|\frac{\varphi^n - \varphi^{n-1}}{\varphi^n}\right| < 10^{-5} \tag{15}$$

where φ is the general dependent variable and the superscript n represents the iteration step number. The time increment is $\Delta\tau = 2\pi/(1000\Omega)$ for high and moderate values of the g-jitter frequency Ω and even smaller for small vales of the g-jitter frequency Ω in order to ensure the converged solution.

4. RESULTS AND DISCUSSION

The algorithm explained in the pervious section is first tested before studying the effect of g-jitter on the free convection from vertical plate. The test is selected to study the laminar transient free convection with a step change in the surface temperature with Pr = 0.72, i.e. ε = 0 in equation (8). At the initial stage of the transient, the transport mechanism is predominantly conduction. On the other hand at larger time, the flow will be at steady-state condition. Therefore the transient results are expected to fall within these two limits. Several time steps are chosen for calculations to demonstrate this fact, which are τ = 0.2, 0.4, 0.8, 1.5, 2.5, 5 and 10. Figure 1 shows the velocity and temperature profiles, at the upper end of the wall, plotted at different time steps until the steady-state is reached ($\tau \geq 5$). The steady-state velocity and temperature profiles obtained from similarity solution [22] are also presented in Figure 1 for comparison. It is important to note that in the similarity solution [22], the non-dimensional velocity is defined as $f'(\eta) = u/\left(2\sqrt{x\beta g\Delta T}\right)$ and $\eta = (y/x)(Gr/4)^{1/4}$, while the definition of the non-dimensional temperature is same as in (6). It can be shown that $U = 2f'(\eta)$ and $Y = \eta\sqrt{2}$ at the upper end of the plate where X = 1. An excellent agreement of the present results with the similarity results is shown in Fig. 1 for both velocity and temperature profiles.

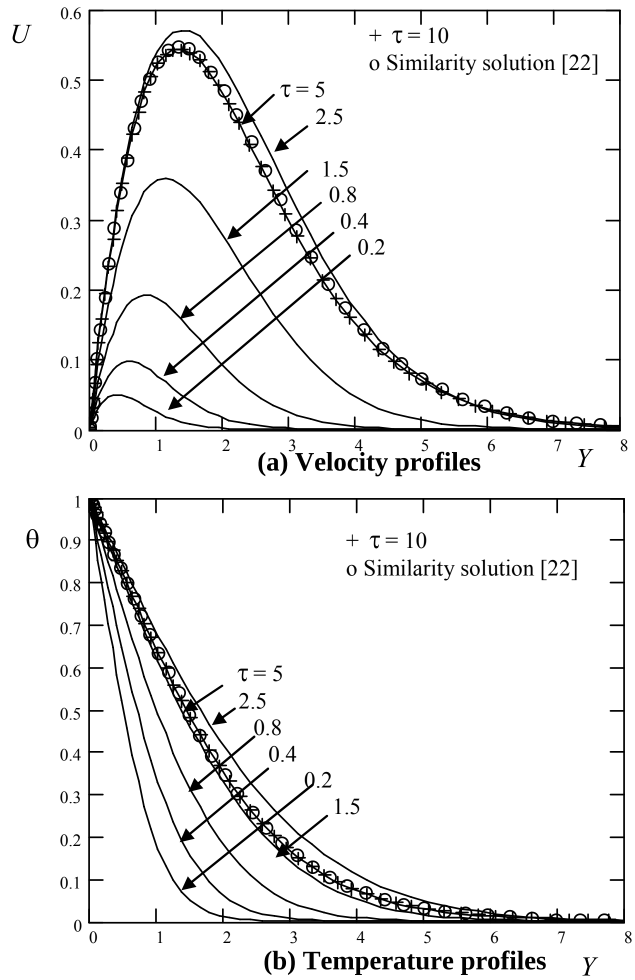


Fig. 1: *Transient velocity and temperature profiles for air (Pr = 0.72)*

It is observed that the dimensionless velocity and temperature, at the upper end of the plate, increase with time to reach maximum value and then decrease to reach the steady-state values. Thus the heat transfer coefficient and hence, the local Nusselt number will decrease with time and it reaches a minimum value and then increase slightly to approach the steady-state value as shown in Fig. 2 for Pr = 0.72 (air) and Pr = 7.00 (water). Figure 2 shows an excellent agreement of the present results of the transient variation of the Nusselt number with that obtained by Hellums and Churchill [18].

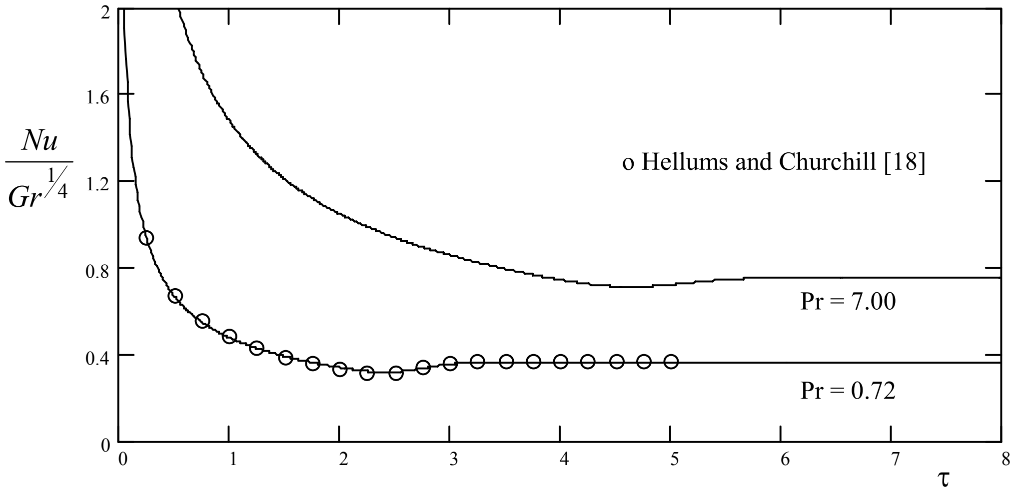


Fig. 2: *Variation of* $Nu/Gr^{1/4}$ *with* τ *for air (Pr = 0.72) and water (Pr =7.00)*

These results provided confidence to the accuracy of the present numerical model to study the effect of the g-jitter on the free convection from a vertical flat plate. The oscillation of g(t) is considered now with amplitude range from $\varepsilon = 0$ to $\varepsilon = 1.0$ and frequency range from $\Omega = 0$ to $\Omega = 5$ where converged solution is obtained. The free convection process starts when the surface temperature increases suddenly from the ambient temperature T_∞ to the surface temperature T_w. At this time the ratio $Nu/Gr^{1/4}$ goes to infinity and $C_f/Gr^{1/2}$ starts from zero as shown in Fig. 3. Then, when g(t) oscillates the ratios $Nu/Gr^{1/4}$ and $C_f/Gr^{1/2}$ are found to oscillate accordingly. This oscillation becomes steady periodic oscillation after some periods. Figure 3 shows the oscillation of the ratios $Nu/Gr^{1/4}$ and $C_f/Gr^{1/2}$ at the upper end of the plate (X = 1) with the non-dimensional time with g-jitter amplitude $\varepsilon = 0.5$ and frequency $\Omega = 5$ for air (Pr = 0.72). The last two oscillating periods in figure 3 are almost similar, which means that the oscillation of $Nu/Gr^{1/4}$ and $C_f/Gr^{1/2}$ become periodic oscillation.

The steady periodic oscillation is achieved when the amplitude and the average values of the ratios $Nu/Gr^{1/4}$ and $C_f/Gr^{1/2}$ becomes constant for different periods. The following condition is considered for the steady periodic oscillation:

$$\frac{A(\Re)^p - A(\Re)^{p-1}}{A(\Re)^p} \le 10^{-3} \quad \text{and} \quad \frac{\overline{\Re}^p - \overline{\Re}^{p-1}}{\overline{\Re}^p} \le 10^{-3} \tag{16}$$

where the superscript p is the period number, and $\Re$ can stand for either $Nu/Gr^{1/4}$ or $C_f/Gr^{1/2}$ and

$$A(\Re) = \tfrac{1}{2}\left[Max(\Re) - Min(\Re)\right] \tag{17}$$

for $\tau_o \le \tau \le \tau_o + (2\pi/\Omega)$ and

$$\overline{\Re} = \frac{1}{(2\pi/\Omega)} \int_{\tau_o}^{\tau_o+(2\pi/\Omega)} \Re d\tau \tag{18}$$

The oscillation of $Nu/Gr^{1/4}$ and $C_f/Gr^{1/2}$ for the steady periodic oscillation in the last period are shown in Figs. 4 and 5 for Pr = 0.72 and Pr = 7.00 respectively. The forcing frequency is chosen Ω = 1 with ε = 0.1 to 1.0. It is observed that the oscillation of $C_f/Gr^{1/2}$ takes the shape of the forcing g-jitter function, which is cosine function with the peak acceleration values at τ = 0 and τ = 2π of each forcing period. While in the oscillation of $Nu/Gr^{1/4}$ there is a small phase shift for Pr = 0.72 and approximately out of phase for Pr = 7.00 at the specified forcing frequency Ω = 1. This is due to delay in the response of the thermal boundary layer to the g-jitter effect. For smaller values of the forcing frequency (Ω) or long period of g-jitter variation, this phase change will be less. Increasing the forcing amplitude (ε) leads to oscillation of $Nu/Gr^{1/4}$ and $C_f/Gr^{1/2}$ with high amplitude (defined in equation (17)). The temporal averaged values (defined in equation (18)) of the oscillating $Nu/Gr^{1/4}$ and $C_f/Gr^{1/2}$ are found with small variations and they are approximately constant (= steady state values) for all the values of the forcing amplitude (ε).

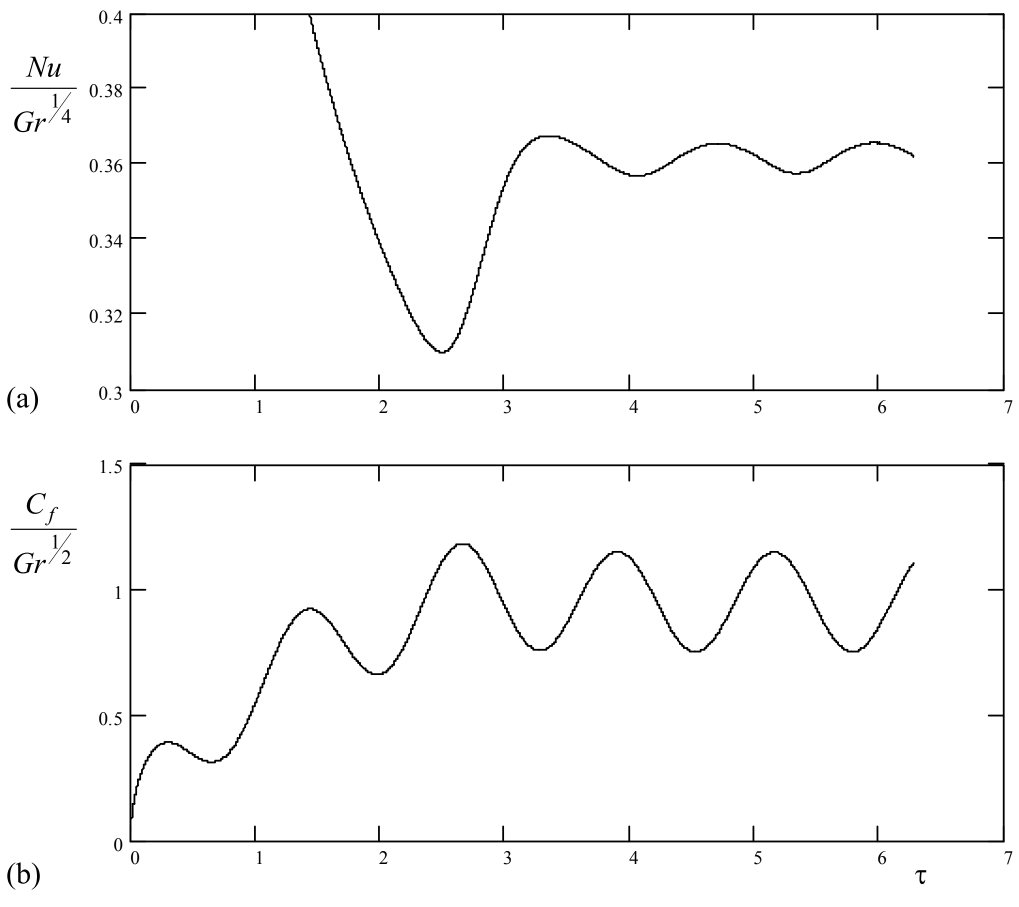
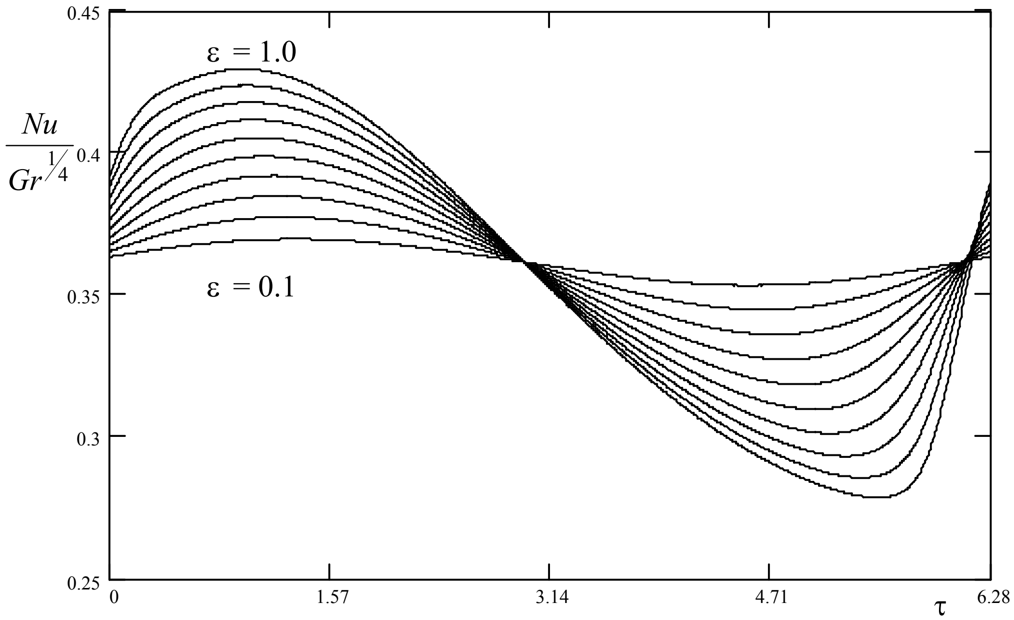
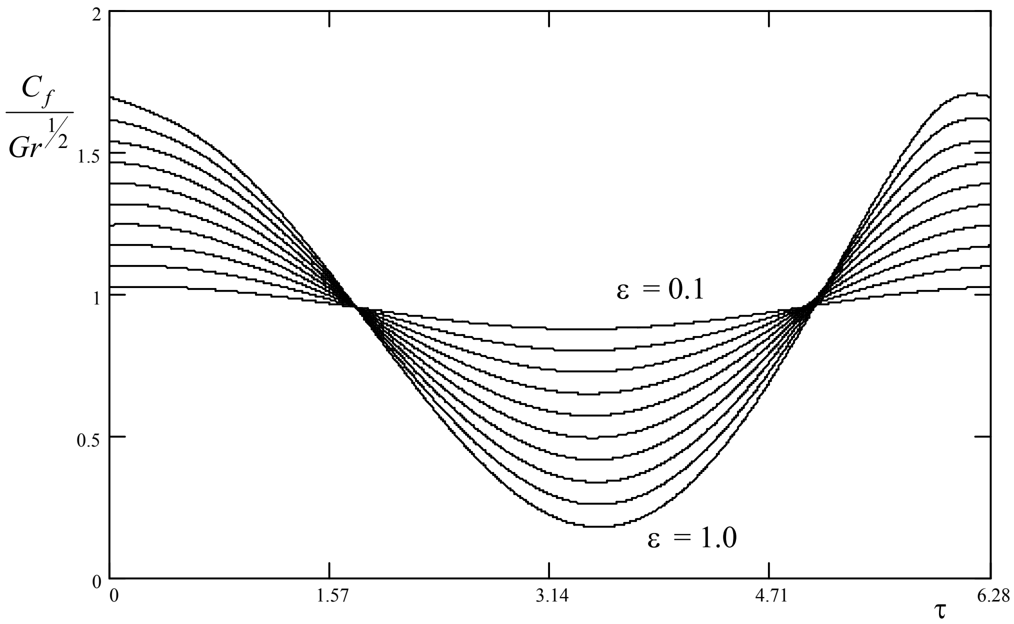


Fig. 3: *Oscillation of (a)* $Nu/Gr^{1/4}$ *(b)* $C_f/Gr^{1/2}$ *with the non-dimensional time for ε = 0.5, Ω= 5 and Pr = 0.72*



(a)



(b)

Fig. 4: *Periodic oscillation of (a) $Nu/Gr^{1/4}$ (b) $C_f/Gr^{1/2}$ with the non-dimensional time for Ω = 1 and Pr = 0.72*

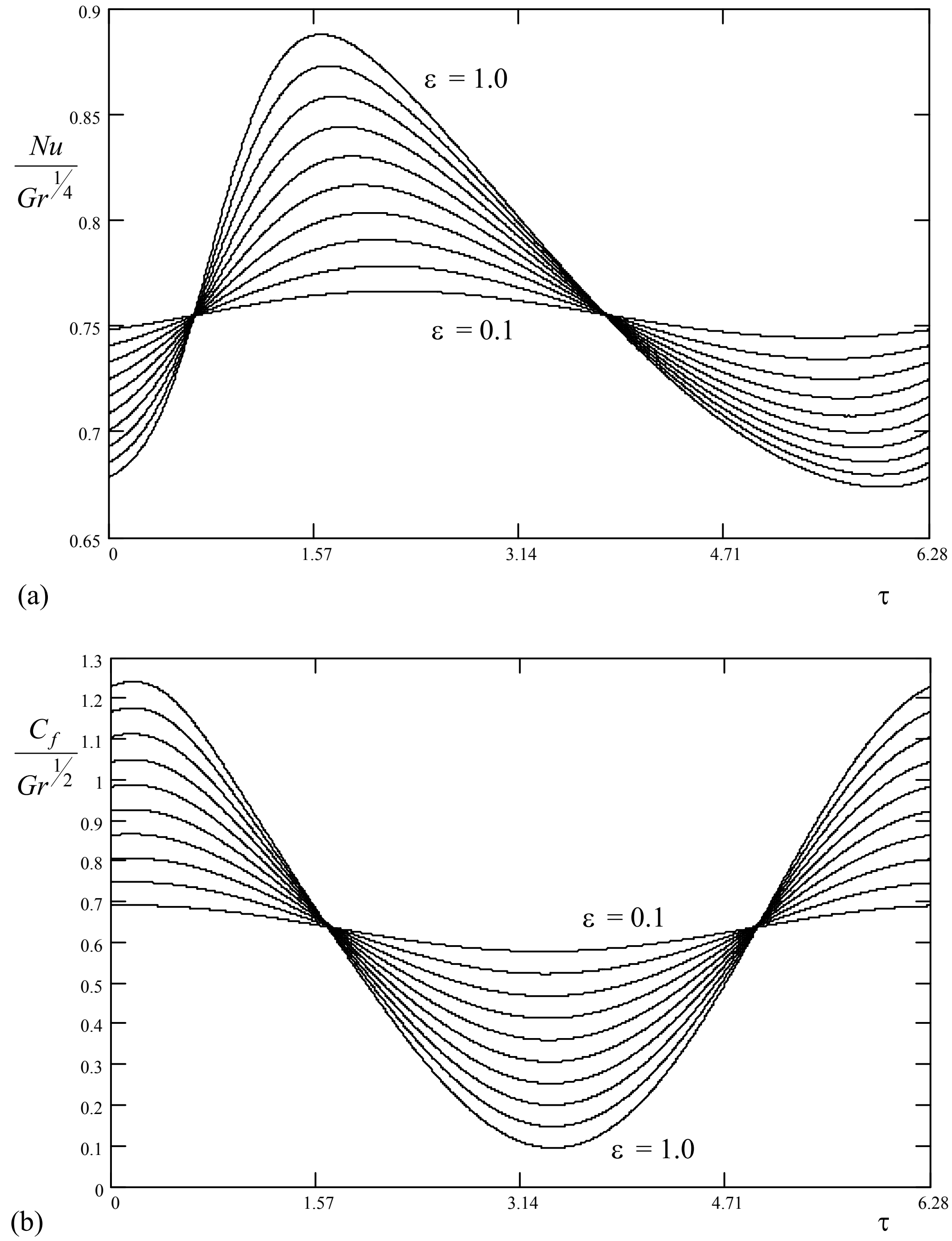


Fig. 5: *Periodic oscillation of (a)* $Nu/Gr^{1/4}$ *(b)* $C_f/Gr^{1/2}$ *with the non-dimensional time for Ω = 1 and Pr = 7.00*

The effect of the g-jitter frequency is studied and the results are shown in Fig. 6. Figure 6 shows clearly how the forcing frequency influences the periodic variation of the Nusselt number for Pr = 0.72 with forcing amplitude ε = 0.5. At low values of Ω there will be enough time for the momentum and heat transfer to follow the effect of the periodic variation of the acceleration. Therefore the periodic Nusselt number is found to follow the g-jitter forcing function (cosine function) for small values of Ω. The amplitude of the Nusselt number oscillation is higher for smaller forcing frequency. Figure 6 shows also, as Ω increases, the peak value of Nusselt number is delayed progressively and the amplitude of the Nusselt number oscillation also decreases. For very high forcing frequency $\Omega > 5$ the Nusselt number oscillation will be approximately constant. For different values of the forcing frequency, the temporal averaged values of the oscillating $Nu/Gr^{1/4}$ and $C_f/Gr^{1/2}$ are found with small variations and they are also approximately constant for both Pr = 0.72 and Pr = 7.00.

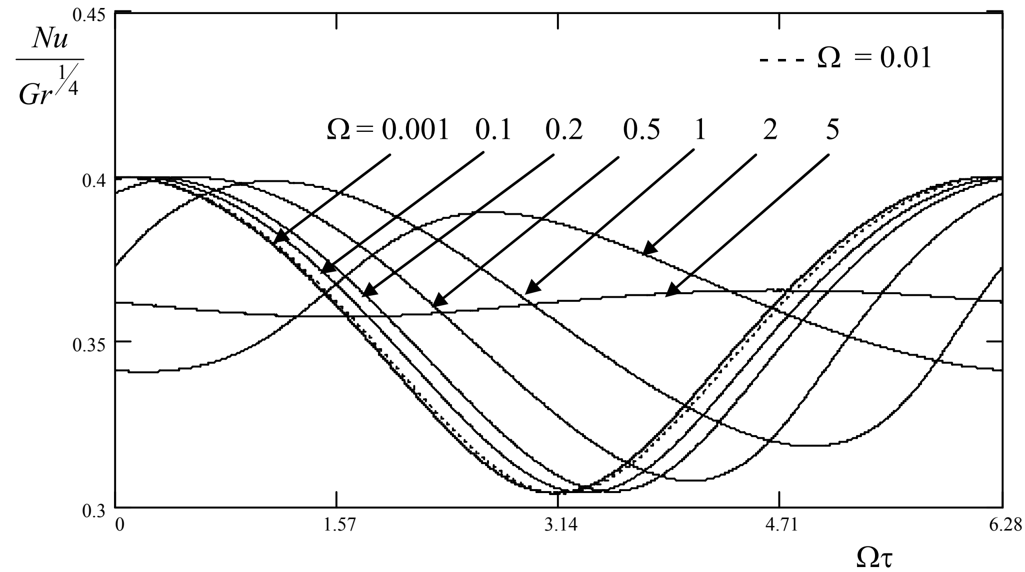


Fig. 6: *Periodic oscillation of* $Nu/Gr^{1/4}$ *for* ε *= 0.5 and Pr = 0.72*

5. CONCLUSIONS

Numerical investigation has been carried out in the present paper to study the effect of the sinusoidal gravity modulation on the free convection from a vertical plate. The step change in plate temperature has been assumed and the two dimensional laminar boundary layer approximation is used in the formulations. The fully implicit finite-difference scheme is used to solve the dimensionless system of the governing equations. After validating the result of the computational code, the results for laminar flow of air (Pr = 0.72) and water (Pr = 7.00) are presented for different values of the amplitudes and frequencies of the g-jitter. It is observed that the oscillation of the skin friction takes the shape of the forcing g-jitter function. While in the oscillation of Nusselt number there is a phase shift due to delay in the response of the thermal boundary layer to the g-jitter effect at moderate and high forcing frequencies. At low values of the forcing frequency there will be enough time for the momentum and heat transfer to follow the effect of the periodic variation of the acceleration. Therefore the periodic Nusselt number is found to follow the g-jitter forcing function.

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