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EFFECT OF THE RECIPROCATING MASS OF SLIDER-CRANK MECHANISM ON TORSIONAL VIBRATIONS OF DIESEL ENGINE SYSTEMS

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ABSTRACT

The torsional vibration phenomenon in the running gear of reciprocating engine systems is usually dealt with by considering a series of constant inertias connected by sections of massless shafting. However in reality, a slider crank mechanism is a vibrating system with varying inertia because the effective inertia of the total oscillating mass of each crank assembly varies twice per revolution of the crankshaft. Large variations in inertia torques can give rise to the phenomenon of secondary resonance in torsional vibration of modern marine diesel engines which can not be explained by conventional theory incorporating only the mean values of the varying inertias. In the past associated secondary resonances and regions of instability tended to be dismissed by most engineers as interesting but of no importance. The situation changed in recent years since there is evidence of the existence of the secondary resonance effects which could have contributed to a number of otherwise inexplicable crankshaft failures in large slow speed marine engines. The cyclic variation of the polar moment of inertia of the reciprocating parts during each revolution causes a periodic variation of frequency and corresponding amplitude of vibration of reciprocating engine systems. It also causes an increase in the speed range over which resonance effects are experienced and only a partial explanation of the behaviour of the systems has been worked out. It is impossible to avoid these instabilities by changes in the design, unless of course the variations in mass and spring constant can be made zero. In the present paper a critical appraisal of the regions of instability as determined from the equation of motion which takes into account variation of inertia is given. The motion in the form of complex waveforms is studied at different speeds of engine rotation. A comparison of theoretical results with Goldsbrough's experimental results and Gregory's analysis is included.

1. INTRODUCTION

It is well known that, in the case of reciprocating engines there are certain critical speeds of running at which the torsional vibrations in the shaft become large in amplitude and introduce an element of danger into the system. In the simple methods used for practical calculation of torsional vibrations, the reciprocating parts of the engine are replaced by an "equivalent mass" which is assumed to contribute to the elastic vibrations of the shaft in exactly the same way as the actual rather complicated slider-crank mechanism. This type of approximate analysis usually

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leads to the design of crankshafts whose behaviour in practice is quite satisfactory. However, a number of engines are now in service whose pistons are so massive that their reciprocation gives rise to inertia variations substantial enough to excite troublesome torsional vibrations. Indeed, a number of unexpected torsional failures of crankshafts in large two-stroke marine diesel engines have occurred in practice and two of the cases are well documented by Archer [1]. The conventional vibration calculations in these engines indicated an *n*th order critical of small equilibrium amplitude occurring at or near resonance with the service speed being excited by large resultant engine excitations of order (*n*-2). This operating condition was considered acceptable, one order being close to the service speed but suitably small and the second being far enough above the service speed not to merit further investigation. Inspection of torsiograph records of the failed crankshafts, however revealed that the actual vibration stresses at service speed were three or four times larger than the calculated values.

Goldsbrough [2] in his investigations has shown that, in the case of reciprocating engines, torsional vibrations of large amplitude occur within a series of ranges of instability, hence increasing the possibility of crankshaft failure in fatigue. Gregory [3] carried out further investigations by constructing solutions of the non-linear equation for an idealized single cylinder engine system. Draminsky [4, 5] attributed the presence of higher than expected stresses to the phenomenon of secondary resonance; that is to say, the possibility of a *n*th order critical of small equilibrium amplitude at or near resonance with the service speed being excited by large resultant engine excitations of order (*n*-2) and (*n*+2). Draminsky developed a theory based on a non-linear analysis and used an equivalent single cylinder engine model in which the crankshaft and all the pistons and connecting rods were represented by an equivalent single crank assembly. It was further suggested that in practice, only large (*n*-2) order excitation reinforced the vibration amplitude at order *n* causing excessive stress.

Failures have not occurred in all cases of engines in service which were considered to be susceptible to secondary resonance. In view of this Carnegie and Pasricha [6] examined the torsional vibration behaviour of a ten-cylinder two-stroke-cycle engine with suspected secondary resonance. It was found that stress magnification due to secondary resonance failed to appear. Following this, Pasricha and Carnegie [7, 8] carried out further investigations to explain the secondary resonance phenomenon. It was realized that Draminsk’s work served only to indicate, in very broad terms, the circumstances in which adverse effects could be anticipated.

It was against this background that the authors studied work done by researchers on the parametrically excited systems by Zevine [9], Zadoks and Midha [10, 11] and Hesterman and Stone [12].These methods tend to be specialized and the amount of information thus gained is inadequate for design situation. Further these methods do not provide waveform solutions at different speeds of the crankshaft. It is important to obtain these solutions as the harmonic analysis of the waveform responses can predict the order number of external excitations that have significant effect on the vibratory motion. Also the stability of such systems is the subject of great deal of current interest to the design engineers.

The results of the present paper predict characteristics of motion, regions of instability and shapes of the complex waveforms at different speeds of the engine rotation. Keeping in view the importance of the variable ε that is the inertia ratio of the engine, two specific values of ε have been examined for their effect on the vibratory behaviour of the system. One value of ε covers nearly the largest of present-day slow-speed marine oil engines and the other on the lighter side for comparison of behaviour of the system. The purpose of this paper is to examine whether the above solutions are confirmed by the corresponding experimental findings of Goldsbrough [2] for regions of instability in the engines and then show the usefulness of theoretical waveform solutions which can predict the possibility of magnification of certain orders of motion leading to crankshaft failures due to secondary resonance.

2. THEORETICAL CONSIDERATIONS

Figure 1 shows diagrammatically a single-cylinder reciprocating engine driving a heavy flywheel A of moment of inertia I_A (a list of symbols is given in the notation). Assume that the reciprocating mass moves with simple harmonic motion and the gas pressure in the cylinder is omitted.

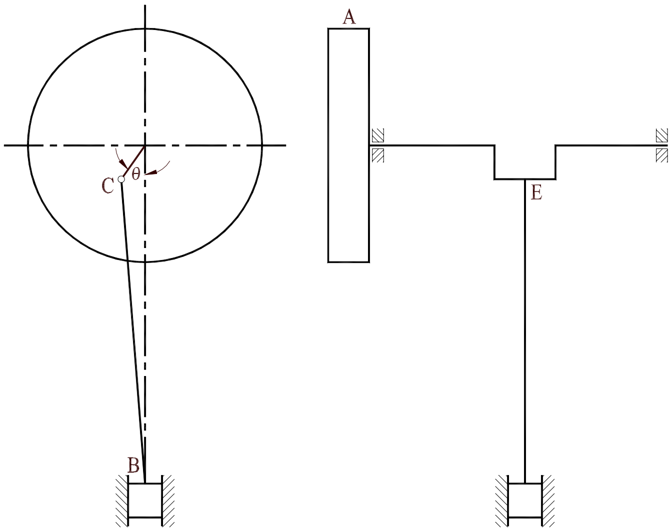

Fig. 1: *Diagrammatic arrangement of engine running gear*

The kinetic energy, T, of the system is given by

$$T = \tfrac{1}{2}\dot{\theta}^2\left(I + \tfrac{1}{2}Ma^2 - \tfrac{1}{2}Ma^2\cos 2\theta\right) + \tfrac{1}{2}I_A\omega^2 \tag{1}$$

The potential energy V of the system is

$$V = \tfrac{1}{2}\mu(\theta - \theta_1)^2 \tag{2}$$

The equation of Lagrange for the co-ordinate θ is

$$d(\partial T/\partial\dot{\theta})/dt - \partial T/\partial\theta + \partial V/\partial\theta = 0 \tag{3}$$

Substituting the relations (1) and (2) in (3) gives

$$\ddot{\theta}\left(I + \tfrac{1}{2}Ma^2 - \tfrac{1}{2}Ma^2\cos 2\theta\right) + \tfrac{1}{2}Ma^2\dot{\theta}^2\sin 2\theta + \mu(\theta - \theta_1) = 0 \tag{4}$$

Making use of the equations

$$\theta_1 = \omega t,\quad \theta = \omega t + \gamma,\quad \tau = \omega t,\quad \varepsilon = \tfrac{1}{2}Ma^2/\left(I + \tfrac{1}{2}Ma^2\right),$$
$$1/r^2 = \mu/\omega^2\left(I + \tfrac{1}{2}Ma^2\right), \tag{5}$$

and neglecting the second and higher order terms, letting dashes represent differentiation with respect to τ , equation (4) becomes

$$(1 - \varepsilon\cos 2\tau)\gamma'' + (2\varepsilon\sin 2\tau)\gamma' + \left(1/r^2 + 2\varepsilon\cos 2\tau\right)\gamma = -\varepsilon\sin 2\tau \tag{6}$$

Pasricha [13] has analysed equation (6) with the terms for external excitations added on to it to investigate the effect of phase angle and inertia ratio. The investigations of the present paper arise out of the importance of the parameter ε for the effect on the torsional vibration characteristics of the system. The quantity ε represents the ratio of the equivalent moment of inertia of the reciprocating mass $(\frac{1}{2}Ma^2)$ to the total equivalent moment of inertia $(I + \frac{1}{2}Ma^2)$. Clearly ε is always less than unity. Two values of ε selected for investigations are $\varepsilon = 0.34$ and $\varepsilon = 0.236$ since at these values corresponding experimental results are obtainable [2].

Figures 2 and 3 show the responses of the system when ratios of angular velocity of the shaft to the natural frequency of the system are r = 1/12 and 1/10 for $\varepsilon = 0.34$. Fig. 4 is the waveform relationship of $\gamma \sim \tau$ for the speed of the engine for r = 1/12 and $\varepsilon = 0.236$. These solutions are determined from equation (6) omitting gas forces with initial conditions $\gamma = 1$ and $\gamma' = 0$ at $\tau = 0$ by use of Runge-Kutta-Merson method as described by Dimarogonas [14]. Fig. 5 shows the solution of the equation for the speed of the engine at r = 0.5 when $\varepsilon = 0.34$.

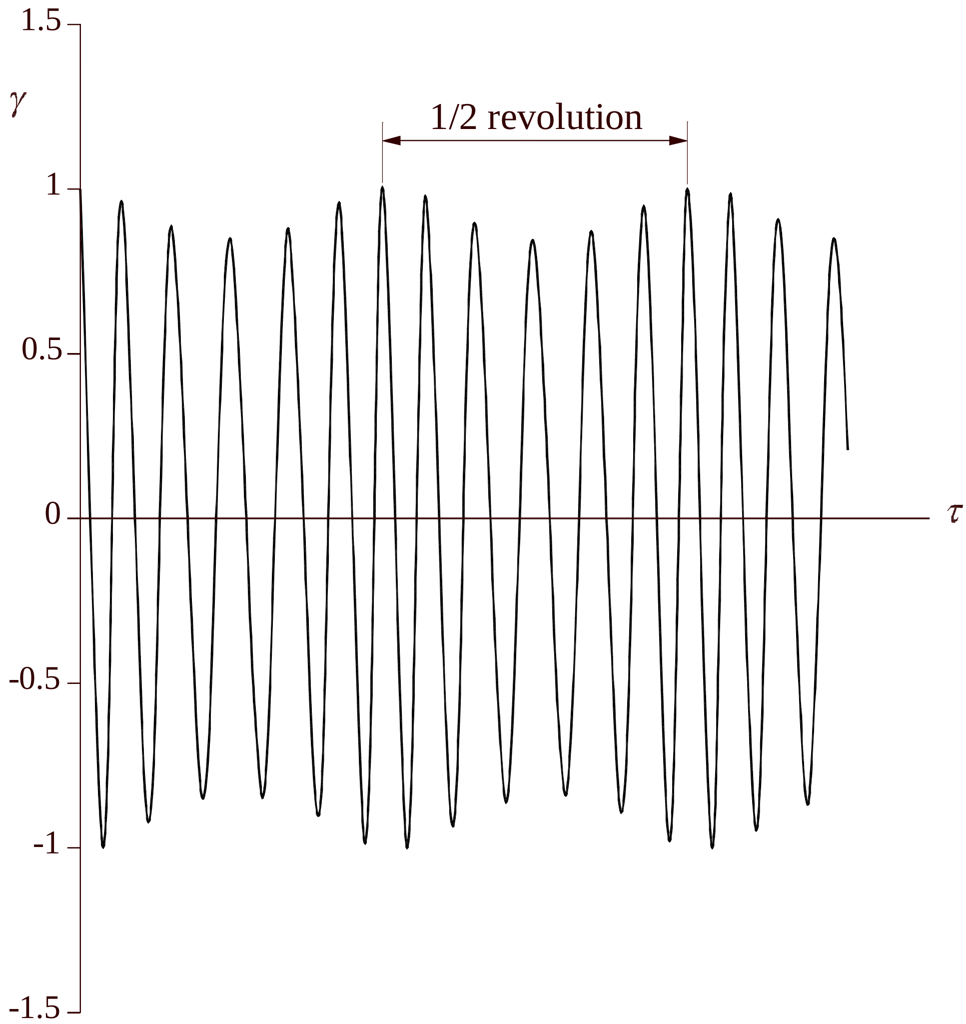

Fig. 2: *Theoretical waveform relationship of $\gamma \sim \tau$ for r = 1/12 and ε = 0.34*

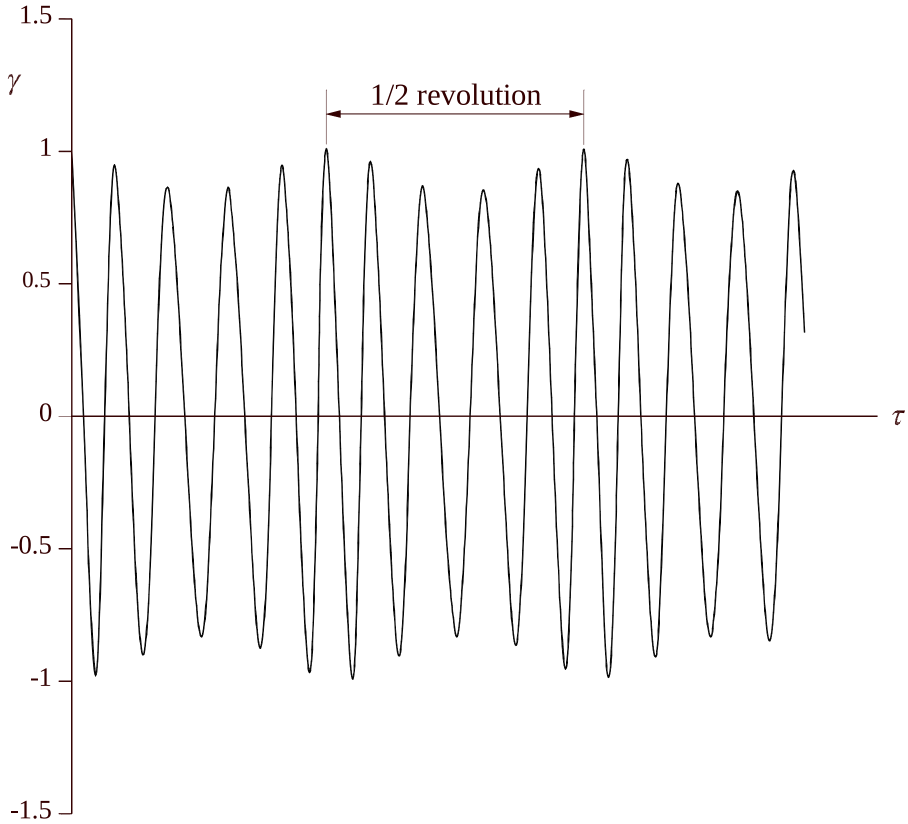

Fig. 3: *Theoretical waveform relationship of $\gamma \sim \tau$ for $r = 1/10$ and $\varepsilon = 0.34$*

3. DISCUSSION OF RESULTS

Time responses determined from equation (6) for $r = 1/12$ and 1/10 for $\varepsilon = 0.34$, as in Figs. 2 and 3 respectively, show the presence of beats and there are two beats in one revolution of the shaft. Similarly the solution of Fig. 4 at engine speed of $r = 1/12$ and $\varepsilon = 0.236$ shows a solution conforming to beat form. Several solutions for specific values of $\varepsilon = 0.34$ and $\varepsilon = 0.236$ are investigated over a range of r varying from 0.02 to 10. At lower values of $r < 0.2$ the solutions show the beat forms as described above. Beats at some values of r > 0.2 show no correlation between the frequency of one cycle of the response envelope and the speed of rotation as exhibited by time response of Fig. 5.

Figures 6 and 7 show the variation of maximum amplitude (γ_{max}) of displacement of torsional motion against the parameter r for $\varepsilon = 0.34$ and $\varepsilon = 0.236$ respectively. These Figures were constructed by observing maximum amplitudes of a number of steady state solutions obtained by direct numerical integration for various values of r. For $r \cong 1$ and $r \cong ½$ the pairs of broken vertical lines bound the regions of instability containing the two critical speed ranges in which the amplitude of vibration grows indefinitely large. Figure 5 shows the time response of the system at $r = 0.5$ and $\varepsilon = 0.34$ depicting the growth of maximum amplitude of motion near the critical speed range $r \cong ½$.

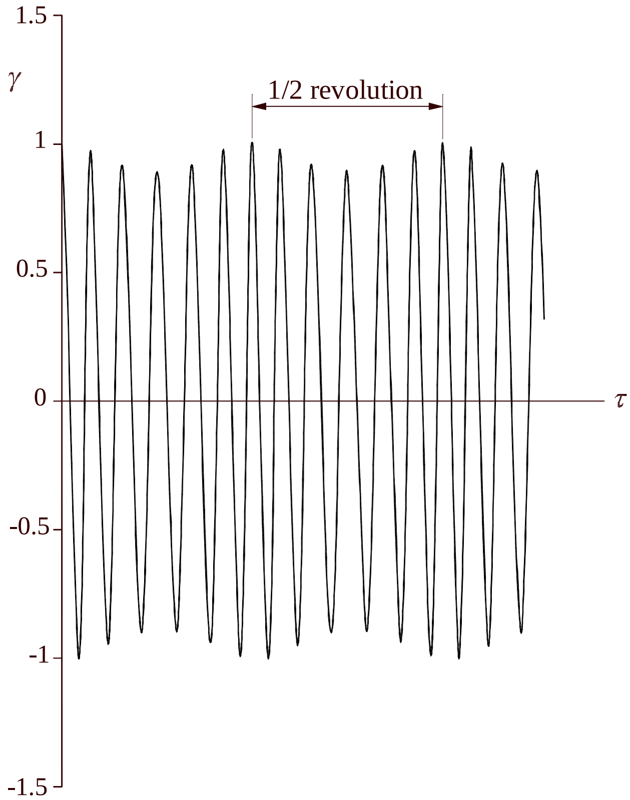

Fig. 4: *Theoretical waveform relationship of γ ~ τ for r = 1/12 and ε = 0.236*

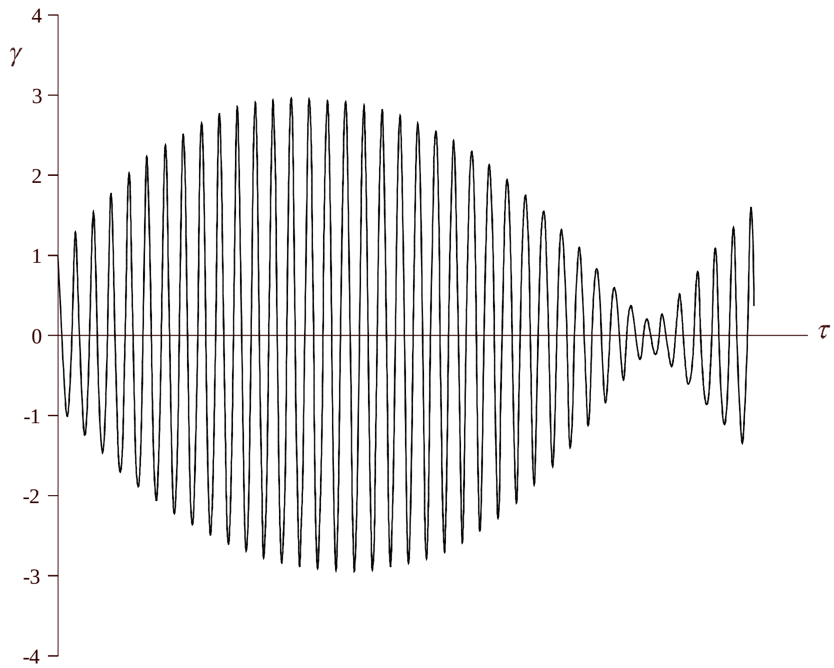

Fig. 5: *Theoretical waveform relationship of γ ~ τ for r = 0.5 and ε = 0.34*

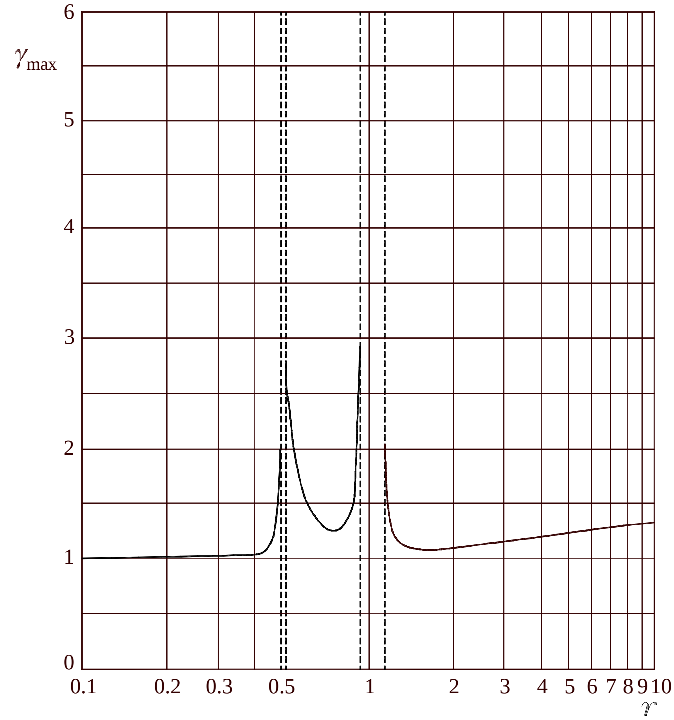

Fig. 6: *Maximum amplitude γ_{max} of the response versus r for ε = 0.34*

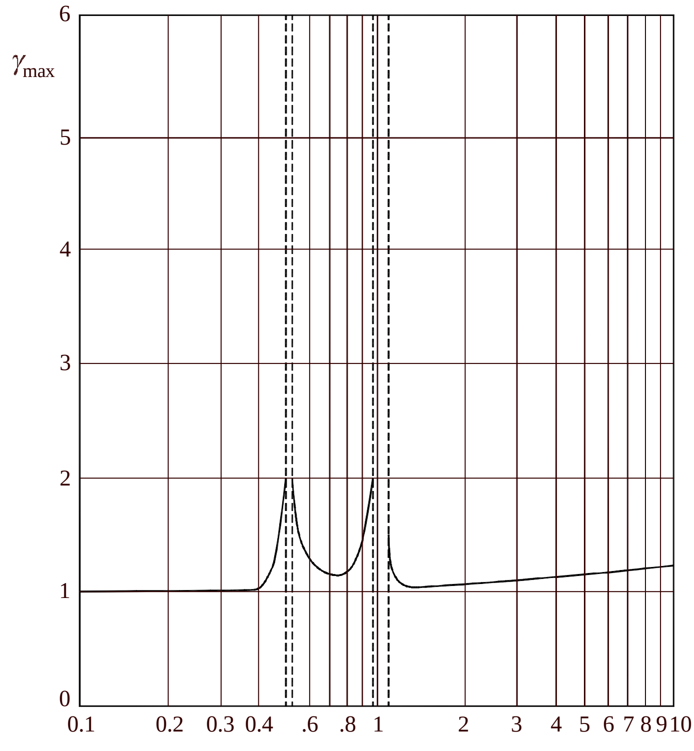

Fig. 7: *Maximum amplitude γ_{max} of the response versus r for ε = 0.236*

Goldsbrough's [2] experimental investigations for $\varepsilon = 0.34$ and $\varepsilon = 0.236$ showing the ranges of instability for $r \cong ½$ and $r \cong 1$, the corresponding theoretically determined values obtained from Figs. 6 and 7, and Gregory's [3] results are tabulated for comparison in Table1. Goldsbrough's experimental results and Gregory's analysis are found to be in close agreement with the theoretical investigations of the present paper. The unstable range which occurs at $r \cong 1$ becomes very large for heavy pistons with substantial inertia variations.

Table 1: *Comparison of experimental and theoretical results for instability ranges when $\varepsilon = 0.34$ and 0.236*

	UNSTABLE RANGE WHEN $\varepsilon = 0.34$					
	For $r \cong ½$			For $r \cong 1$		
I	Goldsbrough's [2] experimental results	Theoretical Values	Gregory's [3] analysis	Goldsbrough's [2] experimental results	Theoretical Values	Gregory's [3] analysis
	Maximum at $r = 0.525$	$r = 0.511$ to $r = 0.521$	$r = 0.512$ to r = 0.516	$r = 0.933$ to $r = 1.154$.	$r = 0.961$ to $r = 1.16$	$r = 0.966$ to r = 1.168
	UNSTABLE RANGE WHEN $\varepsilon = 0.236$					
	For r ≅ 1/2			For ≅ 1		
II	Goldsbrough's [2] Experimental results	Theoretical values	Gregory's [3] analysis	Goldsbrough's [2] Experimental results	Theoretical values	Gregory's [3] analysis
	Maximum at $r = 0.507$	$r = 0.502$ to $r = 0.508$	$r = 0.506$ to $r = 0.508$	$r = 0.964$ to $r = 1.107$	$r = 0.963$ to $r = 1.08$	$r = 0.967$ to $r = 1.09$

Table 2: *Harmonic analysis results when $\varepsilon = 0.34$ and $r = 1/10$*

Harmonic No.[a]	Resultant Magnitude	Harmonic No.[a]	Resultant Magnitude
1	7.57×10^{-3}	11	3.16×10^{-1}
2	2.46×10^{-2}	12	2.79×10^{-1}
3	2.57×10^{-2}	13	3.80×10^{-1}
4	2.76×10^{-2}	14	4.46×10^{-2}
5	5.94×10^{-2}	15	2.27×10^{-1}
6	2.34×10^{-2}	16	1.07×10^{-1}
7	6.24×10^{-2}	17	1.16×10^{-1}
8	3.36×10^{-1}	18	1.05×10^{-1}
9	7.44×10^{-1}	19	5.45×10^{-2}
10	8.03×10^{-1}	20	8.91×10^{-2}

[a]Order No. = 2 × Harmonic No.

In cases of failures which have occurred in practice, the working speeds of the engines have been such that $r < 0.2$ and hence the growth of amplitudes as shown in Fig. 5 near the region $r \cong ½$ should not have played a part in these failures. With this in view, harmonic analysis was carried out for the time responses of the system when $r = 1/12$ and $r = 1/10$ for $\varepsilon = 0.34$ as given in Fig. 2 and 3 respectively. The results of the analysis of the Figure for $r = 1/10$ are given in Table 2 for illustration. The analysis gives harmonic number and the resultant magnitude of each harmonic in radians. Since there are two beats in one revolution of the shaft at all speeds for $r < 0.2$, the order number is twice the harmonic number.

The harmonic analysis of the waveform in Fig. 3 for $r = 1/10$ as given in Table 2 suggests that it is composed of a component of order 10 and the secondary components of order 6, 8, 12, 14, 16, 18, etc. A similar analysis of the waveform solution of Fig. 2 for the speed of the engine for $r = 1/12$ and $\varepsilon = 0.34$, shows that the waveform contains a component of order 12 and secondary components of order 10, 14, 16, 18, 20 etc. Such an analysis gives the order of the principal component at a specific speed of the engine which can be excited to increased amplitudes by the exciting torques of the same orders as those of the secondary components.

4. CONCLUSIONS

The results presented in this paper give the analysis of the response of the variable inertia system representing a single-cylinder engine. The solutions thus determined, show the time response waveform shapes, maximum amplitudes and speeds of the engine at which the solutions are unstable and that the effects of variation in inertia are far more general than commonly suggested.

Due to the effect of the cyclic variation of engine inertia of the reciprocating parts, unstable conditions can occur over an appreciable range of engine r.p.m., in two regions at $r \cong ½$ and $r \cong 1$ without any externally applied excitation such as harmonic torque components arising from the cylinder gas pressure. For the same value of ε the unstable range which occurs at $r \cong ½$ is smaller compared with that at $r \cong 1$. For larger value of ε representing substantial increase in inertia variation, the unstable ranges become bigger in size. When the speeds of the engine are close to instability regions, the amplitudes become larger and grow indefinitely bigger within the regions. The regions of instability as determined in the analysis of this paper are in close agreement with Goldsbrough's corresponding experimental results and Gregory's analysis.

In cases of failures which have occurred in practice, the working speeds of engine have been such that $r < 0.2$. For the speeds of engine within this range, the waveform solutions show the occurrence of a modulation of both amplitude and frequency conforming to the form of beats. It is also shown that there are two beats in one revolution of the shaft.

The orders of the harmonic components of motion through which the energy can be transferred to the system from external excitations of the same orders can be determined from the harmonic analysis of the waveform solutions at the specific speed of rotation. This explains the possibility of an otherwise harmless principal component being magnified by the interaction with powerful secondary excitations. Thus dangerous vibrations may be evoked due to a secondary resonance phenomenon in marine diesel engines.

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NOTATION

γ_{max}	maximum amplitude of γ
a	crank radius
I	moment of inertia of rotating parts
M	mass of reciprocating parts
θ	angular displacement of crank from its datum position
θ_1	angular displacement of flywheel A
r	ratio of angular velocity ω of crankshaft to natural frequency ω_n of system, with variable inertia effects neglected
γ	displacement of torsional motion
ε	$\frac{1}{2}Ma^2/(I+\frac{1}{2}Ma^2)$
μ	torsional stiffness of crankshaft
ω	angular velocity of crankshaft
ω_n	natural frequency of system, with variable inertia effects neglected, $= \left[\mu/(I+\frac{1}{2}Ma^2)\right]^{1/2}$