

Extended radial point interpolation method for dynamic crack analysis in functionally graded materials

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ABSTRACT:

Functionally graded materials (FGMs) have been widely used as advanced materials characterized by variation in properties as the dimension varies. Studies on their physical responses under in-serve or external loading conditions are necessary. Especially, crack behavior analysis for these advanced material is one of the most essential in engineering. In this present, an extended meshfree radial point interpolation method (RPIM) is applied for calculating static and dynamic stress intensity factors (SIFs) in functionally graded materials. Typical

advantages of RPIM shape function are the satisfactions of the Kronecker's delta property and the high-order continuity. To assess the static and dynamic stress intensity factors, non-homogeneous form of interaction integral with the non-homogeneous asymptotic near crack tip fields is used. Several benchmark examples in 2D crack problem are performed such as static and dynamic crack parameters calculation. The obtained results are compared with other existing solutions to illustrate the correction of the presented approach.

Key words: FGMs, crack, stress intensity factors, meshless, RPIM

1. INTRODUCTION

Functionally graded materials (FGMs) are types of advanced composite that have been made based on the concept of continuous variation of microstructures. The non-uniform distributions of the reinforcement phase cause different material properties in one or more specified directions [1, 2]. In recent years, the FGMs hold promising for applications that require extra high material performance [3]. For example, FGMs are used in thermal protection systems because they evolve

the advantage of typical ceramics such as heat and corrosion resistance and typical of metal such as stiffness and mechanical strength. FGMs can be applied to generate thermal barrier coating for space applications, thermal-electric and piezoelectric devices, optical materials with graded reflective indices, bone and dental implants in medicine and so on. In many cases, FGMs structure are brittle and prone to cracking due to hard working conditions such as overload,

vibration, fatigue, and so on. For the reason that, crack behaviors of such FGMs has become an interesting study subject.

In this work, we focus on fracture behaviors of FGMs under static and dynamic loading. There are several analytical and also numerical studies that have been performed to obtain the fracture behavior of FGMs structures. Delale and Erdogan *et al* considered the stress field at crack tip in FGM which has the same square root singularity as that in the homogenous materials [4]. In 1987, Eischen *et al* present his mixed-mode crack analysis in non-homogenous materials using finite element method (FEM) [5]. Gu P. *et al* (1999) used domain J-integral to calculate the crack tip field of FGM [6]. In 2002, Kim and Paulino used FEM to calculate the mixed-mode SIFs in FGMs with some modifies for path-independent integral [7]. In 2005, Menouillard *et al* applied extended finite element method (XFEM) to calculate mixed-mode stress intensity factors for graded materials [8]. In the next year, Song *et al* applied FEM to compute the dynamic SIFs for heterogeneous materials [9]. In 2007, Kim and Paulino performed crack propagation problems in FGMs using XFEM [10]. Recently, in the last year, Chiong *et al* presented the scaled boundary FEM using polygon element for dynamic SIFs calculation for FGMs [11].

Over the ensuing decades, the so-called meshless or meshfree methods have developed. Different from FEM, meshfree methods do not require a mesh connect data points of the simulation domain. Since no finite mesh is required in the approximation, meshfree methods are very suitable for modeling crack growth problems [12, 13, 14, 15]. There are a few studies about meshless method for fracture problems in FGMs in recent years. Rao and Rahman (2003)

used EFG method for calculating SIFs in isotropic FGMs [16]. In 2006, Sladek *et al* applied meshless local Petrov-Galerkin method to evaluate fracture parameters for crack problems in FGM [17]. In 2009, Koohkan *et al* presented a new technique with J-integral to calculate the SIF values for FGM crack problems [18].

In this study, we propose an extended meshfree method based on the radial point interpolation method (XRPIM) associated with the vector level set method for modeling the crack problem in functionally graded materials under static and dynamic loading conditions. To calculate the SIFs, the dynamic form of interaction integral formulation for nonhomogeneous materials is used. Several numerical examples including static and dynamic SIFs calculation are performed and investigated to highlight the accuracy of the proposed method.

2. XRPIM FORMULATION FOR CRACK PROBLEMS

2.1. Weak-form formulation

Consider a 2D solid with domain Ω and bounded by Γ , the initial crack face is denoted by boundary Γ_c , the body is subjected to a body force $\mathbf{b}$ and traction $\bar{\mathbf{t}}$ on Γ_t as depicted in Fig. 1. The weak-form obtained for this elasto-dynamic problem can be written as

$$\int_{\Omega} \delta \mathbf{u}^T \ddot{\mathbf{u}} \rho d\Omega + \int_{\Omega} \delta \boldsymbol{\varepsilon}^T \boldsymbol{\sigma} d\Omega - \int_{\Omega} \delta \mathbf{u}^T \mathbf{b} d\Omega - \int_{\Gamma_t} \delta \mathbf{u}^T \bar{\mathbf{t}} d\Gamma = 0 \quad (1)$$

where $\mathbf{u}$, $\ddot{\mathbf{u}}$ are the vectors of displacements and acceleration, $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are stress and strain tensors, respectively. These unknowns are functions of location and time: $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$, $\ddot{\mathbf{u}} = \ddot{\mathbf{u}}(\mathbf{x}, t)$, $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\mathbf{x}, t)$ and $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(\mathbf{x}, t)$.

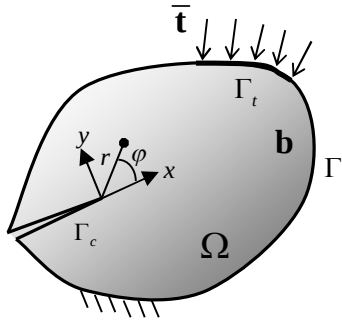


Figure 1. A FGM crack model

2.2. Meshless X-RPIM discretization and vector level set method

Base on the extrinsic enrichment technique, the displacement approximation is rewritten in terms of the signed distance function f and the distance from the crack tip as follow:

$$u^h(\mathbf{x}, t) = \sum_{I \in W(\mathbf{x})} \phi_I(\mathbf{x}) u_I + \sum_{I \in W_b(\mathbf{x})} \phi_I(\mathbf{x}) \alpha_I H(f(\mathbf{x})) + \sum_{I \in W_s(\mathbf{x})} \phi_I(\mathbf{x}) \sum_{j=1}^4 B_j(\mathbf{x}) \beta_{Ij} \quad (2)$$

where ϕ_I is the RPIM shape functions [19] and $f(\mathbf{x})$ is the signed distance from the crack line. The jump enrichment functions $H(f(\mathbf{x}))$ and the vector of branch enrichment functions $B_j(\mathbf{x})$ ($j = 1, 2, 3, 4$) are defined respectively by

$$H(f(\mathbf{x})) = \begin{cases} +1 & \text{if } f(\mathbf{x}) > 0 \\ -1 & \text{if } f(\mathbf{x}) < 0 \end{cases} \quad (3)$$

$$B(\mathbf{x}) = \left(\sqrt{r} \sin \frac{\varphi}{2}, \sqrt{r} \cos \frac{\varphi}{2}, \sqrt{r} \sin \frac{\varphi}{2} \sin \varphi, \sqrt{r} \cos \frac{\varphi}{2} \sin \varphi \right) \quad (4)$$

where r is the distance from $\mathbf{x}$ to the crack tip $\mathbf{x}_{TP}$ and φ is the angle between the tangent to the crack line and the segment $\mathbf{x} - \mathbf{x}_{TP}$ as shown in Fig. 2. W_b denotes the set of nodes whose support contains the point $\mathbf{x}$ and is bisected by the crack line and W_s is the set of nodes whose support contains the point $\mathbf{x}$ and is slit by the crack line and contains the crack tip. α_I, β_{Ij} are

additional variables in the variational formulation.

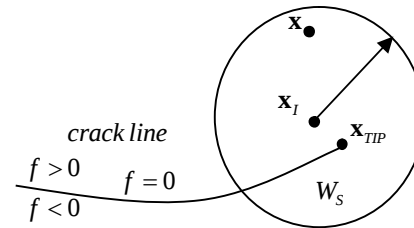
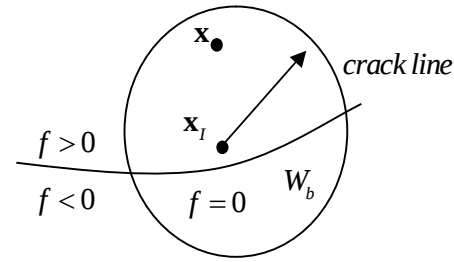


Figure 2. Sets of enriched nodes

2.3. Discrete equations

Substituting the approximation (2) into the well-known weak form for solid problem (1), using the meshless procedure, a linear system of equation can be written as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F} \quad (5)$$

with $\mathbf{M}, \mathbf{K}$ being the mass and stiffness matrices, respectively, and $\mathbf{F}$ being the vector of force, they can be defined by

$$\mathbf{M}_{IJ} = \rho \int_{\Omega} \Phi_I^T \Phi_J d\Omega \quad (6)$$

$$\mathbf{K}_{IJ} = \int_{\Omega} \mathbf{B}_I^T \mathbf{D} \mathbf{B}_J d\Omega \quad (7)$$

$$\mathbf{F}_I = \int_{\Omega} \Phi_I^T \mathbf{b}_I d\Omega + \int_{\Gamma_t} \Phi_I^T \bar{\mathbf{t}}_I d\Gamma \quad (8)$$

where Φ is the vector of enriched RPIM shape functions; the displacement gradient matrix $\mathbf{B}$ must be calculated appropriately dependent upon enriched or non-enriched nodes.

3. J-INTEGRAL FOR DYNAMIC SIFS IMPLEMENTATION

The dynamic stress intensity factors are important parameters, and they are used to calculate the positive maximum hoop stress to evaluate dynamic crack propagation properties. The dynamic form of J -integral for nonhomogeneous materials is written as [9]

$$J = \int_{V_f} (\sigma_{ij} u_{i,1} - W \delta_{1j}) q_{,j} dA + \int_{V_f} (\rho \ddot{u}_i u_{i,1} - \frac{1}{2} C_{ijkl} \varepsilon_{ij} \varepsilon_{kl}) q_{,j} dA \quad (9)$$

where $W = \frac{1}{2} \sigma_{ij} \varepsilon_{ij}$ is strain energy density; q is a weight function, changing from $q = 1$ near a crack-tip and $q = 0$ at the exterior boundary of the J domain.

In this paper, the interaction integral technique is applied to extract SIFs. After some mathematical transformations, the path independent integration can be written as

$$M = \int_A (\sigma_{ij} u_{i,1}^{aux} + \sigma_{ij}^{aux} u_{i,1} - \sigma_{ij}^{aux} \varepsilon_{ij} \delta_{1j}) q_{,j} dA + \int_A (\sigma_{ij,j}^{aux} u_{i,1} + \rho \ddot{u}_i u_{i,1}^{aux} - C_{ijkl} \varepsilon_{ij}^{aux} \varepsilon_{kl}) q_{,j} dA \quad (10)$$

The stress intensity factors can then be evaluated by solving a system of linear algebraic equations:

$$K_I = M^{(mod\ell)} E_{tip}^* / 2 \quad (11)$$

$$K_{II} = M^{(mod\ell)} E_{tip}^* / 2 \quad (12)$$

where $E_{tip}^* = E_{tip} / (1 - \nu_{tip}^2)$ for plain strain state

4. NUMERICAL EXAMPLES

4.1. Single mode in infinite edge crack FGM plate

In the first example, we consider a rectangular FGM plate with an edge crack. The plate is subjected to a far field tensile stress as shown in Fig. 3. To imply the infinity boundary, the dimensions are set as $H/W = 10$. Various values of crack length and ratio of E_2/E_1 are

chosen to investigate the static mode I SIF of the model.

The elastic modulus is assumed to follow an exponential function as in (13) and the Poisson's ratio is held constant at $\nu = 0.3$

$$E(x_1) = E_1 e^{\gamma x_1}, \quad 0 \leq x_1 \leq W \quad (13)$$

where $E_1 = E(0)$, $E_2 = E(W)$ and $\gamma = (1/W) \log(E_2/E_1)$

A model with 16×160 regular distributed nodes is used for calculation. The obtained results are compared with available analytical solution given by Erdogan and Wu [20] and XFEM solution given by Dolbow and Gosz [21].

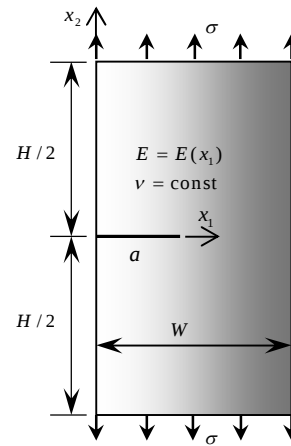


Figure 3. Infinite edge crack FGM plate

There are two crack length ratios are investigated ($a/W = 0.2, 0.4$).

Table 1 and Table 2 summarize the acceptable results obtained by XRPIM in the comparison with other numerical solutions.

Table 1. Normalized SIFs for plate with edge crack ($a/W = 0.2$)

E_2/E_1	XRPIM (proposed)	Analytical [20]	XFEM [21]
0.1	1.286	1.2965	1.279
0.2	1.378	1.396	1.381
1.0	1.331	1.373	1.363
5.0	1.080	1.132	1.133
10.0	0.948	1.024	1.004

Table 2. Normalized SIFs for plate with edge crack ($a/W = 0.4$)

E_2 / E_1	XRPIM (proposed)	Analytical [20]	XFEM [21]
0.1	2.564	2.570	2.552
0.2	2.428	2.443	2.438
1.0	2.068	2.107	2.116
5.0	1.679	1.748	1.752
10.0	1.512	1.626	1.590

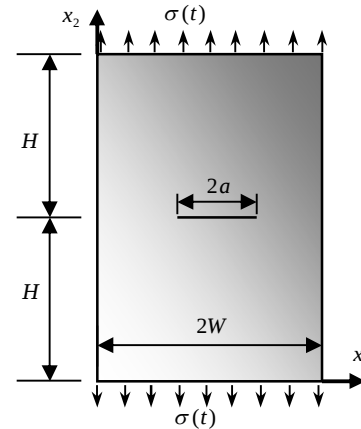
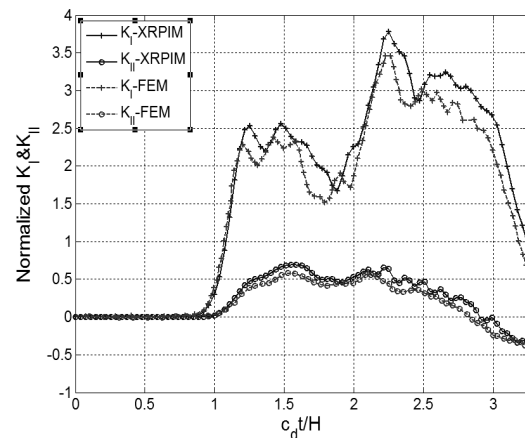
4.2. Center crack FGM plate under dynamic tensile loading

In the next example, a FGM plate with a central crack is considered as shown in Fig. 4. The dimensions are given as $2H = 40\text{ mm}$; $2W = 20\text{ mm}$ and $2a = 4.8\text{ mm}$. The plate is subjected to a step tensile load at the top and the bottom edges. The Poisson's ratio taken is 0.3, the Young's modulus and density are assumed to vary through the exponential functions of both x_1 and x_2 coordinates as follows:

$$E = E_0 e^{(\beta_1 x_1 + \beta_2 x_2)}, \quad \rho = \rho_0 e^{(\beta_1 x_1 + \beta_2 x_2)} \quad (14)$$

Where $E_0 = 199.992\text{ GPa}$, $\rho_0 = 5000\text{ kg/m}^3$, $\beta_1 = \beta_2 = 0.1$

There are 30×60 scattered nodes are used for the problem. A time step $\Delta t = 0.1\mu\text{s}$ is used for Newmark integration calculation. Fig. 5 shows the normalized dynamic SIFs ($K_{I,II} / (\sigma\sqrt{\pi a})$) at the right crack tip versus normalized time ($t c_d / H$) where $c_d = 7.34\text{ mm}/\mu\text{s}$ is the dilatational wave velocity. The XRPIM results are compared with the FEM results given by Seong *et al* [9] and the charts show a good agreement. It can be seen in the results that after the time of H/c_d , the both SIFs start to increase. The amplitude of the mode-I SIF is much larger than that of the mode-II SIF.

**Figure 4.** Center crack FGM plate with material distribution in x_1, x_2 - directions**Figure 5.** Normalized dynamic SIFs results

4.3. Center crack FGM plate under dynamic tensile loading

The last example deals with a center crack FGM plate that has the same geometry and load condition with the one in 5.2. section. However, in this problem, as shown in Fig. 6, the material distribution is different from the previous case in which $\beta_1 = 0$ and three values of β_2 are considered ($\beta_2 = 0, 0.05, 0.1$).

Because of the symmetry of geometry, load and material, a half model is considered with the symmetry boundary condition at $x_1 = W$. A distribution of 10×40 nodes is used for the

XRPIM model. The plots in Fig. 7 and Fig. 8 show the XRPIM solutions with several cases of β_2 values. In the comparison with the report of Seong *et al* [9], the XRPIM dynamic SIFs results are acceptable. It can be seen that the values of mode-I SIF are much larger than mode-II. The material value $\beta_2 = 0.1$ gives maximum stress intensity factors in both modes. In the case of $\beta_2 = 0$ (homogenous), the model is single mode so mode-II SIF is equal to zero during the time.

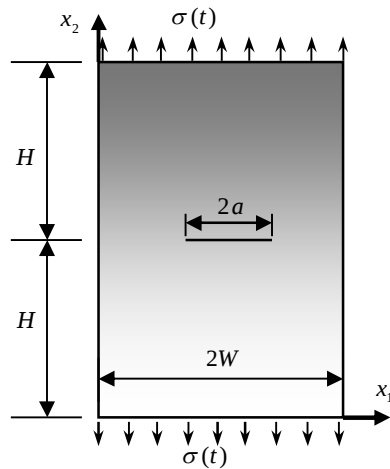


Figure 6. Center crack FGM plate with material distribution in x_2 - directions

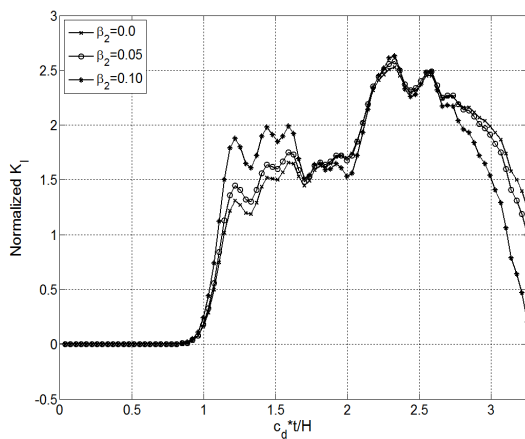


Figure 7. Normalized dynamic SIFs results for mode-I

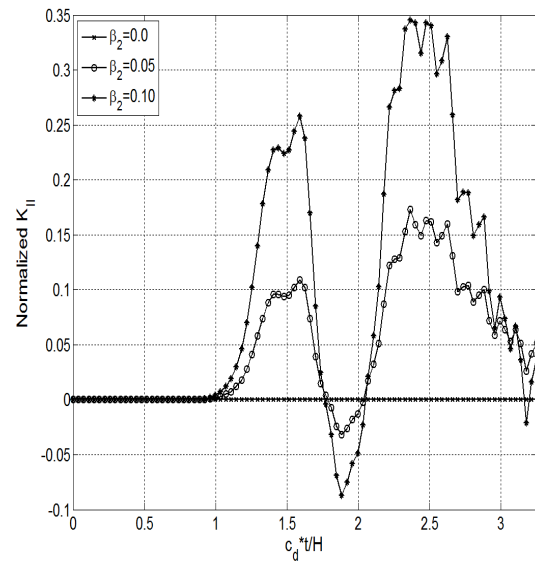


Figure 8. Normalized dynamic SIFs results for mode-II

5. CONCLUSION

An extended radial point interpolation method (XRPIM) has been proposed for static and dynamic cracks analysis in functionally graded models. This method is convenient in treating the Dirichlet boundary conditions because of the RPIM shape functions satisfying the Kronecker's delta property. Three numerical examples are investigated with different material models and crack modes. The obtained solutions show a good agreement of between the presented method and the references. The presented approach has shown several advantages and it is promising to be extended to more complicated problems such as dynamic crack propagation problems for functionally graded materials.

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Phương pháp không lưới RPIM mở rộng cho bài toán nứt động trong vật liệu phân lớp chức năng

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TÓM TẮT:

Vật liệu phân lớp chức năng (FGM) ngày nay được sử dụng rộng rãi trong những kết cấu đòi hỏi tính năng ứng xử phức tạp của vật liệu cấu tạo. Điều này có được từ đặc trưng tính chất vật liệu thay đổi theo vị trí của vật liệu FGM. Việc nghiên cứu đáp ứng vật lý của vật liệu FGM ứng với các điều kiện làm việc, tải trọng là rất cần thiết. Đặc biệt, việc phân tích ứng xử nứt cho những vật liệu này là vô cùng quan trọng trong kỹ thuật. Trong báo cáo này, phương pháp không lưới mở rộng sử dụng phép nội suy điểm hướng kính (XRPIM) được áp dụng để tính các hệ số cường độ ứng suất tại đỉnh vết nứt với tải tĩnh và động trong vật

liệu phân lớp chức năng. Hàm dạng RPIM có các ưu điểm như thỏa mãn thuộc tính Kronecker's delta và liên tục bậc cao. Để tính toán các hệ số cường độ ứng suất tĩnh và động trong vật liệu FGM, tác giả sử dụng dạng không thuần nhất của tích phân tương tác với trường phụ trợ ở lân cận đỉnh vết nứt cho vật liệu không thuần nhất. Một số ví dụ kiểm chứng cho bài toán nứt tĩnh và động trong không gian hai chiều được thực hiện và so sánh với các kết quả tham khảo từ các công bố trước đây. Sự phù hợp giữa các kết quả cho thấy sự đúng đắn của phương pháp được giới thiệu.

Từ khóa: vật liệu FGM, hệ số cường độ ứng suất, phương pháp không lưới RPIM

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